



Analysis of Various Wavelets Transform Techniques

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Abstract: The paper present an overview of different types wavelet transform techniques. Wavelet analysis is used for data encoding, channel, and source encoding, signal denoising etc. As wavelet analysis is done using both time and frequency domain, the signal analysis becomes easy and more accurate. Wavelet uses shifting and scaling property of the signal for analysis of the signal. It is beneficial in the case where signal frequency changes over time. Various types of wavelet using Haar, Daubechies, Symlet and Coiflet has been studied for their performance.

Keywords: Wavelet transform techniques, Haar, Daubechies, Symlet and Coiflet wavelet, Multi Input Multi Output (MIMO).

1. INTRODUCTION

Wireless communication is a long range communication, that are impossible to implement with the use of wires. The term is commonly used i.e telecommunication system which use some form of energy to transfer information without wires. Information is transfered for both short and long distance. The wireless communication channel suffers from much impairment which leads degradation of the overall system performance. There are many performance degradation factor in wireless communication channel but fading problem is the

major impairment problem. Diversity technique is used to decreased the fading effect and improve system performance in fading channel. Among the various diversity techniques spatial diversity is best suitable for the wireless communication Multi Input Multi Output (MIMO) wireless communication uses spatial diversity techniques. In wireless communication, MIMO uses multiple antenna at the transmitter and the receiver. The two main forms of MIMO are diversity coding and spatial multiplexing. In diversity coding, multiple version of the same data are encoded and transmitted over multiple antennas. In spatial multiplexing, Instead of transmitting multiple version of the same signal, the data to be sent is split into multiple streams and transmitted using multiple antennas. Full duplex communication is the most efficient way to utilize the bandwidth in wireless communication system. In full duplex communication, signals are transmitted and received at same time. Wavelets have been applied in all aspects of wireless communication system including data compression, source and channel coding, signal denoising etc[4]. Wavelet transform is very much usefull for examining the signal in time and as well as in frequency domain. It is used because it owns better orthogonality[3].

Wavelet analysis is done to obtain information about the time domain and frequency domain

component of signal. As it provides time localization and frequency localization of the information, it is commonly used in signal processing application. It is effectively used for representation better orthogonality. It is used because it owns by the process of decomposition and reconstruction of the signal. Wavelet transform break the input signal into small functions called wavelets. Wavelets are generated by scaling and shifting of the main basic signal. These wavelets are represented in time-frequency tile for efficient signal analysis.

The DWT has both low pass filter (lpf) and high pass filter (hpf) and it acts as a quadrature mirror filters. The lpf and hpf coefficient are called as approximate and detailed coefficients[2].

2. WAVELET TRANSFORM TECHNIQUES

Wavelet transforms are mathematical means for performing signal analysis when signal frequency varies over time. For certain signals and images, wavelet analysis provides more precise information about signal data than other signal analysis techniques. Wavelet transform analysis represents image as a sum of wavelet functions with different locations and scales. Wavelet transforms exhibit high decorrelation and energy compaction properties. These properties made wavelets suitable for image compression.

2.1. Haar Wavelets

Haar is the simplest and very fast wavelet transform. Haar matrix is sequentially ordered. In mathematics, the Haar wavelet is a sequence of rescaled —square-shaped functions. The wavelet theorems are most popular methods of image processing, de-noising and compression[1]. Considering that the Haar functions are the simplest wavelets, these forms are used in many methods of discrete image transforms and

processing. Technical disadvantage of Haar wavelet is that it is not continuous and therefore not differentiable. Haar Transform has poor energy compaction for images[5].

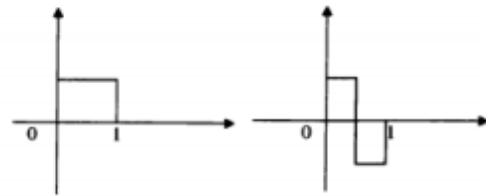


Figure 1. Scaling and wavelet functions of Haar [1].

It is the most basic wavelet form. It is a group of square waves with magnitude of ± 1 in the interval $[0,1]$. Haar function can be defined as

$$\varphi_0(t) = \begin{cases} 1 & \text{for } 0 \leq t < 1 \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

$$\varphi_1(t) = \begin{cases} 1 & \text{for } 0 \leq t < \frac{1}{2} \\ -1 & \text{for } \frac{1}{2} \leq t < 1 \\ 0 & \text{otherwise} \end{cases} \quad (2)$$

All the other subsequent function are generated from $\varphi_1(t)$ which means where $i = 2j + k$, $j \geq 0$ & $0 \leq k < 2^j$. Haar wavelets are orthogonal to each other. The main drawback of Haar wavelet is that it is not continuous and therefore not differentiable. The matrix associated with Haar wavelet is given by

$$H_2 = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \quad (3)$$

If we have a sequence of length a multiple of 4, we can build block of 4 elements and transform them in a 4x4 Haar matrix.

$$H_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix} \quad (4)$$

It (H4) combines two stages of a fast Haar wavelet transform. Haar transform is derived

from the above Haar matrix. A 4x4 Haar transformation matrix can be given as

$$H_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ \sqrt{2} & -\sqrt{2} & 0 & 0 \\ 0 & 0 & \sqrt{2} & -\sqrt{2} \end{bmatrix} \quad (5)$$

Haar transform is a form of a sampling process in which rows of the transformation matrix act as samples of finer resolution. In Haar transform, we do not require multiplication as it requires only addition of elements with mostly zero elements. It is having equal length of input and output signal.

2.2. Daubechies Wavelets

A major problem in the development of wavelets was the search for scaling functions that are compactly supported, orthogonal and continuous. Ingrid Daubechies were constructed these scaling functions and finding the low pass filter h . For high order Daubechies wavelets DbN , N is the number of vanishing moments and denotes the order of wavelet. Daubechies wavelets have highest number of N for given support width $N=2A[1]$. Daubechies wavelets are continuous but they are more computational and expensive to use. They have balanced frequency responses but non linear phase resembles a step function. responses. Db wavelets use overlapping windows, so the high frequency coefficient spectrum reflects all high frequency changes. There are four wavelet and scaling function coefficients for Daubechies transform. The scaling function coefficient are

$$h_0 = \frac{1+\sqrt{3}}{4\sqrt{2}} \quad (6)$$

$$h_1 = \frac{3+\sqrt{3}}{4\sqrt{2}} \quad (7)$$

$$h_2 = \frac{3-\sqrt{3}}{4\sqrt{2}} \quad (8)$$

$$h_3 = \frac{1-\sqrt{3}}{4\sqrt{2}} \quad (9)$$

Scaling function is applied to the data input with each step of wavelet transform. If we have N values in original data set the scaling function will be applied in each step of wavelet transform to calculate $N/2$ smoothed values. The smoothed values are stored in the lower half of N element input vector. Wavelet function is applied to the input data in each step of wavelet transform. The wavelet function coefficient values are

$$g_0 = h_3 \quad (10)$$

$$g_1 = -h_2 \quad (11)$$

$$g_2 = -h_1 \quad (12)$$

$$g_3 = -h_0 \quad (13)$$

For a original data set having N values, the wavelet function will be applied to obtain $N/2$ differences which will reflect the change in data.

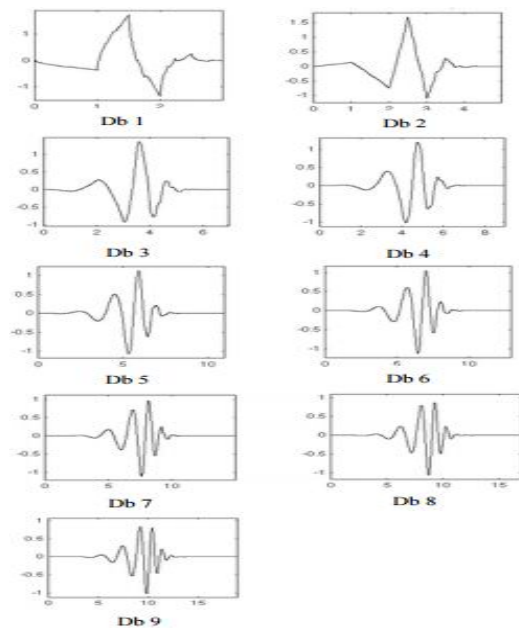


Figure 2. Wavelet Functions of Daubechies [1].

Figure 2 shows the nine member of the wavelet function. In which x-axis represent the time and y-axis represent the frequency. Daubechies wavelet transforms are same as that of the Haar wavelet and defined in the same way. For high order Daubechies wavelets DbN, N is the number of vanishing moments and denotes the order of wavelet. The wavelet values are stored in the upper half of the N element input vector. The scaling and wavelet function are calculated by taking inner product of coefficient of four data values. Daubechies wavelets have highest number of N for given support width $N=2A$. Wavelets give less smoothing with fewer vanishing moment and produces distortion with more vanishing moments. Daubechies wavelets are used to solve the problems like self-similarity Properties of a signal discontinuities.

D4 scaling function:

$$a[i] = h_0 s[2i] + h_1 s[2i + 1] + h_2 s[2i + 2] \quad (14)$$

D4 wavelet transform:

$$c[i] = g_0 s[2i] + g_1 s[2i + 1] + g_2 s[2i + 2] \quad (15)$$

If the signal f is approximately linear over the support of a D4 wavelet, then the corresponding fluctuating value is approx zero.

2.3. Symlet Wavelets

Daubechies wavelets select the minimum phase square root because the Wavelet functions of Daubechies wavelets are far from symmetry. Symlets are approximately symmetrical wavelets Proposed by Daubechies as modifications to the Db family, apart from the symmetry, other properties of Daubechies and Symlet families are similar.

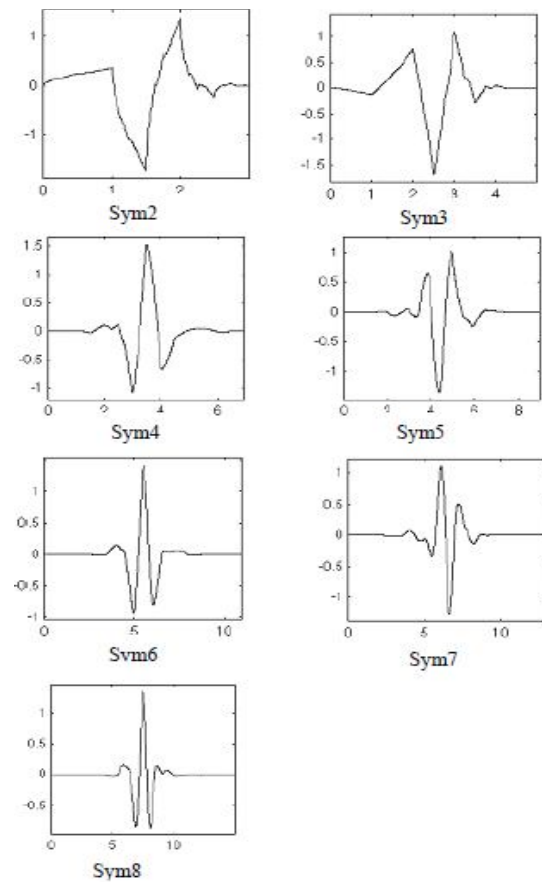


Figure 3. Wavelet functions of Symlets [1].

Wavelet function of Symlets shows the eight member in the Figure 3. When applied to the signal for better performance, the Signal to Noise Ratio (SNR) of de-noised signal is improved[1].

2.4. Coiflet Wavelets

Coiflets and Daubechies wavelets are similar for certain level, but the Coiflet was constructed with vanishing moments for scaling function $\phi(x)$, but not only for wavelet function $\psi(x)$. Coiflets wavelet functions have $N/3$ vanishing moments and $(N/3)-1$ scaling functions. Coiflet is less flexible in visualizing any frequency and its discrete form is useful for digital implementation. The scaling function of coiflets allows a good approximation of polynomial function at different resolutions.

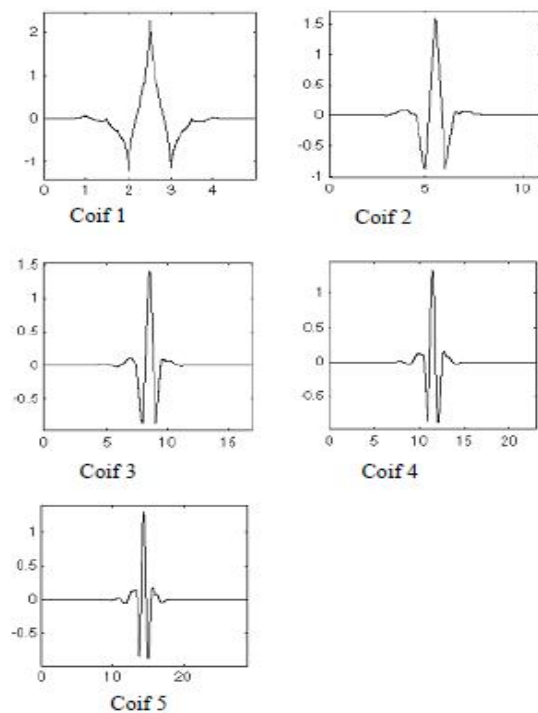


Figure 4. Wavelet functions of Coiflets [1].

Coiflet wavelets have oscillatory structure and non-zero for limited interval of time[7]. Due to the linear phase of the transfer function, the property of symmetry in coiflets is desirable in signal analysis work. Coiflet transform can present both frequency and time information in an integral scheme. Visualization of signal characteristics can be easily acquired[1].

3. CONCLUSION

In this paper we have studied various types of wavelet transform techniques. Wavelet transformation decomposition and reconstruction of the signal and the wavelets can be represented in time and frequency domain for better analysis of the signal. The various types of wavelets studied are Haar, Daubechies, Symlet and Coiflet wavelets. Haar is the simplest to impliment and uses simple mathematical computations. Daubechies wavelets are usually used to solve problems like self similarity of the signal. Symlet wavelets are used to improve the SNR of

denoised signal and gives the better performance. And the Coiflet wavelets are the best having symmetry property and gives a linear phase of the transfer function.

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