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Three Dimensional Free Convective Heat and Mass Transfer Flow past a Porous Plate with Sinusoidal Plate Temperature and Plate Concentration

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Abstract: The problem of three-dimensional free convective heat and mass transfer flow of an incompressible viscous fluid past an infinite vertical porous plate through porous medium with uniform free steam velocity and sinusoidal plate temperature and concentration are discussed. The porous medium is bounded by a vertical plane surface. The surface absorbs the fluid with a periodic transverse suction velocity. The governing equations are solved by regular perturbation technique. The expressions for velocity field, temperature field, concentration field, skin friction at the plate in the direction of the main flow, the rate of heat transfer in terms of Nusselt number, the rate of mass transfer in terms of Sherwood number, first order skin friction, Nusselt number and Sherwood number are obtained and are demonstrated in graphs for different values of parameters involved.

Key words: Incompressible Viscous Fluid, Three- Dimensional Free Convective Flow, Wall Shear Stress

INTRODUCTION

A number of scholars have devoted their extensive research on the study of free and force convective three-dimensional flow with heat and mass transfer due to its day-to-day applications in science and technology. The phenomenon of heat and mass transfer are observed in buoyancy induced motions in the atmosphere, in water bodies, quasi solid-bodies such as earth and so on. The free convective heat transfer flows play an important rule in chemical engineering, turbo-machinery and aerospace technology. In industrial applications many transport exists where the transfer of heat and mass takes place simultaneously as a result of combined buoyancy effects due to thermal diffusion and chemical species diffusion. The study of such flows was initiated by Lighthill [1] who studied the effects of free steam oscillations on the flow of a viscous incompressible fluid past an infinite plate. Stuart [2] further extended it to study a two-dimensional oscillatory flow past an infinite, porous plate with constant suction. Soundalgekar [3] studied the flow past an infinite vertical plate oscillating in its own plane and with the wall temperature. Also Messiha [4] investigated the two dimensional oscillatory flow when the plate is subjected to a time-dependent suction. The effects of different arrangements and configurations of the suction holes and slits have been studied extensively by various scholars and have been complied by Lachmann [5]. Oscillatory three-dimensional flow past an infinite vertical porous plate with thermal diffusion and chemical reaction in presence of heat sink has been presented by Ahmed et al. [6]. The effect of transverse sinusoidal suction on the steady flow along a plane wall has been presented by Gersten and Gross [7]. The flow in the boundary layer becomes three-dimensional by considering this type of suction. Singh et al. has investigated the boundary layer flow and heat transfer on a horizontal plane whose temperature differs from that ambient fluid.

Rapits [8] investigated the problem of unsteady flow through a porous medium bounded by an infinite porous plate sbjected to a constant suction and variable temperature. Further Rapids and Perdikis [9] studied the unsteady two-dimensional free convective flows through highly porous medium. The problem of three-dimensional fluctuating flow and heat transfer through a porous medium with variable permeability was represented by Singh et al. [10]. Ahmed et al. [11] studied the three-dimensional free convective flow and heat transfer through a porous medium.

Further Singh and Thakar [12] have analyzed the effects of periodic suction velocity on three-dimensional viscous fluid with heat and mass transfer. Also Guria and Jana [13] studied the effect of buoyancy forces and time dependent periodic suction on three-dimensional flow past a vertical porous plate. Jain and Sharma [14] and Jain and Gupta [15] have studied three-dimensional coutte flow past with slip boundary conditions and suction velocity vary sinusoidaly. Aboeldahab and Azzam [15] have studied unsteady three-dimensional combined heat and mass transfer for convective flow over a stretching surface.

The aim of the present work is to investigate the free convection, the heat and mass transfer effects on the steady three dimensional flow of a viscous incompressible fluid past a vertical porous plate with periodic suction along the breadth, in the presence of a heat sink. We assume that the free stream velocity is uniform. We further assume that the temperature and species concentration at the plate are sinusoidal along the breadth.

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BASIC EQUATIONS

A coordinate system is introduced with the plate lying vertically along the XZ plane, such that the X-axis is oriented along the length of the plate in the direction of the buoyancy force and the Y-axis is perpendicular to the plane of the plate and directed into the fluid region. The plate is assumed to have a variable suction velocity distribution (along the width of the plate) of the form:

$$\bar{v}_w(\bar{z}) = -V_0\left[1+\varepsilon\cos\frac{\pi\bar{z}}{L}\right] \tag{1}$$

where $V_0>0$ and $0<\varepsilon<<1$. Here V_0 is the undisturbed part of the suction velocity, L is the wavelength of the periodic suction and the negative sign indicates that the suction is towards the plate. All the fluid properties are assumed to be independent of x, except possibly the the pressure. However, the flow remains three dimensional due to the form of the suction velocity distribution given above. Let $\vec{q} = u\hat{i}+v\hat{j}+w\hat{k}$ be the fluid velocity at the point (x,y,z) where $\hat{i},\hat{j},\hat{k}$ are the unit vectors along X-axis, Y-axis and Z-axis respectively. The free stream velocity U is assumed to be uniform.

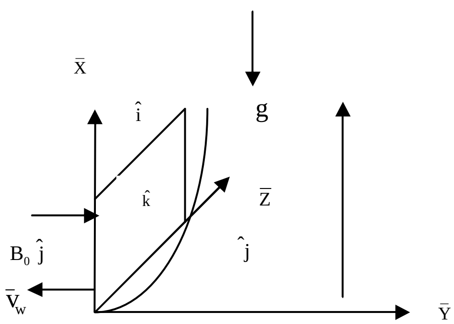

Fig-1. Flow configuration

The equations governing the fluid motion are

Equation of continuity

$$\frac{\partial\bar{v}}{\partial\bar{y}}+\frac{\partial\bar{w}}{\partial\bar{z}}=0 \tag{2}$$

Momentum equations

$$\bar{v}\frac{\partial\bar{u}}{\partial\bar{y}}+\bar{w}\frac{\partial\bar{u}}{\partial\bar{z}} = g\beta(\bar{T}-\bar{T}_\infty)+g\bar{\beta}(\bar{C}-\bar{C}_\infty)+ \nu\left(\frac{\partial^2\bar{u}}{\partial\bar{y}^2}+\frac{\partial^2\bar{u}}{\partial\bar{z}^2}\right)+\frac{\nu}{K^*}(\bar{U}-u) \tag{3}$$

$$\bar{v}\frac{\partial\bar{v}}{\partial\bar{y}}+\bar{w}\frac{\partial\bar{v}}{\partial\bar{z}} = -\frac{1}{\rho}\frac{\partial\bar{p}}{\partial\bar{y}}+\nu\left(\frac{\partial^2\bar{u}}{\partial\bar{y}^2}+\frac{\partial^2\bar{u}}{\partial\bar{z}^2}\right)-\frac{\nu}{K^*}\bar{v} \tag{4}$$

$$\bar{v}\frac{\partial\bar{w}}{\partial\bar{y}}+\bar{w}\frac{\partial\bar{w}}{\partial\bar{z}} = -\frac{1}{\rho}\frac{\partial\bar{p}}{\partial\bar{z}}+\nu\left(\frac{\partial^2\bar{w}}{\partial\bar{y}^2}+\frac{\partial^2\bar{w}}{\partial\bar{z}^2}\right)-\frac{\nu}{K^*}\bar{w} \tag{5}$$

Energy equation

$$\bar{v}\frac{\partial\bar{T}}{\partial\bar{y}}+\bar{w}\frac{\partial\bar{T}}{\partial\bar{z}} = \alpha\left(\frac{\partial^2\bar{T}}{\partial\bar{y}^2}+\frac{\partial^2\bar{T}}{\partial\bar{z}^2}\right)+\frac{Q_0(\bar{T}_\infty-\bar{T})}{\rho C_p} \tag{6}$$

Species continuity equation:

$$\bar{v}\frac{\partial\bar{C}}{\partial\bar{y}}+\bar{w}\frac{\partial\bar{C}}{\partial\bar{z}} = D_M\left(\frac{\partial^2\bar{C}}{\partial\bar{y}^2}+\frac{\partial^2\bar{C}}{\partial\bar{z}^2}\right)+ D_T\left(\frac{\partial^2\bar{T}}{\partial\bar{y}^2}+\frac{\partial^2\bar{T}}{\partial\bar{z}^2}\right) \tag{7}$$

The symbols are defined in the nomenclature.

The relevant boundary conditions are:

$$\text{At } \bar{y}=0:\quad \bar{u}=0,\ \bar{v}=\bar{v}_w,\ \bar{w}=0,\ \&\ \bar{T}=\bar{T}_\infty+(\bar{T}_w-\bar{T}_\infty)\left(1+\varepsilon A\cos\frac{\pi\bar{z}}{L}\right),\ \bar{C}=\bar{C}_\infty+(\bar{C}_w-\bar{C}_\infty)\left(1+\varepsilon A\cos\frac{\pi\bar{z}}{L}\right) \tag{8}$$

$$\text{At } \bar{y}\to\infty:\ \bar{u}=\bar{U},\ \bar{v}=-V_0,\ \bar{w}=0,\ \bar{T}=\bar{T}_\infty\ \&\ ,\ \bar{C}=\bar{C}_\infty,\ \bar{p}=\bar{p}_\infty \tag{9}$$

We introduce the following non dimensional quantities:

$$y=\frac{\bar{y}}{L},\ z=\frac{\bar{z}}{L},\ u=\frac{\bar{u}}{V_0},\ v=\frac{\bar{v}}{V_0},\ U=\frac{\bar{U}}{V_0},\ w=\frac{\bar{w}}{V_0},$$

$$\theta=\frac{\bar{T}-\bar{T}_\infty}{\bar{T}_w-\bar{T}_\infty},\ Q=\frac{Q_0L}{\rho C_pV_0},\ \phi=\frac{\bar{C}-\bar{C}_\infty}{\bar{C}_w-\bar{C}_\infty},\ Pr=\frac{\nu}{\alpha},$$

$$Sc=\frac{\nu}{D_M},\ Sr=\frac{D_T(\bar{T}_w-\bar{T}_\infty)}{\nu(\bar{C}_w-\bar{C}_\infty)},\ Gr=\frac{Lg\beta(\bar{T}_w-\bar{T}_\infty)}{V_0^2}, \tag{10}$$

$$Gm=\frac{Lg\bar{\beta}(\bar{C}_w-\bar{C}_\infty)}{V_0^2},\ Re=\frac{V_0L}{\nu},\ p=\frac{\bar{p}}{\rho\left(\frac{\nu}{L}\right)^2},$$

$$p_\infty=\frac{\bar{p}_\infty}{\rho\left(\frac{\nu}{L}\right)^2},\ K=\frac{V_0K^*}{\nu L}$$

The non dimensional forms of the equations (2) to (7) are

$$\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \tag{11}$$

$$v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}=Gr\,\theta+Gm\,\phi+\frac{1}{Re}\left(\frac{\partial^2u}{\partial y^2}+\frac{\partial^2w}{\partial z^2}\right)+\frac{1}{K}(U-u) \tag{12}$$

$$v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}=-\frac{1}{Re^2}\frac{\partial p}{\partial y}+\frac{1}{Re}\left(\frac{\partial^2v}{\partial y^2}+\frac{\partial^2v}{\partial z^2}\right)-\frac{v}{K} \tag{13}$$

$$v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}=-\frac{1}{Re^2}\frac{\partial p}{\partial z}+\frac{1}{Re}\left(\frac{\partial^2w}{\partial y^2}+\frac{\partial^2w}{\partial z^2}\right)-\frac{w}{K} \tag{14}$$

$$v\frac{\partial\theta}{\partial y}+w\frac{\partial\theta}{\partial z}=\frac{1}{Pr\,Re}\left(\frac{\partial^2\theta}{\partial y^2}+\frac{\partial^2\theta}{\partial z^2}\right)-Q\theta \tag{15}$$

$$v\frac{\partial\phi}{\partial y}+w\frac{\partial\phi}{\partial z}=\frac{1}{Sc\,Re}\left(\frac{\partial^2\phi}{\partial y^2}+\frac{\partial^2\phi}{\partial z^2}\right)+\frac{Sr}{Re}\left(\frac{\partial^2\theta}{\partial y^2}+\frac{\partial^2\theta}{\partial z^2}\right) \tag{16}$$

with relevant boundary conditions

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$$\left.\begin{array}{l} y=0: u=0\,,\ v=-(1+\varepsilon \operatorname{Cos}\pi z)\,,\ w=0\,, \\ \quad \theta=1+\varepsilon A\cos\pi z\,,\ \phi=1+\varepsilon A\cos\pi z \\ y\to\infty:\ u=U\,,\ v=-1\,,\ w=0\,,\quad \theta=0 \\ \quad ,\ \phi=0\,,\quad p=p_\infty \end{array}\right\} \tag{17}$$

METHOD OF SOLUTION:

We assume the solution of the equations (11) to (16) to be of the form:

$$u=u_0(y)+\varepsilon u_1(y,z)+0(\varepsilon^2) \tag{18}$$

$$v=v_0(y)+\varepsilon v_1(y,z)+0(\varepsilon^2) \tag{19}$$

$$w=w_0(y)+\varepsilon w_1(y,z)+0(\varepsilon^2) \tag{20}$$

$$p=p_0(y)+\varepsilon p_1(y,z)+0(\varepsilon^2) \tag{21}$$

$$\theta=\theta_0(y)+\varepsilon \theta_1(y,z)+0(\varepsilon^2) \tag{22}$$

$$\phi=\phi_0(y)+\varepsilon \phi_1(y,z)+0(\varepsilon^2) \tag{23}$$

$$p_0=p_\infty\,,\quad w_0=0 \tag{24}$$

Substituting these in the equations (11) to (16) and equating the harmonic terms and neglecting ε^2 we get the following set of the differential equations

Zeroth-order equations

$$\frac{d v_0}{d y}=0 \tag{25}$$

$$v_0\frac{d u_0}{d y}=Gr\,\theta_0+Gm\,\phi_0+\frac{1}{Re}\frac{d^2u_0}{dy^2}+\frac{1}{K}(U-u) \tag{26}$$

$$v_0\frac{d\theta_0}{dy}=\frac{1}{Pr\,Re}\frac{d^2\theta_0}{dy^2}-Q\theta_0 \tag{27}$$

$$v_0\frac{d\phi_0}{dy}=\frac{1}{Sc\,Re}\frac{d^2\phi_0}{dy^2}+\frac{Sr}{Re}\frac{d^2\theta_0}{dy^2} \tag{28}$$

First-order equations

$$\frac{\partial v_1}{\partial y}+\frac{\partial w_1}{\partial z}=0 \tag{29}$$

$$-\frac{\partial u_1}{\partial y}+v_1\frac{d u_0}{d y}=Gr\,\theta_1+Gm\,\phi_1+\frac{1}{Re}\left(\frac{\partial^2u_1}{\partial y^2}+\frac{\partial^2u_1}{\partial z^2}\right)-\frac{1}{K}u_1 \tag{30}$$

$$-\frac{\partial v_1}{\partial y}=-\frac{1}{Re^2}\frac{\partial p_1}{\partial y}+\frac{1}{Re}\left(\frac{\partial^2v_1}{\partial y^2}+\frac{\partial^2v_1}{\partial z^2}\right)-\frac{1}{K}v_1 \tag{31}$$

$$-\frac{\partial w_1}{\partial y}=-\frac{1}{Re^2}\frac{\partial p_1}{\partial z}+\frac{1}{Re}\left(\frac{\partial^2w_1}{\partial y^2}+\frac{\partial^2w_1}{\partial z^2}\right)-\frac{1}{K}w_1 \tag{32}$$

$$-\frac{\partial\theta_1}{\partial y}+v_1\frac{d\theta_0}{dy}=\frac{1}{Pr\,Re}\left(\frac{\partial^2\theta_1}{\partial y^2}+\frac{\partial^2\theta_1}{\partial z^2}\right)-Q\theta_1 \tag{33}$$

$$-\frac{\partial\phi_1}{\partial y}+v_1\frac{d\phi_0}{dy}=\frac{1}{Sc\,Re}\left(\frac{\partial^2\phi_1}{\partial y^2}+\frac{\partial^2\phi_1}{\partial z^2}\right)+\frac{Sr}{Re}\left(\frac{\partial^2\theta_1}{\partial y^2}+\frac{\partial^2\theta_1}{\partial z^2}\right) \tag{34}$$

Subject to boundary conditions

$$\begin{array}{l} y=0:\ u_0=0\,,\ v_0=-1\,,\ \theta_0=1\,,\ \phi_0=1\,, \\ \quad u_1=0\,,\ v_1=-\operatorname{Cos}\pi z \\ w_1=0\,,\ \theta_1=A\cos\pi z\,,\ \phi_1=A\cos\pi z \end{array} \tag{35}$$

$$\begin{array}{l} y\to\infty:\ u_0=U\,,\ v_0=-1\,,\ \theta_0=0\,,\ \phi_0=0 \\ \quad ,\ u_1=0\,,\ v_1=0 \\ ,\ w_1=0\,,\ p_1=0\,,\ \theta_1=0\,,\ \phi_1=0 \end{array} \tag{36}$$

The solution of the equations (25) to (28) under the boundary conditions (35) and (36) are

$$v_0=-1 \tag{37}$$

$$\theta_0=e^{-ay} \tag{38}$$

$$\phi_0=(1-a_1)e^{-Sc\,Re\,y}+a_1e^{-ay} \tag{39}$$

$$u_0=U+A_1e^{-ay}+A_2e^{-Sc\,Re\,y}+(-A_1-A_2-U)e^{-\lambda\,Re\,y} \tag{40}$$

where

$$a=\frac{PrRe+\sqrt{Pr^2\,Re^2+4\,Pr\,Re\,Q}}{2}\,,\quad a_1=\frac{a\,SrSc}{Sc\,Re-a}\,,$$

$$A_1=\frac{-Gm\,a_1\,Re}{a^2-Re\,a-\frac{Re}{K}}-\frac{Gr\,Re}{a^2-Re\,a-\frac{Re}{K}},\lambda=\frac{1+\sqrt{1+\frac{4}{Re\,K}}}{2}$$

$$A_2=\frac{-Gm(1-a_1)}{Sc^2\,Re-Sc\,Re-\frac{1}{K}},$$

CROSS FLOW SOLUTION

We shall first consider the equations (29), (31), (32) for $v_1(y,z)$, $w_1(y,z)$ and $p_1(y,z)$ which are independent of main flow component u_1, temperature field θ_1 and concentration field ϕ_1.

The suction velocity $v_w=-(1+\varepsilon\operatorname{Cos}\pi z)$ consists of a uniform distribution -1 with superimposed weak sinusoidal distribution $\varepsilon\operatorname{Cos}\pi z$. Hence the velocity components v, w and p are also separated in to mean and small sinusoidal components v_1, w_1 and p_1.

We assume v_1, w_1 and p_1 to be of the following forms:

$$v_1=-\pi v_{11}(y)\operatorname{Cos}\pi z \tag{41}$$

$$w_1=v'_{11}(y)\operatorname{Sin}\pi z \tag{42}$$

$$p_1=Re^2\,p_{11}(y)\operatorname{Cos}\pi z \tag{43}$$

On substitution of (41), (42) and (43), the equation (29) is satisfied and the equations (31) and (32) reduce to the following ordinary differential equations

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$$v''_{11}+R_e\,v'_{11}-\left(\pi^2+\frac{Re}{K}\right)v_{11}=-\frac{Re\,p'_{11}}{\pi} \tag{44}$$

$$v'''_{11}+Re\,v''_{11}-\left(\pi^2+\frac{Re}{K}\right)v'_{11}=-Re\,\pi\,p_{11} \tag{45}$$

with relevant boundary conditions

$$y=0 \quad : \quad v_{11}=\frac{1}{\pi}\ ,\ v'_{11}=0 \tag{46}$$

$$y\to\infty\ :\ v_{11}=0\ ,\ v'_{11}=0\ ,\ p_{11}=0 \tag{47}$$

The solutions of these equations are:

$$v_{11}=\frac{\pi\xi}{b}e^{-by}-\xi e^{-\pi y} \tag{48}$$

$$p_{11}=A\,e^{-\pi y} \tag{49}$$

where, $A=\dfrac{b\left(\pi+\dfrac{1}{K}\right)}{\pi(\pi-b)}$,

$b=\dfrac{Re+\sqrt{Re^2+4\left(\pi^2+\dfrac{Re}{K}\right)}}{2}$, $\xi=\dfrac{b}{\pi(\pi-b)}$

Hence the solutions for the velocity components v_1 , w_1 and pressure p_1 are as follows

$$v_1=\pi\xi\left(e^{-\pi y}-\frac{\pi}{b}e^{-by}\right)\cos\pi z \tag{50}$$

$$w_1=\pi\xi\left(e^{-\pi y}-e^{-by}\right)\sin\pi z \tag{51}$$

$$p_1=R_e^2\,A\,e^{-\pi y}\cos\pi z \tag{52}$$

SOLUTION FOR FIRST ORDER FLOW, CONCENTRATION AND TEMPERATURE FIELD

We now consider the equations (30), (33) and (34). The solutions for velocity component u, temperature field θ and concentration field ϕ are also separated in to mean and sinusoidal components u_1 , θ_1 and ϕ_1. To reduce the partial differential equations (30), (33), (34) in to ordinary differential equations, we consider the following forms for u_1 , θ_1 and ϕ_1 .

$$u_1=u_{11}(y)\cos\pi z \tag{53}$$

$$\theta_1=\theta_{11}(y)\cos\pi z \tag{54}$$

$$\phi_1=\phi_{11}(y)\cos\pi z \tag{55}$$

Using the expressions for v_1 , u_1 , θ_1 , ϕ_1 in (30), (33) and (34) we get the following differential equations:

$$u''_{11}+Re\,u'_{11}-\left(\pi^2+\frac{Re}{K}\right)u_{11}=-\pi\,Re\,v_{11}\,u'_0-Re\,Gr\,\theta_{11}-Re\,Gm\,\phi_{11} \tag{56}$$

$$\theta''_{11}+Pr\,Re\,\theta'_{11}-\left(\pi^2+Pr\,Re\,Q\right)\theta_{11}=-\pi\,Pr\,Re\,v_{11}\,\theta'_0 \tag{57}$$

$$\phi''_{11}+Sc\,Re\,\phi'_{11}-\pi^2\phi_{11}=-\pi\,Sc\,Re\,v_{11}\,\phi'_0-Sc\,Sr\left(\theta''_{11}-\pi^2\theta_{11}\right) \tag{58}$$

with the boundary conditions

$$y=0\ :\ u_{11}=0\ ,\ \theta_{11}=A\ ,\ \phi_{11}=A \tag{59}$$

$$y\to\infty\ :\ u_{11}=0\ ,\ \theta_{11}=0\ ,\ \phi_{11}=0 \tag{60}$$

The solutions of the (56), (57) and (58) subject to boundary conditions (59) and (60) are

$$\theta_{11}=G_0\,e^{-hy}+G_1\,e^{-(a+b)y}+G_2\,e^{-(\pi+a)y} \tag{61}$$

$$\phi_{11}=H_0\,e^{-my}+H_1\,e^{-(a+\pi)y}+H_2\,e^{-(b+a)y}+H_3\,e^{-(Sc\,Re+\pi)y}+H_4\,e^{-(b+Sc\,Re)y}+H_5\,e^{-hy} \tag{62}$$

$$\begin{aligned}u_{11}=\ &M_0\,e^{-by}+M_1\,e^{-hy}+M_2\,e^{-my}+M_3\,e^{-(a+b)y}+\\&M_4\,e^{-(b+Sc\,Re)y}+M_5\,e^{-(\pi+a)y}++M_6\,e^{-(\pi+Sc\,Re)y}\\&+M_5\,e^{-(\pi+a)y}++M_6\,e^{-(\pi+Sc\,Re)y}+M_7\,e^{-(b+\lambda\,Re)y}\\&+M_8\,e^{-(\pi+\lambda\,Re)y}\end{aligned} \tag{63}$$

where

$$G_1=\frac{a\pi^2\,Pr\,Re\,\xi}{b\left\{(a+b)^2-Pr\,Re(a+b)-\left(\pi^2+Pr\,Re\,Q\right)\right\}}$$

$$G_2=\frac{-a\pi\,Pr\,Re\,\xi}{(\pi+a)^2-Pr\,Re(\pi+a)-\left(\pi^2+Pr\,Re\,Q\right)}$$

$$G_0=A-(G_1+G_2)$$

$$h=\frac{Pr\,Re+\sqrt{Pr^2\,Re^2+4\left(\pi^2+Pr\,Re\,Q\right)}}{2}$$

$$m=\frac{Sc\,Re+\sqrt{Sc^2\,Re^2+4\pi^2}}{2}$$

$$B_1=\frac{(1-a_1)\pi^2 Sc^2\,Re^2\,\xi}{b}\ ,\ B_2=-(1-a_1)\,Sc\,Re\,\xi$$

$$B_3=\frac{a\,a_1\pi^2 Sc\,Re\,\xi}{b},$$

$$B_4=-a\,a_1\pi Sc\,Re\,\xi$$

$$E_1=-Sc Sr\,G_0\left(h^2-\pi^2\right)\ ,\ E_2=-Sc Sr G_1\left\{(a+b)^2-\pi^2\right\}$$

$$E_3=-Sc Sr G_2\left\{(\pi+b)^2-\pi^2\right\}$$

$$H_1=\frac{B_4+E_3}{(\pi+a)^2-Sc\,Re(\pi+a)-\pi^2},$$

$$H_2=\frac{B_3+E_2}{(b+a)^2-Sc\,Re(b+a)-\pi^2}$$

$$H_3=\frac{B_2}{(\pi+Sc\,Re)^2-Sc\,Re(\pi+Sc\,Re)-\pi^2}$$

$$H_4=\frac{B_1}{(b+Sc\,Re)^2-Sc\,Re(b+Sc\,Re)-\pi^2}$$

$$H_5=\frac{E_1}{h^2-Sc\,Re\,h-\pi^2}\ ,\ H_0=A-\sum_{i=1}^{5}H_i\ ,$$

$$K_1=\frac{aA_1\,Re\,\xi\pi^2}{b},\ K_2=\frac{A_2 Sc\,Re^2\,\xi\pi^2}{b},\ K_3=-\frac{\lambda\,Re^2\,\xi\pi^2\,(A_1+A_2+U)}{b},$$

$$K_4=-a\,A_1\,Re\,\xi\,\pi\ ,\ K_5=-A_2 Sc\,Re^2\,\xi\,\pi,$$

$$K_6=\lambda\,Re^2\,\xi\,\pi\left(A_1+A_2+U\right),$$

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$L_1 = -GrG_0 \,Re - GmH_5 \,Re$, $L_2 = -GmH_0 \,Re$,

$L_3 = K_1 - GrG_1 \,Re - GmH_2 \,Re$,

$L_4 = K_2 - GmH_4 \,Re$, $L_5 = K_4 - GrG_2 \,Re - GmH_1 \,Re$,

$$L_6 = K_5 - GmH_3 \,Re \ , \ M_1 = \frac{L_1}{h^2 - h\,Re - \left(\pi^2 + \frac{Re}{K}\right)} ,$$

$$M_2 = \frac{L_2}{m^2 - m\,Re - \left(\pi^2 + \frac{Re}{K}\right)}$$

$$M_3 = \frac{L_3}{(a+b)^2 - (a+b)Re - \left(\pi^2 + \frac{Re}{K}\right)},$$

$$M_4 = \frac{L_4}{(b+Sc\,Re)^2 - Re(b+Sc\,Re) - \left(\pi^2 + \frac{Re}{K}\right)} ,$$

$$M_5 = \frac{L_5}{(a+\pi)^2 - Re(a+\pi) - \left(\pi^2 + \frac{Re}{K}\right)},$$

$$M_6 = \frac{L_6}{(\pi+Sc\,Re)^2 - Re(\pi+Sc\,Re) - \left(\pi^2 + \frac{Re}{K}\right)}$$

$$M_7 = \frac{K_3}{(b+\lambda\,Re)^2 - Re(b+\lambda\,Re) - \left(\pi^2 + \frac{Re}{K}\right)} ,$$

$$M_8 = \frac{K_6}{(\pi+\lambda\,Re)^2 - Re(\pi+\lambda\,Re) - \left(\pi^2 + \frac{Re}{K}\right)} ,$$

$$M_0 = -\sum_{i=1}^{8} M_i$$

SKIN FRICTION AT THE PLATE

The non-dimensional skin-friction at the plate in direction of the free steam is given by

$$\tau = \frac{\mu \left(\frac{\partial \bar{u}}{\partial \bar{y}}\right)_{\bar{y}=0}}{\rho V_0^2} = -\frac{1}{Re}\left[u_0'(0) + \varepsilon\, u_{11}'(0) \cos \pi z\right] \quad (64)$$

$$= \tau_0 + \varepsilon Q_1 \cos \pi z$$

where

$$\tau_0 = -\frac{1}{Re} u_0'(0) = \frac{a A_1}{R_e} + S_c A_2 - \lambda(A_1 + A_2 + U) \quad (65)$$

and

$$Q_1 = -\frac{1}{Re} u_{11}'(0)$$

$$= \frac{1}{Re}\begin{bmatrix} bM_0 + h M_1 + m M_2 + (a+b) M_3 + \\ (Sc\,Re + b) M_4 + (a+\pi) M_5 + (Sc\,Re + \pi) M_6 \\ + (\lambda\,Re + b) M_7 + (\lambda\,Re + \pi) M_8 \end{bmatrix} \quad (66)$$

THE CO-EFFICIENT OF RATE OF HEAT TRANSFER

The heat flux from the plate to the in terms of Nusselt number Nu is given by

$$Nu = -\frac{k}{\rho V_0 C_p (\bar{T}_w - \bar{T}_\infty)}\left(\frac{\partial \bar{T}}{\partial \bar{y}}\right)_{y=0}$$
$$= -\frac{1}{Pr\,Re}\frac{\partial \theta}{\partial y}\bigg]_{y=0} = Nu_0 + \varepsilon Q_2 \cos \pi z \quad (67)$$

where

$$Nu_0 = -\frac{\theta_0'(0)}{Pr\,Re} = \frac{a}{Pr\,Re} \quad (68)$$

and

$$Q_2 = -\frac{\theta_{11}'(0)}{Pr\,Re} = \frac{1}{Pr\,Re}\left[h G_0 + (a+b) G_1 + (\pi + a) G_2\right] \quad (69)$$

THE COEFFICIENT OF MASS TRANSFER

The mass flux at the wall y = 0 in terms of Sherwood number Sh is given by

$$Sh = \frac{-D_M}{V_0 (\bar{C}_w - \bar{C}_\infty)}\left(\frac{\partial \bar{C}}{\partial \bar{y}}\right)_{y=0} = -\frac{1}{Sc\,Re}\frac{\partial \phi}{\partial y}\bigg]_{y=0}$$
$$= -\frac{1}{Sc\,Re}\left[\phi_0'(0) + \varepsilon\, \phi_{11}' \cos \pi z\right]$$
$$= Sh_0 + \varepsilon Q_3 \cos \pi z \quad (70)$$

where

$$Sh_0 = -\frac{1}{Sc\,Re}\left[Sc\,Re(1 - a_1) + a\, a_1\right] \quad (71)$$

and

$$Q_3 = -\frac{1}{Sc\,Re}\phi_{11}'(0)$$
$$= \frac{1}{Sc\,Re}\begin{bmatrix} m H_0 + (\pi + a) H_1 + (a+b) H_2 + \\ (\pi + Sc\,Re) H_3 + (b + Sc\,Re) H_4 + h H_5 \end{bmatrix} \quad (72)$$

RESULTS AND DISCUSSION

In order to get physical insight in to the problem, we have carried out numerical computations from the analytical solutions for non-dimensional velocity field, temperature field, species concentration field, Q_1, Q_2, Q_3 which are respectively the amplitudes of the first order skin friction, Nusselt number and Sherwood number and their values are demonstrated in graphs and tables. We choose air as the medium of diffusion. The Prandtl number for air given by Pr = 0.71. We consider separately the gases: Helium, Steam and Ammonia in ascending order of their Schmidt numbers given by Sc = 0.30, 0.60, and 0.78 respectively. These gases readily diffuse to form dilute mixtures with air. The values of the Grashof number Gr for heat transfer has been chosen as 10 (externally cooled plate) whereas the values of Grashof number Gm for mass transfer is considered to be 15, the free steam velocity (U) is selected to be 1, the small reference parameter ε is chosen as .1, z = 1/3, heat sink parameter Q =

International Journal of Science and Applied Information Technology (IJSAIT), Vol. 3 , No.4, Pages : 39 - 47 (2014)
Special Issue of ICECT 2014 - Held during September 01, 2014 in The Golkonda Hotel, Hyderabad, India

1, Soret number Sr = .5 and the remaining parameters namely porosity parameter (K), Reynolds number (Re) and suction parameter A are chosen arbitrarily.

The effects of Reynolds number on the velocity field u, temperature and concentration against the normal co-ordinate y have been presented in figures 2, 3 & 4. It is

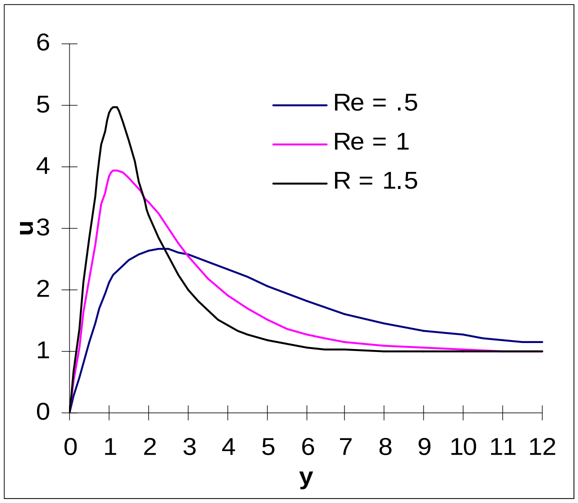

Fig-2 Velocity profile u versus y when Sc = .6, Pr = .71, K = .5

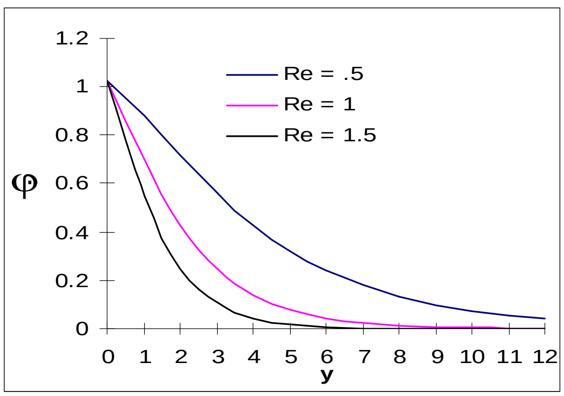

Fig-3 Concentration profile u versus y when Sc = .6, Pr = .71

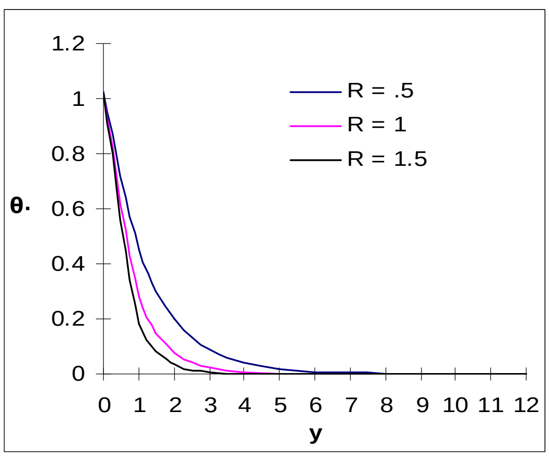

Fig-4 Temperature profile versus y when Pr = .71

noticed that an increase in Reynolds number leads to increase in the fluid velocity u near the plate and it has a reverse effect far away from the plate. This is due to the dominance of viscous forces over the inertia force near the plate and reverse effect far away from the plate. Moreover, figures 3 and 4 indicate that there is a steady fall in the temperature and concentration profile for increasing Reynolds number.

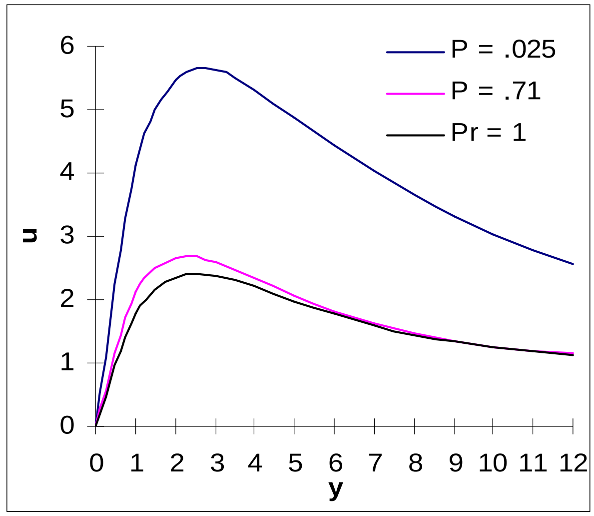

Fig-5 Velocity profile u versus y when Sc = .6, Re = .5, K = .5

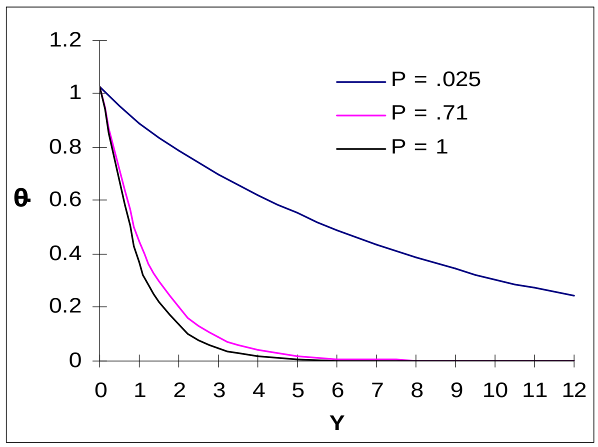

Fig- 6 Velocity profile u versus y when Pr = .71, Re = .5, K = .5

Figures 5 & 6 illustrates the variation of velocity profile and temperature with various Prandtl numbers for diffusing steam in air (species diffusivity > momentum diffusivity, Sc = .6). With increasing values of Prandtl number, there is clearly a decrease in fluid velocity i.e. the flow is decelerated through the boundary layer transverse to the plate when the plate is cooled by free convection (Gr > 0). Pr encapsulates the ratio of momentum diffusivity to thermal diffusivity for a given fluid. It is also product of dynamic viscosity and specific heat capacity divided by thermal conductivity. Higher Pr fluids will therefore posses higher viscosities (and lower thermal diffusivities) implying that such fluids will flow slower than lower Pr fluids. As a result the velocity will be decreased substantially with increasing Prandtl number.

Also it is observed that (Figure 6), the fluid temperature is reduced asymptotically when the Prandtl

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Special Issue of ICECT 2014 - Held during September 01, 2014 in The Golkonda Hotel, Hyderabad, India

number is increased. As the smaller values of Prandtl numbers indicate an increase in the thermal conductivity of the fluid and therefore, heat is able to diffuse away from the fluid more rapidly for higher values of Prandtl number. Moreover Figure 6 also shows that the effect of higher Pr results in the thinner thermal boundary layer as the higher Prandtl number fluid has a lower thermal diffusivity.

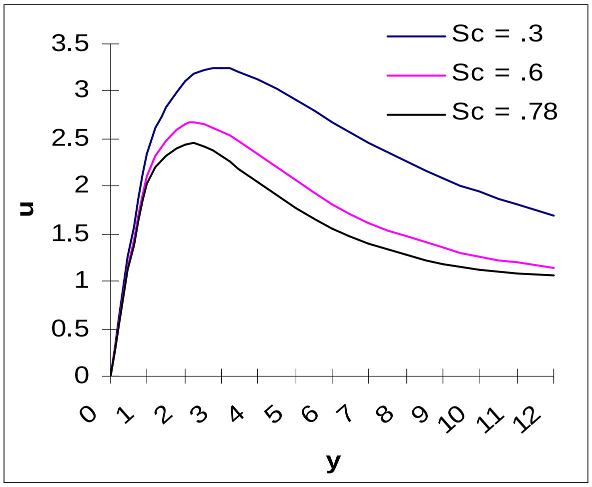

Fig- 7 Velocity profile u versus y when Pr = .71, Re = .5, K = .5

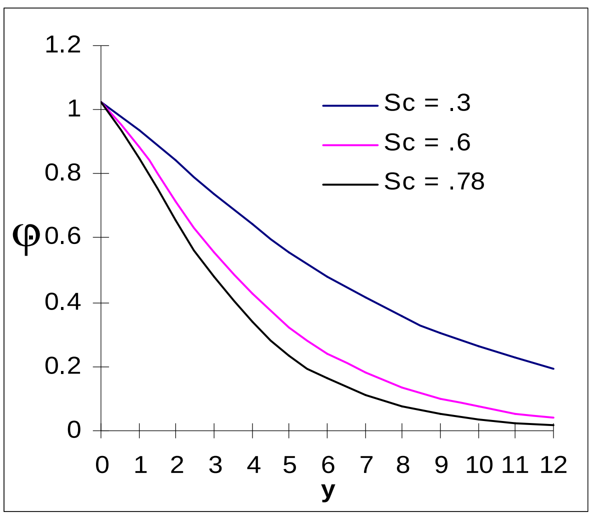

Fig-8 Concentration profile versus y when Re = .5

The velocity profile u and concentration has been exhibited in the respective Figures 7 and 8 with different values of Schmidt number (Sc). Sc quantifies the relative effectiveness of momentum and mass transport by diffusion. Higher values of Sc amount to fall in the chemical molecular diffusivity i.e. less diffusion take place by species transfer. In the present study we have performed calculation for Prandtl number (Pr = .71), so that Pr ≠ Sc. Physically this implies that the thermal species diffusion regions are of different extents. An increase in Sc will suppress concentration in the boundary layer thickness. Lower Sc will result in higher concentrations i.e. greater molecular (species) diffusivity causing an increase in concentration boundary layer thickness. Velocity u as shown in Figure 7 is found to decrease strongly with an increase in Schmidt number (Sc). Similarly there is a strong reduction in species concentration values (ϕ) as shown in Figure 8 with a rise in Schmidt number (Sc). Concentration profiles follow a smooth decay from the wall (plate) to the edge of the boundary layer; velocity profiles however, as in earlier graphs, peak close to the plate and then descend thereafter towards the free steam.

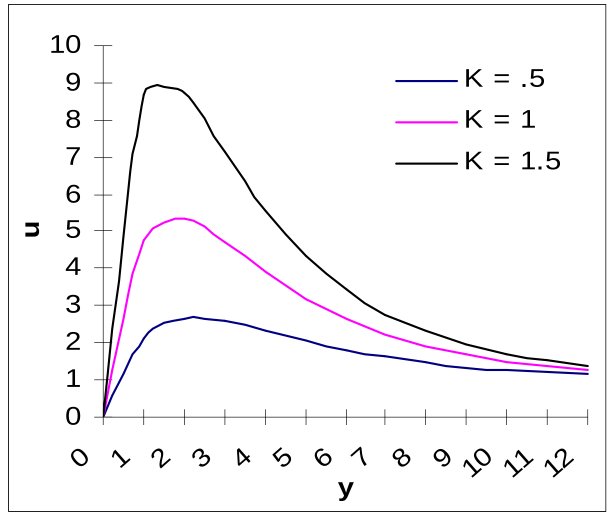

Fig- 9 Velocity profile u versus y when Pr = .71, Re = .5, Sc = .6

The behaviour of permeability parameter (K) on the fluid velocity u has been shown in Figure 9. A strong acceleration in the flow is induced with a rise in permeability parameter (K). This is due to the fact that increase in permeability of the medium implies that the resistance of the medium decreases, as a result of which the fluid velocity is accelerated.

Table- 1 Velocity versus y for A when Re = .5, Sc = .6, Pr = .71, K = .5

y	u (A = .2)	u (A = .5)	u (A = .7)
0	0	0	0
1	2.11149786	2.11191648	2.1125639
2	2.64465461	2.6446839	2.64472666
3	2.58331034	2.58331242	2.58331455
4	2.33498759	2.33498772	2.33498781
5	2.06092687	2.06092688	2.06092688
6	1.81789408	1.81789408	1.81789408

Table- 2 Temperature profile versus y for Pr = .71, Re = .5

y	θ (A = .2)	θ (A = .5)	θ (A = .7)
0	1.01	1.025	1.035
1	0.449892	0.450404	0.450745
2	0.202223	0.202241	0.202248
3	0.090937	0.0909375	0.09093791
4	0.040893	0.04089348	0.04089349
5	0.018389	0.01838939	0.01838939
6	0.00826953	0.00826953	0.00826953

The effects of suction parameter on the velocity field u, temperature and concentration against the normal

International Journal of Science and Applied Information Technology (IJSAIT), Vol. 3 , No.4, Pages : 39 - 47 (2014)
Special Issue of ICECT 2014 - Held during September 01, 2014 in The Golkonda Hotel, Hyderabad, India

co-ordinate y have been presented in Tables 1, 2 and 3 respectively. It is inferred from these Tables that velocity u, concentration and temperature increase with the increasing values of suction parameter.

Table-3 Concentration profile versus y when Sc = .6, Re = .5, Pr = .71

y	ϕ (A = .2)	ϕ (A = .5)	ϕ (A = .7)
0	1.01	1.025	1.035
1	0.880814019	0.881409896	0.881807147
2	0.715275097	0.715298549	0.715314183
3	0.558164565	0.558165481	0.558166092
4	0.42621399	0.426214026	0.426214049
5	0.321465064	0.321465065	0.321465066
6	0.240718489	0.240718489	0.240718489

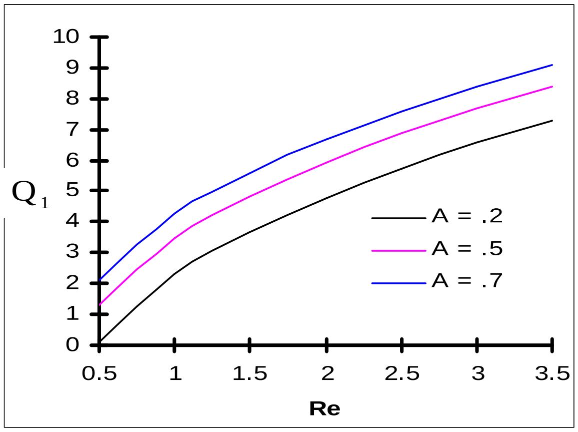

Fig-10 The amplitude Q_1 of the first order skin friction versus Re for K = .5, Sc = .6, Pr = .71

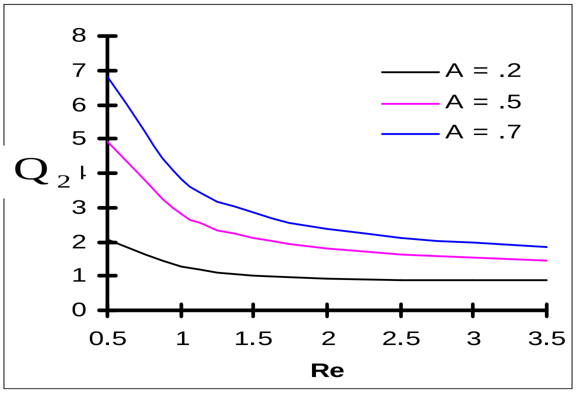

Fig-11 The amplitude Q_2 of the first order Nusselt number versus Re for Sc = .6, Pr = .71

Figures 10, 11 & 12 demonstrate how the amplitude of the perturbed part of the skin-friction Q_1, the amplitude Q_2 of the first order Nusselt number and the change of behaviour of Q_3 the amplitude of the first order Sherwood number are effected by the suction parameter A. It is noticed that first order skin-friction Q_1, the amplitude of the first order Nusselt number Q_2 and the amplitude of the first order Sherwood number Q_3 increase with the increasing values of suction parameter A.

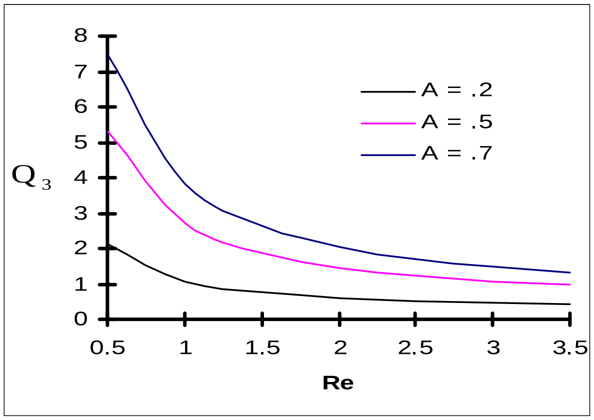

Fig-12 The amplitude Q_3 of the first order Sherwood number versus Re for Sc = .6

CONCLUSIONS:

1. An increase in Reynolds number leads to increase in the fluid velocity u near the plate and it has a reverse effect far away from the plate.
2. There is a steady fall in the temperature and concentration profile for increasing Reynolds number.
3. With increasing values of Prandtl number, there is a marked decrease in fluid velocity i.e. the flow is decelerated through the boundary layer transverse to the plate when the plate is cooled by free convection.
4. The fluid temperature is reduced asymptotically when the Prandtl number increases.
5. Velocity is found to decrease sharply with an increase in Schmidt number.
6. There is a sharp reduction in species concentration values with a rise in Schmidt number.
7. A marked acceleration in the flow is induced with a rise in permeability parameter.
8. Velocity, concentration and temperature increase with the increasing values of suction parameter.
9. The first order skin-friction Q_1, the amplitude of the first order Nusselt number Q_2 and the amplitude of the first order Sherwood number Q_3 increase with the increasing values of suction parameter.

NOMENCLATURE

A is the suction parameter
$\bar{C}$ is the species concentration
$\bar{C}_\infty$ is the species concentration in the free stream
$\bar{C}_w$ is the species concentration at the plate
C_p is the specific heat at constant pressure
D_M is the chemical molecular diffusivity
D_T is the chemical thermal diffusivity
g is the acceleration due to gravity
Gr is the Grashof number for heat transfer
Gm is the Grashof number for mass transfer
k is the thermal conductivity

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K^* is the permeability of porous medium
K is the permeability parameter
L is the wave length of the periodic suction
$\overline{p}$ is the pressure
$\overline{p}_\infty$ is the pressure in the free steam
p is the non dimensional pressure
p_∞ is the non dimensional in the free steam
$\overline{Q}$ is the first order heat sink
Q is the non dimensional first order heat sink
Re is the Reynolds number
Sr is the Soret number
Pr is the Prandtl number
Sc is the Schmidt number
$\overline{T}$ is the temperature in the boundary layer
$\overline{T}_w$ is the temperature at the plate
$\overline{T}_\infty$ is the fluid temperature at the free steam
$\overline{U}$ is the free steam velocity
U is the non dimensional free steam velocity
$(\overline{u}, \overline{v}, \overline{w})$ are the components of the fluid velocity
(u, v, w) are the non dimensional components of the fluid velocity
V_0 is the mean suction velocity
$(\overline{x}, \overline{y}, \overline{z})$ is the coordinate system
$\hat{i}, \hat{j}, \hat{k}$ are the unit vectors in the increasing direction of $\overline{x}, \overline{y}, \overline{z}$

Greek symbols:

α is the thermal diffusivity
β is the coefficient of volume expansion for heat transfer
$\overline{\beta}$ is the coefficient of volume expansion for mass transfer
ν is the kinematic viscosity
ρ is the density of the fluid
ε is a small reference parameter
θ is the non dimensional temperature
ϕ is the non dimensional concentration
μ is the coefficient of viscosity

and the other symbols have their usual meanings.

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