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Simulated Annealing for Multi Objective Stochastic Optimization

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Abstract:This paperaddresses the multi-objective stochastic optimization problem that arises in many real-world applications, especially in supply chain management and optimization.To this end, a simulated annealing algorithm is presented and used for solving this problem. The algorithm uses the hill-climbing criterionin order to escape from local minimality trap. The paper also introduces a new Pareto set for stochastic optimization problems and demonstrates the application of simulated annealing on this Pareto set. Finally, the proposed algorithm is applied on an inventory example that is solved by optimizing three objectives. Numericalresults indicate that the algorithm is capable of constructing a Pareto set of non-dominated solutions.

Keywords: Multi Objective Optimization, Simulated Annealing, Hill-climbing algorithm, Supply Chain Management.

INTRODUCTION

Multi-objective optimization (MOO) problems arise in many applications, especially in supply chain management and optimization. Supply chain performance is concerned with optimizing a multi-attribute decision. Altiparmak*et al.* [1] formulated the supply chain design as a multi-objective optimization problem in which the objective is not only to minimize supply chain costs, but also to maximize customer service while at the same time maximizing the capacity utilization and balance at the distribution centers. Azaron*et al.* [2] considered three objectives:(i) the minimization of the sum of current investment costs and the expected future processing, transportation, shortage, and capacity expansion costs, (ii) the minimization of the variance of the total cost, and (iii) the minimization of the financial risk or the probability of not meeting a certain budget. Franca *et al.* [3] considered the multi-objective supply chain that maximizes the profit and minimizes supplier defects (increasing the quality level). Moncayo-Martínez*et al.* [4] consideredtwo objectives: minimizing the total cost and lead-time simultaneously for a product or a family of products. The complexity of the supply chain network largely contributes to the difficulty of optimizing supply chain performance. Jayaraman and Ross [5] described the PLOT (Production, Logistics, Outbound, Transportation) system to address network design problems involving a central manufacturing plant, multiple distribution centers and cross-docking sites, and retail outlets stocking multiple products.

Simulated Annealing, in particular, plays a critical role in keeping the time required to optimize the model manageable for practical problems. Ulungu*et al.* [6] conceived a Multi-Objective Simulated Annealing (MO-SA) algorithm for solving combinatorial optimization problems. Alrefaei and Diabat [7] also proposed a simulated annealing algorithm for solving a multi-objective optimization problem and implemented it toan inventory problem. Another example of using SA for SCM optimization can be found in [8].

MULTI-OBJECTIVE OPTIMIZATION

Consider an n-vector function $f = (f_1, \dots, f_n)$, where $f_j$is the j^{th} objective function, $j = 1, 2, \dots, n$, usually f_j is the expected performance of a complex stochastic system whose evaluation encounters some noise;

$$f_j(s) = E\left[h_j(s;\, Y_s)\right] \qquad (1)$$

Assume that we are interested in selecting a system that optimizes all the objective functions $f_1, \dots, f_n$. We need to keep in mind that there may be no single optimal solution that solves all objective functions and the optimization objective maybe conflicting. One way of solving this problem is to give a weight for the various objective functions based on their importance and then aggregating them into a single objective optimization problem as follows:

$$f(s) = \sum_{j=1}^{n} w_j f_j(s), \text{ where} \sum_{j=1}^{n} w_j = 1, \quad w_j \geq 0 \qquad (2)$$

Then, one can use any optimization problem to solve the aggregated problem.This approach was used by Alrefaei and Diabat [13] to solve a multi-objective inventory optimization problem. Santé-Riveira*et al.* [9] alsoused this approach for optimizing the land use allocation. A key challenge for this method is in determining the weights $w_j, j = 1, 2, \dots, n$.

Another way of solving the MOO is to construct a Pareto set, which is a set of alternative solutions that are not dominated by any other solution. A solution x is said to dominate solution x' if all components of x ; $f_j(x), j = 1, \dots, n$ are at least as good as those of x' (with at least one strictly better component). In other words

$$f_j(x) \leq f_j(x'), \quad \text{for all } j = 1, \dots, n$$
$$\text{and } f_j(x) < f_j(x'), \text{ for at least one } j.$$

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Solution x belongs to the Pareto optimal set if it is not dominated by any other feasible solution.

Other approaches that were suggested to solve MOO problems include thetwo well-known approaches ofPAES (Knowles and Corne [10]) and NSGA-II (Deb *et al*. [11]). The first popular algorithm is called the Non-dominated Sorting Genetic Algorithm (NSGA-II) proposed by Deb *et al.* [11]. Initially, a random parent population is generated andsorted based on the non-domination relation. A fitness function is then assigned to each solution based on itsnon-domination level. A child of population smaller than the parent population is generated from the parent population and all the solutions are sorted based on their non-domination status. The new population is now used as a parent. The other known approach in this regard is proposed by Knowles and Corne [10].They suggested a simple multi-objective Evolutionary Approach (EA) using a single parent, single child EA. Both the parent and the child are compared;, the one that dominates the other is added to the Pareto set, and if they do not dominate each other, then they are compared with the solutions in the Pareto set.

MULTI OBJECTIVE SIMULATED ANNEALING ALGORITHMS

Simulated Annealing is a heuristic optimization technique that was developed earlier by Kirkpatrick*et al*.[12] for solving deterministic combinatorial optimization problems. It is assumed that the solution space is well organized. The simulated annealing uses a hill-climbing criteria in order to escape the local minimality. Given a current solution,a new candidate is selected from the neighborhood of the current state and compared with the current solution, if its function value is better than it is accepted as the new solution. If its function value is larger, then there is a chance for selecting it as a new solution with probability that depends on the difference in their objective function values. Suppose that the current solution is X and the candidate solution is Z with objective function values $f(X)$ and$f(Z)$, then the probability of accepting the candidate solution is given by

$$p = exp\left[\frac{-[f(Z)-f(X)]^{+}}{T_k}\right]$$

where T_k is a control parameter, that is decreased to zero with a predetermined rate in order to converge to optimal solution, and

$$[f(Z)-f(X)]^{+} = \begin{cases} 0 \ if\ f(Z) \leq f(X) \\ f(Z)-f(X), otherwise \end{cases}$$

Note that if $f(Z) \leq f(X)$ then $p = 1$ and the candidate solution Z is accepted.
The simulated annealing has been modified for solving stochasticoptimization problems. Alrefaei and Andrad´ottir [13] propose a SA algorithm that uses a constant parameter T instead of a decreasing sequence. Ahmed and Alkhamis [14] use a simulated annealing approach that uses the Ranking and Selection procedure to compare the current solution with its neighboring solution.
Several authors consider the use of simulated annealing for multi-objective optimization. Suppapitnarm *et al.*[15] used the simulated annealing algorithm for solving multi-objective problems.Their method startswith an initial solution X in the Pareto set, then selects a candidate Z from the neighborhood of X and uses the acceptance probability $P = \min\left\{1, \prod_{j=1}^{n} exp\left[\frac{-[f_j(Z)-f_j(X)]^{+}}{T_k}\right]\right\}$ for moving to a candidate solution Z, where $f_j(t)$ is the j^{th} objective function, and $T_k$is a decreasing sequence. If Z is accepted, then the algorithm adds it to the Pareto set. Suman [16] has also used a Pareto dominated simulated annealing for multi-objective optimization problems where he has made extensive comparisons of multi-objective simulated annealing algorithms.

THE PROPOSED α-MOSA

The underlying MOO problem in this paperis a stochastic optimization problem that has no exact objective function values. Instead, optimization values are estimated using simulation. Thus, we cannot check the dominance criterion with certainty. To address this issue, we define the α-dominance as follows:

Definition 1:For two solutions x and z, we say that x is α-dominated byz if for all objectives j, the confidence interval $C_{\alpha}^{j} = (1-\alpha)100\%$ for the difference $f_j(Z) - f_j(x)$ are contained in the interval $(-\infty, 0]$ and at least one $C_{\alpha}^{j} = (1-\alpha)100\%$ is contained in the interval $(-\infty, 0)$. If x is not α-dominatedby z, then we say that x is α-non-dominated by z.

In the proposed method, we construct a Pareto optimality set that consists of α-non-dominated solutions. All the previous simulated annealing methods use a control parameter T_k (called the temperature) that is decreased to 0 in order to make the probability of moving to a new solution small when the algorithm becomes closer to the optimal solution. However, in our approach, we let the temperature T be constant all the time.In this way, we give more chancesforthe algorithm to explore the set of feasible solutions in order to locate an optimal or a near optimal solution. It is assumed that the solution set is well structured, that is, for each feasible solution X, there is a neighborhood $N(X)$, the neighborhood $N(X)$ is assumed to be symmetric, i.e., if $Z \in N(X)$, then $X \in N(Z)$, and each feasible solution x' is reachable from X ;i.e. there is a sequence of feasible points $X_0 = X, X_1, X_2, \dots, X_r = X'$ such that $X_1 \in N(X_0), X_2 \in N(X_1), \dots, X' \in N(X_{r-1})$. At the beginning, a set of size n is selected randomly as the initial Pareto set. The Pareto set is then updated using the hill-climbing algorithminthe simulated annealing method. At the beginning, the algorithm starts by initial Pareto setP_0. At any iteration k, a new candidate Z is selected from the neighborhood of the current solution X. All the objective functions at the current and at the candidate solutions are estimated and compared through the probability selection

$$p = \prod_{j=1}^{n} exp\left[\frac{-\left[f_j(Z)-f_j(X)\right]^{+}}{T}\right] \quad (3)$$

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If the new candidate is accepted, then it is compared with the Pareto set P_k. If the new candidate α-dominates one of the solutions in P_k, then the candidate solution enters the Pareto set and the solution that is dominated is removed from P_k.

The α–MOSA algorithm:

Step0: Select a starting set P_0 as the initial Pareto set. Let $k = 0$. Select X_k from P_k

Step1: Given $X_k = X$, select a candidate Z_k from theneighborhood of X; $N(X)$.

Step2: Given $X_x = X$ and $Z_k = Z$,get estimates of all objective function values $f_1, \ldots, f_n$ at X and Z as $(f_1(X), \ldots, f_n(X))$ and $(f_1(Z), \ldots, f_n(Z))$ through simulation.

Step3: Generate a uniform random number U_k in [0,1], If $U_k \leq p$, where the probability selection P is given by (3), compare Z with the solutions in the Pareto set P_k, if there exists $x' \in P_k$ at which $x'$is α-dominated by z then replace x' by the candidate Z in P_k.

Step4: If the stopping criterion is not met, let $k = k + 1$, Go to Step 1.

NUMERICAL EXAMPLE

The proposed approach is tested on an inventory multi-objective stochastic optimization problem. Consider the following as a (s, S) multi-objective inventory model presented in Alrefaei and Diabat [7]. In (s, S)policy, an order is placed as soon as the inventory position declines to or below thereorder point s. The order size is chosen so that the inventory position increases to S. A stockout occurs if the sum of theundershoot plus the total lead time demand goes below s (see Fig. 1). We assume that the (s, S) inventory model consists of three minimizing objectives: the average orderingcost, the average holding cost (storage cost), and the average shortage cost. We assume that the time between successivedemands is exponentially distributed with a mean interarrival time of k days, and that the size of each demand is a discreterandom variable with the following probabilities

Table 1: The demand size distribution

Demand size	1	2	3	4	5	6	7
Probability	0.15	0.25	0.3	0.1	0.1	0.05	0.05

The lead time (the time between placing an order and receiving it) is assumed to be uniformly distributed between1 and 2 days. The inventory is reviewed every month. The setup cost to place an order is K = $50 and the cost of each itemordered is $C_i = \$5$. The holding cost per item per month is $h_i = \$2$, and the shortage cost per item per month is assumed to be$\pi = \$5$.

We are interested in constructing a Pareto set that consists of three solutions that are not dominated by other solutions. It is assumed that the feasible solution set consists of

$$S = \{(s, S): 20 \leq s \leq 115;\ 45 \leq S \leq 235;\ S - s \leq 25,\ \ s, S \text{ are multiples of } 5\}.$$

We coded the algorithm by Fortran Power station and ran the algorithm to the described inventory model for 500 iterations. Table 2 summarizes the obtained results, it shows the three values of the objective function in the final Pareto set $P = \{(30, 95), (25, 90), (25,95)\}$.

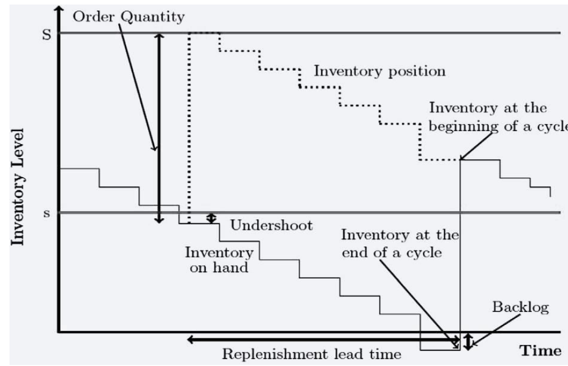

Figure 1: The (s, S) inventory model

Table 2: The three costs in the final Pareto set

$P(j)$	(s, S)	ORDER COST	HOLDING COST	STORAGE COST	Total COST
1	(30 95)	103.93	33.51	8.83	146.28
2	(25 90)	103.57	29.37	12.73	145.67
3	(25 95)	103.54	31.41	12.55	147.50

The function values for the first solution in the Pareto set is depicted in Figure 2: the horizontal axis represents the iteration number and the vertical axis represents the function value. It is clear that the obtained holding cost decreased over the iterations, but the other two costs increased slightly. The results for the other 2 solutions in the Pareto set are depicted in Figures 3and 4, and, as can be seen again in these figures, the holding cost decreases. The decrease is because the holding cost varies more than the other two costs.

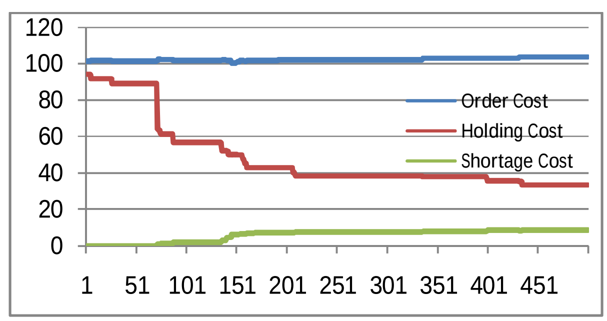

Figure 2: The objective function values for the first solution in the Pareto set

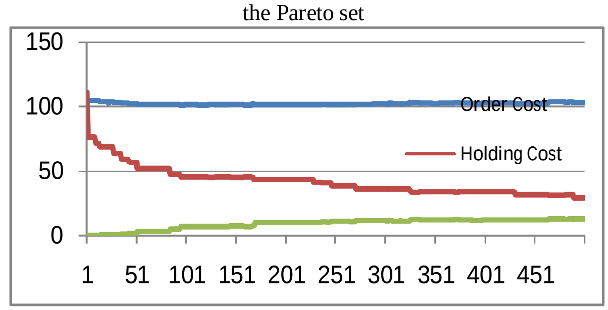

Figure 3: The objective function values for the second solution in the Pareto set

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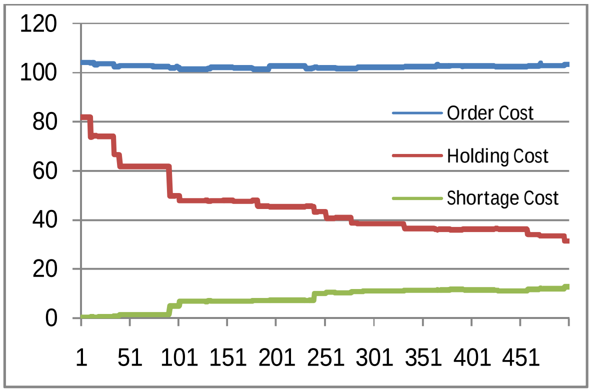

Figure 4: The objective function values for the third solution in the Pareto set

CONCLUSION

This paper presented a simulated annealing algorithm for solving multi-objective stochastic optimization problem. The algorithm uses the hill-climbing feature to escape the local minimality trap. A new Pareto set for multi-objective stochastic optimization is also introduced to assist the simulated annealing algorithm attain optimality. Simulation is used to estimate the objective function values. The algorithm is applied to an (s,S) inventory model where the numerical results showed that the algorithm is capable of selecting a set of non-dominated solution. In the future, the proposed algorithm will be applied to a large-scale supply chain optimization problem at a major steel producer.

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