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A Novel Chaotic Multi-Objective Brain Storm Optimization Approach for Multi-Plate Disk Brake Design Problem

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ABSTRACT

Brain Storm Optimization (BSO) is recent swarm intelligence algorithm that mimics the social intelligence of a group of humans facing a problem via brainstorming process. The main three processes done by the algorithm; clustering, convergence, and generation of new solutions make the standalone algorithm is flexible and can be easily modified to fit various problems. In this paper, a novel chaotic based Multi-Objective version of BSO is presented to tackle a well-known engineering design problem so-called Multi-Plate Disk Brake Design Problem. Some modifications are done to the original BSO in order to fit the multi-objective problem. A novel updating of the solutions' repository so called external archive by the aid of special set of two-dimensional chaotic maps. The employment of two-dimensional chaotic maps in Multi-Objective Optimization (MOO) is a pioneer work in the field.

Key words: Chaos; Brain Storm Optimization; Evolutionary Algorithms; Multi-Objective Optimization; Multi-Plate Disc Brake Problem; Swarm Intelligence.

1. INTRODUCTION

Daily people seek optimal actions in decisions' situations which require well established methods and techniques to help them in searching them. Most of real life situations often involve multiple conflicting criteria or objectives. There must be a compromise obtained between them because an increase of one objective's value will result in decrease of another's. These specific features of such decisions' situations create a special category of problems so-called Mutli-Objective Problems (MOPs) in which its solution methods are seeking set of solution points not a single point like in single objective version. The philosophy of the solution methods is to determine the set of best trade-offs between the conflicting objectives, the so called Pareto optimal set. Now a day, optimization is used in all the fields of Engineering like construction, manufacturing, controlling, and Design Models. Mechanical Engineering is one of the most fields that require developing of optimizations models as well as solution methods. Most of the mechanical design problems found in literature are nonlinear constrained problems which involve non differentiable functions in some cases which requires special treatment and modifications in additional to being one time done or non-repetitive models [3].

Real world engineering design problems are usually multi-objective problems by nature which contain many conflicting objectives. Normally it has mixed (e.g., continuous and discrete) design variables, nonlinear objective functions and nonlinear constraints which make them tough to be tackled.Multi-plate Disk brake finds its applications in airplanes, to apply effective braking while landing. The Multi-plate disk brake design problem has mixed constraints and was proposed by Ray and Liew [6].

In last few decades, bio-inspired algorithms gain a lot of attention among researchers. These algorithms mimic a lot of natural phenomena and surroundings like food hunting, mating, and co-evolution relationships. A number of Swarm Intelligence (SI) algorithms have been proposed and developed for real-world applications. SI algorithms have additional comparative advantage over other Artificial Intelligence (AI) techniques which is the collective intelligence of the flock or swarm in reaching a solution during search. The process of sharing information among the swarm members speed up capturing solutions and provides better exploration capabilities. The processes of this co-evolution sharing of information makes SI algorithms are eligible candidates for solving MOPs. In MOP we search the pareto optimal set which contains the best solutions which can be attained by swarm members which makes SI methods fit this type of problems.

Brain Storm Optimization (BSO) was proposed in 2011 by Professor Shi Yuhui [7]. The social-based algorithm belongs to Swarm Intelligence (SI) algorithms but with a special manner to simulate the process of humans' brainstorming. The algorithm is composed of clusters with centers and individuals. The balance between diversity and convergence of the algorithm is based on clusters' centers and individuals updating as well as the migration done between clusters.

Chaos is a kind of characteristic of nonlinear systems and it has been extensively studied and applied in many fields. Recently many researchers are trying to employ its apparent irregular behavior to generate randomness as well as its hidden underlying rules to control Evolutionary Algorithms (EAs) during search which guide them to escape from local optimal points. The two aforementioned characteristics of chaos make the resulted hybrid chaotic EAs' performance is expected to outrank the standalone EAs in many applications. Although it appears to be stochastic, it occurs in a deterministic nonlinear system under deterministic conditions [4].

In this paper, a new Multi-Objective Evolutionary Algorithm (MOEA) based on chaos search and Brain Storm Optimization (BSO) algorithm is constructed to solve Multi-Objective problems efficiently. The proposed approach is adopted to solve the Multi-plate disk brake design problem. The rest of paper is structured as following; section 2 is made for preliminaries and concepts. Section 3 is made for the proposed approach; it shows the modifications done to the original algorithm in order to explore the multi-objective version. Section 4 is devoted to the Multi-plate disk brake design problem; finally section 5 is for conclusion.

2. PRELIMINARIES AND CONCEPTS

In this section, the mathematical model of MOP and its relevant definitions will be explored in the first sub-section. The BSO will be illustrated in a separate subsection, and finally the last subsection is made for chaotic functions and illustrations.

2.1 Multi-Objective Optimization

A general Multi-objective Optimization Problem (MOP) consists of a number of objectives to be optimized simultaneously in general, a k-objective minimization problem can be written as [8]:

$$\text{Minimize} \quad F(x) = \{f1(x), f2(x), ..., fk(x)\}, \qquad (1)$$
$$\text{Subject to} \quad x \in S$$

where F(x) is the k-dimensional objective vector, $f_i(x)$ is the i^{th} objective function to be minimized, x is the decision vector, and S is the feasible region in the decision space. In MOP, addition to the variable space there is another space so called objective space composed from all objectives. For each solution x in the variable space, there exists a point in the objective space, denoted by $f(x) = z = (z_1, z_2, \ldots, z_k)^T$. The nature of Evolutionary Algorithms (EAs) makes them good candidates to solve MOPs, because they can deal with a set of possible solutions (or the so-called population) simultaneously.

2.2 Brain Storm Optimization

The main idea behind Swarm Intelligence (SI) algorithm (figure 1) is the collective knowledge the population attains based on their sharing of information. The world of Birds, ants, wolves, fish, whales, and other flocks are mapped into search algorithms capturing one main feature which is the population intelligence. Human beings interactions also can inspire many researchers to develop search algorithms coping the same idea of collaboration resulted in social intelligence.

Input: Population: *n*, Maxiterations : Max, number of clusters: *m*
Output: Globally optimal individual of each iteration.
1. Generate *n* individuals randomly.
2. Evaluate the fitness of *n* individuals.
3. While inter Max do
4. Cluster the *n* individuals into *m* clusters; choose the centers of each cluster.
5. **For** each individuals:
6. **If** rand() < *P*_one **then**
7.   **If** rand() < *P*_center
8.   Select a center for updating
9.   **Else**
10.   Select a normal individual for updating
11. **Else**
12.   **If** rand() <*P*_center
13.   Select two centers for updating
14.   **Else**
15.   Select two normal individuals for updating
16. **End**
17. The offspring individuals are compared with parents individuals, the better individuals will be reserved.
18. **Return:** Best individuals.

Figure 1: Procedure of the original BSO

Basically, when people face a problem they share experience and collect information which accumulates social experience and better actions. It is common that human beings formulate groups, panels, committees, organizations, entities, and other formal or informal gatherings to face certain problem or share ideas. One of the most motivating frame of human beings' gatherings the brain storming process done by set of persons to take a decision or generate ideas.

There are some rules govern this discussion mode in a way to facilitate generating ideas and organizing the discussion and time. A master facilitator is being in charge and manages the discussion flow and questions presented. Group participators gathered from different backgrounds and cultures to impose divergent thoughts and create novel inspirations. BSO captures the divergence of thoughts introduced and the features of brain storming process and reflects it into new search algorithm to solve complex problems. There are three main processes in original BSO, including the clustering which plays the role of the solutions' convergence, the generation and mutation which is the divergence aspect of the algorithm to explore more areas, and the selection process [2]. The following algorithm shows the steps of the original BSO for single objective.

2.3 Chaos

Many researchers were motivated to use chaos in optimization based on one crucial feature; in brief its ability to generate both regularity and randomness. The nonlinear dynamics shows its power to increase convergence and divergence capabilities in search. There is a highly motivated research stream arouse last few years is to assess the process of combing chaos to various categories of EAs including Swarm Intelligence techniques and other bio-inspired techniques and to Examine various chaotic maps in the proposed chaotic algorithms to stand over their suitability to various optimization tasks during search. Mathematically, a map can be defined by a function with the same range anddomain. The chaotic maps can be classified into one dimension or two dimension maps. In this sub-section, some well-known one dimension maps are introduced. Then, some of two dimension chaotic maps are presented after [1].

First: One dimension chaotic maps

1. Logistic map

$$x_{n+1} = \mu\, x_n\,(1 - x_n) \tag{2}$$

Obviously, $x_n \in [0,1]$ under the conditions that the initial $x_0 \in [0,1]$, where n is the iteration number and $\mu = 4$.

2. Circle map

$$x_{n+1} = x_n + d - (c / 2\pi)\sin(2\pi z_n) \bmod(1) \tag{3}$$

where $c = 0.5$, $d = 0.2$, and $x_0 \in [0,1]$ generates chaotic sequence in [0,1].

3. Tent map

It generates chaotic sequences in [0,1] assuming the following form:

$$X_{n+1} = \begin{cases} X_n/0.7, & X_n < 0.7, \\ 10/3X_n(1 - X_n), & \text{otherwise.} \end{cases} \tag{4}$$

4. Sinusoidal map

$$X_{n+1} = ax_n^2 \sin(\pi x_n). \tag{5}$$

when $a = 2.3$ and $x_0 = 0.7$ it has the simplified form represented by

$$X_{n+1} = \sin(\pi x_n). \tag{6}$$

It generates chaotic sequence in [0,1].

5. Sinus map

$$X_{n+1} = 2.3(X_n)^{2\ Sin(\pi X_n)}. \tag{7}$$

Second: Two dimension chaotic maps

6. Arnold's cat map

The Arnold's cat map is named after Vladimir Arnold, who demonstrated its effects in the 1960s using an image of a cat. It is represented by

$$\begin{aligned} X_{n+1} &= X_n + Y_n \bmod(1) \\ Y_{n+1} &= X_n + 2Y_n \bmod(1) \end{aligned} \tag{8}$$

It is clear that the sequences $X_n \in [0, 1]$ and $Y_n \in [0, 1]$.

7. Sinai map

$$\begin{aligned} X_{n+1} &= X_n + Y_n + a\cos(2\pi Y_n) \bmod(1) \\ Y_{n+1} &= X_n + 2Y_n \bmod \end{aligned} \tag{9}$$

when $a = 1$ it generates chaotic sequences in (0, 1).

8. Zaslavskii map

$$\begin{aligned} X_{n+1} &= (X_n + v + aY_{n+1}) \bmod(1) \\ Y_{n+1} &= \cos(2\pi X_n) + e^{-r} Y_n \end{aligned} \tag{10}$$

For $v = 400$, $r = 3$, $a = 12$.
Note that in this case, $Y_{n+1} \in [-1.0512, 1.0512]$.

9. Hénon map

$$\begin{cases} x_{n+1} = 1 - ax_n^2 + y_n \\ y_{n+1} = bx_n \end{cases} \tag{11}$$

where $a = 1.4$ and $b = 0.3$

10. Lozi map

$$X_{n+1} = 1 - \alpha |X_n| + Y_n \qquad (12)$$
$$y_{n+1} = \beta X_n$$

where α = 1.4 and β = 0.3.

3.CHAOTIC MULTI-OBJECTIVE BRAIN STORM OPTIMIZATION

Multi-Objective Evolutionary Algorithms (MOEAs) have the advantage of capturing a whole set of optimal solutions. When Solving MOP using MOEAs, some general fashion points should be considered; to minimize the distance to the global pareto front, maximize the spread of solutions found, and finally maximize the elements of the pareto. front produced. Such solutions found along the search are usually stored in a different place which called external archive. Furthermore, the contents of the external archive are also usually reported as the final output of the algorithm. Many modifications should be done to the original BSO to explore the MOPs [9],[10]. The main flow chart of the Multi-Objective Brain Storm Optimization MOBSO is shown in Figure 2. After, the proposed approach is illustrated and the modifications introduced are presented.

1. **Initialization operation:** Randomly generate *n* individuals, evaluate the *n* individuals;
2. **While** termination not satisfied **do:**
3. **Disturbance step:** randomly replace an individual depend on the probability.
4. **Clustering step:** All of the individuals are sorted according to Pareto dominance and crowding distance, Select the first *n* individuals as elite sets; the others belong to normal set.
5. **New individual generated step:**

```
        Randomly generate a probability value P1.
    If  P1< pa  then
        Generate a probability value P2;
          If  P2< pb then
               Choose a randomly individual from normal set as the selected one.
          Else then
               Choose a randomly individual from elite sets as the selected one.
          End
    Else
        Generate a probability value P3
          If P3< pc
              Randomly Choose the combination of two more individuals in the normal set as the selected one.
          Else
              Randomly Choose the combination of two more individuals in the elite set as the selected one.
          End
     End
```

5. **Re-initialize:** When the number of individuals stagnation updates reaches a threshold, Reinitialize the individual.
6. **Update step**: Reorder the collection of parent individuals and offspring individuals using two sets of indicators. Reserve better solutions.
7. **Archive updating step:** when all individuals have been generated, each new non-dominated solution obtained in current iteration will be compared with all members in the Archive. The non dominated solution will be reserved.

Figure 2: Procedure of the MOBSO [10]

The new method so-called Chaotic Multi-Objective Brain Storm Optimization (CMOBSO) employs chaos to increase the efficiency of the previous MOBSO in several parts of the algorithm. The illustrative Flow chart found in Figure 3 indicates these positions.

Through the rest of this subsection, the modifications done will be illustrated:

1) The **initialization** is done chaotically by running the suitable one–dimension map many times to generate the population.
2) Previously the clustering is done by applying k-means clustering algorithm [5], but it has well-known critical drawbacks which will directly affect the computation time. In the proposed approach **clustering** is done by applying the two-dimension Arnold's cat map which significantly will minimize the computation time and also escape from local optimum for each cluster.
3) For the **disturbance** the two-dimension Hénon map is applied for replacement.
4) The **New individuals' generation** is done by running suitable one-dimension maps. The values of P_1, P_2, P_3, p_a, p_b, and p_c are updated chaotically.
5) Updating the external archive where the nondominated solutions are kept and at the end of problem printed as the final solution is the most faced challenge in designing a new MOEA. The solution kept must maintain diversity

to explore all regions of the pareto front as well as closeness to it. The convergence of any MOEA sometimes keeps it stuck into certain regions while neglecting other regions of the pareto front. In the introduced approach the **external archive updating** is done by running the two-dimension Sinai map to attain a well diversified pareto front.

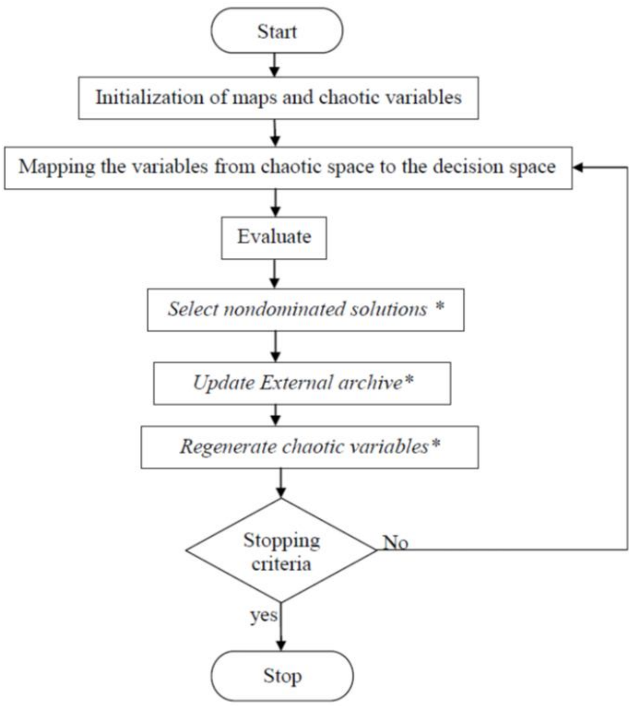

Figure 3: Boxes containing an asterisk (*) are potential positions for chaos incorporation

The aforementioned modifications gain a lot of benefits over the original MOBSO; First less computations by eliminating the k-means algorithm in the clustering process. Second the premature convergence problem suffered by most of MOEAs is tackled by introducing the well generated sequence of random numbers and updating the external archive chaotically. The two-dimension maps are really fit the sequential probabilistic procedures of the algorithm.

4. MULTI-PLATE DISK BRAKE DESIGN PROBLEM

Multi-plate disk brake finds its applications in airplanes, to apply effective braking while landing. The exploded view of the multi-plate disk brake is shown in Figure. 4 and 5 .

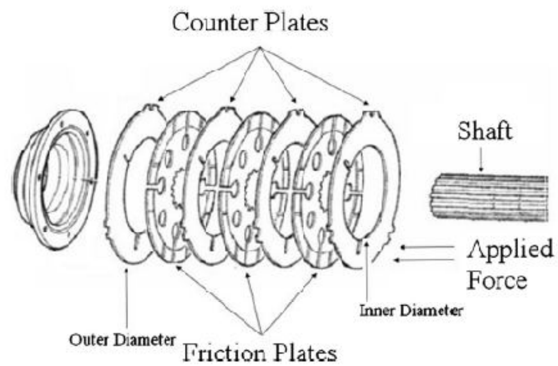

Figure 4: Exploded view for the multi-plate disk brake [6]

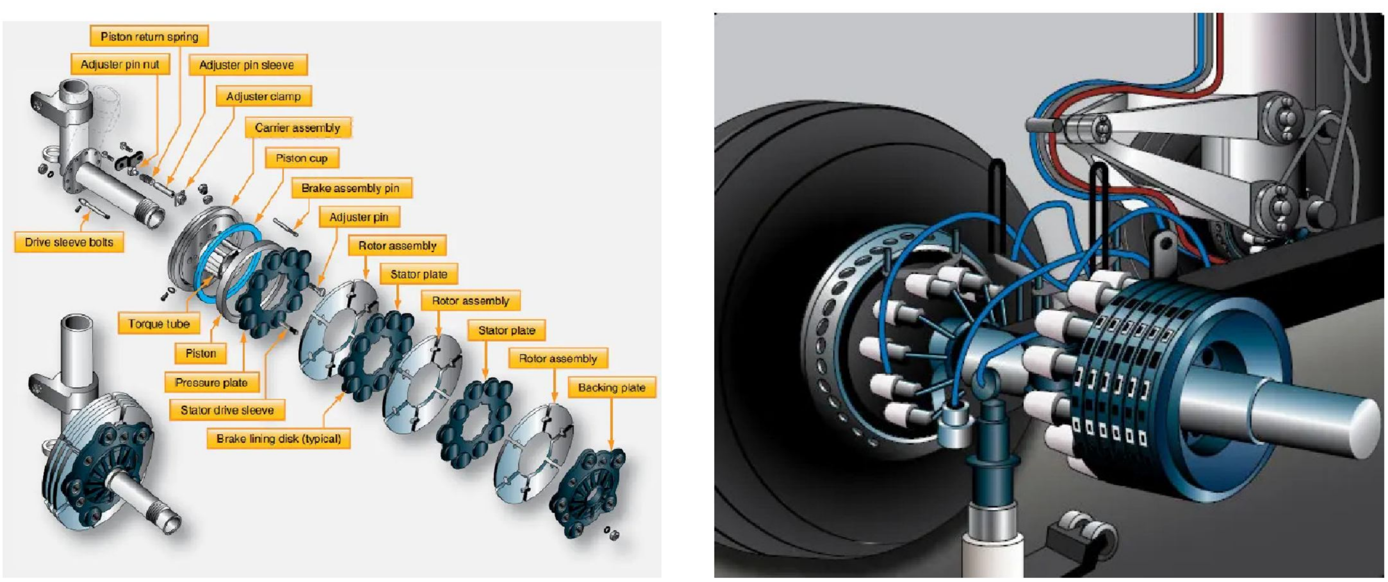

Figure 5: The multi-plate disk brake design

The purpose of the problem is to simultaneously minimize the mass of the brake and the stopping time. There are four design variables for the inner radius, outer radius, the engaging force (applied force) and the number of friction surfaces (number of friction plates). The problem is a constrained mixed integer problem for which five different restrictions are introduced for the distance between the radii of the friction plates, length of the brake, pressure sustained by the plates, maximum limitation for the temperature generated and the braking torque. The problem can be stated as follows:

$$\begin{aligned}
&\text{Minimize} \quad f_1(x) = 4.9e - 5\left(x_2^2 - x_1^2\right)(x_4 - 1)\\
&\text{Minimize} \quad f_2(x) = (9.82e6)\,\frac{x_2^2 - x_1^2}{x_3 x_4\left(x_2^3 - x_1^3\right)}\\
&g_1 = 20 + x_1 - x_2\\
&g_2 = 2.5(x_4 + 1) - 30\\
&g_3 = \frac{x_3}{3.14\left(x_2^2 - x_1^2\right)^2} - 0.4\\
&g_4 = 2.22e - 3x_3\,\frac{x_2^3 - x_1^3}{\left(x_2^2 - x_1^2\right)^2} - 1\\
&g_5 = 900 - \left(\frac{2.66e - 2x_3 x_4\left(x_3^3 - x_1^3\right)}{x_2^2 - x_1^2}\right)\\
&\forall g \leq 0\\
&55 \leq x_1 \leq 80, 75 \leq x_2 \leq 110, 1000 \leq x_3 \leq 3000,\\
&\quad 2 \leq x_4 \leq 20, x_4 \in I
\end{aligned} \tag{13}$$

The parameters of the proposed approach are set as following; n = 50, number of iterations will are set at 100, $p_a \in [0.7,0.9]$, $p_b \in$ [0.4,0.6], and $p_c \in$ [0.45,0.55]. The Pareto front obtained by using CMOBSO is compared to the true Pareto front as shown in Figure 6. The extreme Pareto solutions obtained are (0.12735, 16.6540) and (2.7905, 2.07199) which is diversified and well distributed. The chaotic maps used in our algorithm are as follows: the logistic map is employed for group's members' initialization. The circle map is used in new individuals' generation process to update the values of P_1, P_2, P_3, p_a, p_b, and p_c. The aforementioned two-dimensional chaotic maps listed in the previous section are also incorporated as illustrated in the modifications done of CMOBSO approach.

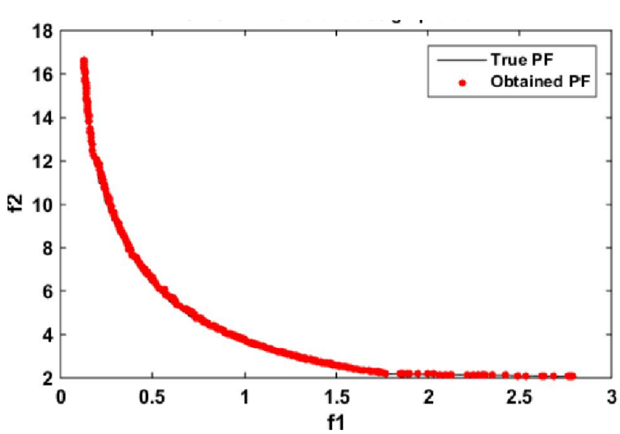

Figure 6: Pareto front obtained by the CMOBSO Algorithm for “Disk brake design problem”

5. CONCLUSION

To the best of knowledge, this proposed approach CMOBSO is the pioneer work done to incorporate chaos to MOBSO. The introduced approach combined one and two dimensional chaotic maps to original BSO which enhances the performance and results more diversified pareto optimal set. The combination of chaos search is done in a new manner to fit MOP. The updating of the external archive is done in a novel way to gain more diversity during search by employing two dimensional chaotic maps. The Multi-Plate Disk Brake Design Problem is presented and solved by the proposed approach efficiently as shown by the pareto front.

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