

The Optimization Technique for Joint Discrete Search and Detection of Observation Objects

Hennadii Khudov¹, Irina Khizhnyak², Iryna Yuzova³, Oleksii Baranik⁴, Galina Semiv⁵, Semen Bondarenko⁶, Olexander Tytarenko⁷

¹Department of Radar Troops Tactic, Ivan Kozhedub Kharkiv National Air Force University, Kharkiv, Ukraine, 2345kh_hg@ukr.net

²Department of Mathematical and Software Automated Control Systems, Ivan Kozhedub Kharkiv National Air Force University, Kharkiv, Ukraine, khizh_ia@ukr.net

³Department of Reserve Officers Training, Ivan Kozhedub Kharkiv National Air Force University, Kharkiv, Ukraine, uzik25@ukr.net

⁴Department of Aviation Armament Complexes, Ivan Kozhedub Kharkiv National Air Force University, Kharkiv, Ukraine, kozaktur@i.ua

⁵Department of Rocket Artillery Armament, Hetman Petro Sahaidachnyi National Army Academy, Lviv, Ukraine, galinasemiv1985@gmail.com

⁶Department of Rocket Artillery Armament, Hetman Petro Sahaidachnyi National Army Academy, Lviv, Ukraine, bondarenko_semen2@ukr.net

⁷Department of Air Forces, Ivan Chernyakhovsky National Defense University of Ukraine, Kyiv, Ukraine, tabzrv@ukr.net

ABSTRACT

Briefly analyzes the main results of solving problems of search and detection of objects of observation in information systems. A discrete search zone is introduced. The task is to find the optimal Bayes decision rule in the current discrete search zone. The Bayesian decision-making rule is clarified when jointly optimizing discrete search and detection of observation objects.

Key words : information system, joint search and detection of objects, discrete search and detection of objects, the time of searching and finding objects, Bayes criterion of minimum average risk.

1. INTRODUCTION

Today, a whole class of information systems is used to solve the problem of searching and detecting objects in conditions of limited search potential [1-7]. This article assesses the possibility of reducing the requirements for search potential due to the joint Bayes optimization of the search and detection of objects.

The goal of research is to reduce the time of searching and finding objects by optimizing the spatio-temporal distribution of the search potential. It takes into account the specifics of the operation of the information system in discrete search mode.

1.1 Problem analysis

The tasks of searching and detecting objects in information systems are currently being solved independently of each other. This applies to both radar [1, 3, 4-8, 10] and optoelectronic [2, 9, 11-16] systems. There are optimization methods that consider search and detection as a single task only in the production plan. Solutions are obtained for the individual components of this task.

In [3], the problem of joint search and detection optimization was solved as follows. For the case of a continuous a certain area of survey Ω , an improved Bayes rule for deciding on the detection of an object is formulated. This rule is as follows. When solving the task of testing a simple hypothesis against a simple alternative, the joint optimization of the search and detection of objects reduces to finding a uniformly optimal search strategy, calculating the maximum of the unconditional likelihood ratio in the current area of view and comparing it with the threshold.

In [17], the results of joint search and detection of objects efficiency are briefly analyzed. The weight criterion of an optimality of detection in a zone of search elementary cell is formulated. The differential characteristics of Bayes criterion of a minimum of average risk are taken into account. The weight criterion of joint optimization of search and detection of objects in the current zone of search is specified. Expression for the ratio of likelihood in the current zone of search is received.

Consider the results obtained in [3, 17, 18] for the case of solving task of the discrete searching and detecting a stationary single object.

2. MAIN MATERIAL

Consider the task of joint Bayesian optimization of discrete search and detection of objects. For this will use the criterion of minimum average risk, namely, the average value of the decision-making fee when testing statistical hypotheses.

For further research, divide the area of search Ω into N sub-areas Ω_i , such that $\sum_{i=1}^N \Omega_i = \Omega$. The average risk in sub-area Ω_j is denoted as R_j .

Introduce the current area of search $\Omega(t_i) = \sum_{j(t_i)} \Omega_j$,

where j – number of sub-area of search and detection of an object at time t . Index j indicates the serial number of the sub-area in which the search and detection of the object at a time t_i .

Set the task of finding the optimal Bayes decision rule in the current discrete area of search $\Omega(t_i)$. With this formulation of the task, an additional optimization parameter appears, namely, the current dimensions and position of the discrete area $\Omega(t_i)$ in the common area Ω .

Therefore, conditions are created for finding the optimal (by the Bayes criterion of minimum average risk) strategy for joint search and detection of an object in the search area of a discrete structure. A solution to such a problem, both in search theory and in detection theory, was previously absent

According to [3, 17, 18], the average risk in the current area of search $\Omega(t_i)$ now can found as:

$$R(t_i) = \sum_{\Omega(t_i)} R_j = R_0 - ((I_{10} - I_{11}) \sum_{\Omega(t_i)} P_{1j}(\gamma_1, t_i) - (I_{01} - I_{00}) \sum_{\Omega(t_i)} P_{0j}(\gamma_1, t_i)), \quad (1)$$

where $P_{1j}(\gamma_1, t_i)$ – the current value of the unconditional probability of the correct detection of objects in the j -th sub-area at a time t ;

$P_{0j}(\gamma_1, t_i)$ – the current value of the unconditional probability of false alarm in the j -th sub-area at a time t .

Assume that R_0 is a non-negative constant for the current area of search $\Omega(t_i)$ at time t . Write the Bayes rule for testing simple hypothesis H_0 against simple alternative H_1 in the current discrete sub-area $\Omega(t_i)$ of area Ω as follows:

$$\frac{\sum_{\Omega(t_i)} P_{1j}(\gamma_1, t_i)}{\sum_{\Omega(t_i)} P_{0j}(\gamma_1, t_i)} \begin{matrix} \gamma_1 > \\ \gamma_0 < \end{matrix} \frac{I_{01} - I_{00}}{I_{10} - I_{11}}, \quad (2)$$

Passing to the unconditional likelihood ratio

$$l(t_i) = \frac{\sum_{\Omega(t_i)} P_{1j}(\gamma_1, t_i)}{\sum_{\Omega(t_i)} P_{0j}(\gamma_1, t_i)},$$

write expression (2) as follows:

$$l(t_i) \begin{matrix} \gamma_1 > \\ \gamma_0 < \end{matrix} \frac{I_{01} - I_{00}}{I_{10} - I_{11}}, \quad (3)$$

Thus, the optimal Bayes rule (3) for testing a simple hypothesis against a simple alternative, which was obtained on the basis of expressions (1) and (2), is to maximize the unconditional likelihood ratio $l(t_i)$ in the current discrete area $\Omega(t_i)$ and compare it with the threshold (4):

$$c_b = \frac{I_{01} - I_{00}}{I_{10} - I_{11}}, \quad (4)$$

At that,

if $l(t) \geq c_b$, then decision γ_1 is made, that is, the hypothesis H_0 is rejected;

if $l(t) < c_b$, then decision γ_0 is made, that is, hypothesis H_0 is accepted.

In accordance with (2) optimization should be carried out by:

parameters of conditional probability of correct detection of $P(\gamma_1 / H_1, x)$ in sub-areas Ω_j ;

parameters of the current discrete area of search $\Omega(t_i)$.

Consider an important special case. Assume that, similarly to the Neumann-Pearson criterion, the value of the unconditional probability of false alarm in sub-area Ω_j at a time t is fixed at a constant level – $P_{0j}(\gamma_1, t_i)$. Then,

according to expression (2), finding the maximum of the unconditional likelihood ratio reduces to finding the maximum of the unconditional probability of the correct detection of the object in the current discrete sub-area $\Omega(t_i)$.

Thus, to find the optimal Bayes decision rule in the current discrete area $\Omega(t_i)$ of the common area Ω , along with the solution of the hypothesis testing problem in this area, the problem of finding the optimal object search strategy (using the Bayes criterion of minimum average risk) must be solved.

Search strategy $\lambda(\Omega_j, t_i)$ is a rule that at any moment of time t_λ establishes in which sub-area Ω_j of the area the search should be carried out and with what energy costs.

For further research, introduce the main restrictions on the search strategy, which are usually used in search theory. Namely, require that search strategy $\lambda(\Omega_j, t_i)$ be T-truncated, that is, $\lambda(\Omega_j, t_i) = 0$ for $t_i > T$ and $x \in \Omega$. That is, the condition of compulsory viewing of area Ω during the search T must be fulfilled.

It's obvious that:

$$\lambda(\Omega_j, t_i) > 0, \quad \text{for } \Omega_j \in \Omega(t_i); \quad (5a)$$

$$\lambda(\tilde{\Omega}, t_i) = 0, \quad \text{for } \tilde{\Omega} \in \Omega / \Omega(t_i). \quad (5b)$$

Assume that the search strategy should be constant for all sub-areas that are viewed at a fixed moment of time t_i . The measure of the current area $\Omega(t_i)$ consists of the sub-areas Ω_j viewed at the moment t_i .

$$\Omega(t_i) = \sum_j \Omega_j. \quad (6)$$

In addition to the above properties of the search strategy, require that it satisfy the optimality condition. This condition is that if each T-truncated strategy $\lambda(\Omega_j, t_i)$ has a functional $P(\lambda(\Omega_j, t_i))$, then strategy $\lambda_{opt}(\Omega_j, t_i)$ will be optimal if:

$$P(\lambda_{opt}(\Omega_j, t_i)) = \sup P(\lambda(\Omega_j, t_i)), \quad (7)$$

where $P(\lambda(\Omega_j, t_i))$ – unconditional probability of correct detection of an object at time t_i at strategy $\lambda(\Omega_j, t_i)$.

Also require that the search strategy be optimal for any moment of time T of the end of the search. That is, at whatever moment of time the search would not be interrupted,

up to this moment of time it should be optimal according to the criterion of the maximum unconditional probability of correct detection.

From the analysis of the results for the selection of search strategies that are studied in the theory of search, of all the strategies, the class of uniformly optimal search strategies most fully satisfies expressions (5)–(7).

Strategy $\lambda(\Omega_j, t_i)$ is uniformly optimal if any T-truncated strategy is optimal, i.e.

$$P(\lambda(\Omega_j, t_i)) = P(\lambda_{opt}(\Omega_j, t_i)), \quad \forall t_i \leq T. \quad (8)$$

Thus, when solving the problem of finding, according to the Bayes criterion of the minimum of the average risk of the search strategy and object detection, the uniformly optimal search strategy is optimal. In accordance with which the current sizes and position of sub-area $\Omega(t_i)$ in the common search area Ω should be selected.

In accordance with expression (2) with a value of the unconditional probability of false alarm fixed at a constant level, the optimization task is formulated as follows:

$$\begin{aligned} P_1(\gamma_1, t_i) &\rightarrow \max; \\ \lambda(\Omega_j, t_i) &\geq 0, \quad t_i > 0; \\ \sum_{\Omega(t_i)} \lambda(\Omega_j, t_i) &= L_0, \quad t_i > 0; \\ \sum_j \lambda(\Omega_j, t_i) &= \varphi(t_i); \\ \sum_{\Omega(t_i)} \varphi(t_i) &= L_0 t_i, \end{aligned} \quad (9)$$

where $P_1(\gamma_1, t_i)$ – the unconditional probability of detecting an object at time t_i in the discrete area of search $\Omega(t_i)$;

L_0 – characterizes the power of a search engine;

$\varphi(t_i)$ – search effort in search area Ω at time t_i .

It is rather difficult to obtain a solution to the optimization task (9) for the discrete case. Therefore, we reformulate the optimization task (9) somewhat and use the well-known approach to solve it.

The main variable will be considered as $0 \leq P_{1j} \leq 1$ – the conditional probability that the object will be detected when viewing the sub-area with the number l of area Ω_j . But

provided that the object is actually located in area Ω_j and has not been previously detected. It is understood that scans with numbers $1, 2, \dots, l-1$ of sub-area Ω_j were carried out earlier, and not necessarily sequentially, one after another.

Introduce the notation $q_{lj} = 1 - P_{lj}$. Let area Ω_j be viewed n_j times. Then the conditional probability that an object in area Ω_j will be detected when viewed with number l is

$$P_{ij} \prod_{i=1}^{l-1} q_{ij}.$$

If all areas $\Omega_j, j = \overline{1, N}$, are viewed, and area Ω_j is viewed n_j times, then there is an unconditional probability of detecting an object during this sequence of views:

$$\sum_{j=1}^N P_j \sum_{l=1}^{n_j} P_{ij} \prod_{i=1}^{l-1} q_{ij}, \tag{10}$$

where P_j – a priori probability of the distribution of the location of the object in area Ω_j .

If condition $\sum_{j=1}^N n_j S_j \leq C$ is introduced,

where C – total search potential allocated at time T ;

$$n_j \in Z = \{0, 1, 2, \dots\};$$

S_j – area of the j -st sub-area of the search area Ω ,

then obtain the following optimization task:

$$\begin{aligned} \sum_{j=1}^N P_j \sum_{l=1}^{n_j} P_{ij} \prod_{i=1}^{l-1} q_{ij} &\rightarrow \max; \\ \sum_{j=1}^N n_j S_j &\leq C; \\ n_j &\in Z, \quad j = \overline{1, N}. \end{aligned} \tag{11}$$

In the optimization task (11), there are two types of unknown quantities:

search efforts expressed in whole numbers n_1, n_2, \dots, n_k ;

areas $\Omega_1, \Omega_2, \dots, \Omega_N$.

Rewrite the optimization task (11) in the form of a dynamic programming task:

$$f_N(z) = \max_{n_j, j=\overline{1, N}} \left\{ \sum_{j=1}^N P_j \sum_{l=1}^{n_j} P_{ij} \prod_{i=1}^{l-1} q_{ij} \right\}, \tag{12}$$

$$\text{where } \sum_{j=1}^N n_j S_j \leq z.$$

Solving (12) using the Bellman optimization theory, obtain the optimal number of views of each area. After finding the optimal number of views of each area, determine the viewing order of these areas.

At the same time, set the condition for ensuring the minimum time for searching and detecting an object. For this we use the relationship:

$$\frac{P_j P(\gamma_1 / H_1, i, j) \prod_{l=1}^{i-1} \overline{P}(\gamma_1 / H_1, l, j)}{S_j}, \tag{13}$$

where $P(\gamma_1 / H_1, i, j) = 1 - \overline{P}(\gamma_1 / H_1, i, j)$;

i – number of views of the j -st area, $i = \overline{1, u_{i-1}^{opt}}$;

$j = \overline{1, k}$, k – total number of areas in the search area Ω .

Having ordered relations (13) in descending order, it is necessary to solve the search and detection task in the same order.

Based on the studies performed, the following refined optimal Bayes decision rule can be formulated. When solving the task of testing a simple hypothesis against a simple alternative, the joint optimization of discrete search and object detection is reduced to:

finding a uniformly optimal search strategy;

calculating the maximum unconditional likelihood ratio in the current discrete area $\Omega(t_i)$;

comparing it with a threshold.

When this rule is fulfilled, the mean time until the object is detected can be calculated by (14):

$$T_m = \int_0^T Q(t) dt, \tag{14}$$

where $Q(t)$ – unconditional probability of non-detection of an object.

Taking into account that $Q(t) = 1 - P_1(\gamma_1, t)$ for discrete search and detection, write expression (15):

$$T_m = \sum_j (1 - P_j P_1(t_j)) \Delta t_j, \quad (15)$$

where $P_1(t_j)$ – conditional probability of detecting an object in a sub-area Ω_j ;

Δt_j – the time allotted to view a sub-area Ω_j ;

$$\sum_j \Delta t_j = T.$$

For quantitative estimates, assume that $P_j = P_1(t)$, $\Delta t_j = \Delta t$.

Consider two special cases:

I. $P_1 = P_2 = 0,5$; $\Delta t = 0,5T$

II. $P_1 = 0,7$; $P_2 = 0,3$; $\Delta t = 0,5T$.

For these cases get:

I. $T_m \approx 0,75T$;

II. $T_m \approx 0,71T$.

Can conclude that with the joint optimization of search and detection, a gain in average time is provided before the detection of an object

3. CONCLUSION

Thus, the joint optimization of discrete search and detection of objects can reduce time and, accordingly, reduce the energy cost of detecting objects.

Introduced the current discrete area of view. The task of finding the optimal Bayes decision-making rule in the introduced current discrete area of view is posed and solved.

The specified Bayes optimal decision rule is formulated. When solving the task of testing a simple hypothesis against a simple alternative, the joint optimization of discrete search and object detection is reduced to:

finding a uniformly optimal strategy for finding an object in discrete cells of the search area;

calculating the maximum unconditional likelihood ratio in the current group of search subdomains;

comparing it with a threshold.

REFERENCES

1. R. Hoseinnezhad, B.-N. Vo, and B.-T. Vo, Visual Tracking in Background Subtracted Image Sequences via Multi-Bernoulli Filtering, *IEEE Trans. Signal Processing*, Vol. 61, No. 2, 2013, pp. 392-397.
2. I. Ruban, H. Khudov, O. Makoveichuk, I. Khizhnyak, V. Khudov, V. Podlipaiev, V. Shumeiko, O. Atrasevych, A. Nikitin, and R. Khudov. **Segmentation of optoelectronic images from on-board systems of remote sensing of the Earth by the artificial bee colony method**, *Eastern-European Journal of Enterprise Technologies*, № 2/9 (98), 2019, pp. 37–45. DOI: <https://doi.org/10.15587/1729-4061.2019.161860>.
3. H. Khudov, I. Khizhnyak, F. Zots, G. Misiyuk, and O. Serdiuk. **The Bayes Rule of Decision Making in Joint Optimization of Search and Detection of Objects in Technical Systems**, *IJETER*, № 8(1), 2020, pp. 7–12. DOI: <https://doi.org/10.30534/ijeter/2020/02812020>.
4. V. Lishchenko, V. Chaliy, G. Khudov, and A. Zvonko. **Proposals for Improving of Air Surveillance Informativity in MIMO Radar Systems Based on Two-Dimensional Radars**, in *Intern. Scient.-Pract. Conf. Problems of Infocommunications. Science and Technology (PIC S&T)*, 2018, pp. 153–156. DOI: <https://doi.org/10.1109/infocommst.2018.8632052>.
5. M. Iasechko, M. Mozhaiev, I. Manzhai, M. Volk, V. Manoylo, O. Ochkurenko, D. Maksuita, V. Larin, A. Markov, and O. Kostyria. **Conditions for Reliable Transmission of Information Over Long Distances Using a Powerful Electromagnetic Radiation**, *IJATCSE*. 8(1), 2020, pp. 138-144. DOI: <https://doi.org/10.30534/ijeter/2020/18812020>.
6. H. Khudov, A. Fedorov, D. Holovniak, and G. Misiyuk. **Improving the Efficiency of Radar Control of Airspace with the Multilateration System Use**, in *Intern. Scient.-Pract. Conf. Problems of Infocommunications. Science and Technology (PIC S&T)*, 2018, pp. 680–684. DOI: <https://doi.org/10.1109/infocommst.2018.8632141>.
7. O. Turinskyi, M. Iasechko, V. Larin, D. Dulenko, V. Kravchenko, O. Golubenko, D. Sorokin, and O. Zolotuin. **Model and development of plasma technology for the protection of radio-electronic means of laser emission**, *IJATCSE*. 8(5), 2019, pp. 2429-2433. doi:10.30534/IJATCSE/2019/85852019.
8. V. Lishchenko, H. Khudov, V. Tiutiunyk, V. Kuprii, F. Zots, and G. Misiyuk. **The Method of Increasing the Detection Range of Unmanned Aerial Vehicles In Multiradar Systems Based on Surveillance Radars**, in *2019 IEEE 39th International Conference on Electronics and Nanotechnology (ELNANO)*, 2019. DOI: <https://doi.org/10.1109/ELNANO.2019.8783263>.
9. H. Khudov, A. Zvonko, S. Kovalevskyi, V. Lishchenko, F. Zots. **Method for the detection of small sized air objects by observational radars**, *Eastern-European Journal of Enterprise Technologies*, № 2/9 (92), 2018, pp.

- 61–68.
DOI:<https://doi.org/10.15587/1729-4061.2018.126509>.
10. M. Iasechko, Y. Gnusov, I. Manzhai, O. Uhrovetskyi, V. Manoylo, A. Iesipov, O. Zaitsev, M. Volk, and O. Volk. **Determination of Requirements for the Protection of Radio-Electronic Equipment from the Terroristic Influence by Electromagnetic Radiation**, *IJATCSE*, 7(12), 2019, pp. 772-777. DOI: <https://doi.org/10.30534/ijeter/2019/077122019>.
11. H. Khudov, I. Ruban, O. Makoveichuk, H. Pevtsov, V. Khudov, I. Khizhnyak, S. Fryz, V. Podlipaiev, Y. Polonskyi, and R. Khudov. **Development of methods for determining the contours of objects for a complex structured color image based on the ant colony optimization algorithm**, *Eureka: Physics and Engineering*, № 1, 2020, pp. 34–47. DOI: <https://doi.org/10.21303/2461-4262.2020.001108>.
12. I. Ruban, V. Khudov, O. Makoveichuk, H. Khudov, and I. Khizhnyak. **A Swarm Method for Segmentation of Images Obtained from On-Board Optoelectronic Surveillance Systems**, in *Intern. Scient.-Pract. Conf. Problems of Infocommunications. Science and Technology (PIC S&T)*, 2018, pp. 613–618. DOI: <https://doi.org/10.1109/infocommst.2018.8632045>.
13. I. Ruban, H. Khudov, V. Khudov, I. Khizhnyak, and O. Makoveichuk. **Segmentation of the images obtained from onboard optoelectronic surveillance systems by the evolutionary method**, *Eastern-European Journal of Enterprise Technologies*, № 5/9 (89), 2017, pp. 49–57. DOI: <https://doi.org/10.15587/1729-4061.2017.109904>.
14. I. Ruban, H. Khudov, O. Makoveichuk, M. Chomik, V. Khudov, I. Khizhnyak, V. Podlipaiev, Y. Sheviakov, O. Baranik, and A. Irkha. **Construction of methods for determining the contours of objects on tonal aerospace images based on the ant algorithms**, *Eastern-European Journal of Enterprise Technologies*, № 5/9 (101), 2019, pp. 25–34. <https://doi.org/10.15587/1729-4061.2019.177817>.
15. I. Ruban, H. Khudov, O. Makoveichuk, I. Khizhnyak, N. Lukova-Chuiko, G. Pevtsov, Y. Sheviakov, I. Yuzova, Y. Drob, and O. Tytarenko, **Method for determining elements of urban infrastructure objects based on the results from air monitoring**, *Eastern-European Journal of Enterprise Technologies*, № 4/9 (100), 2019, pp. 52–61. DOI: <https://doi.org/10.15587/1729-4061.2019.174576>.
16. I. Ruban, V. Khudov, H. Khudov, and I. Khizhnyak. **An Improved Method for Segmentation of a Multiscale Sequence of Optoelectronic Images**, in *Intern. Scient.-Pract. Conf. Problems of Infocommunications. Science and Technology (PIC S&T)*, 2017, pp. 137–140. DOI: <https://doi.org/10.1109/INFOCOMMST.2017.8246367>.
17. G. V. Khudov, **Features of optimization of two-alternative decisions by joint search and detection of objects**. *Problemy Upravleniya I Informatiki (Avtomatika)*, 2003, № 5, pp. 51–59.
18. T. D. Wickens. *Elementary signal detection theory*, New York, NY: Oxford University Press, 2002, 277 p. <https://doi.org/10.1093/acprof:oso/9780195092509.001.0001>