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An efficient reduced model applied to the study of the mechanical reliability of an Aircraft's Wing

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ABSTRACT

This paper presents the reliability study of an aircraft wing in the presence of random uncertainties due to manufacturing tolerances or material inhomogeneity. This effect generally leads to an increase in vibration amplitudes, resulting in dynamic deformations much larger than those estimated at the design stage. A study of the dynamic behaviour was carried out using ANSYS. This model was coupled to the MCS procedure. In order to reduce the computational effort required for the reliability analysis by the finite element model, a reduced order model formulation based on modal analysis is proposed to examine the response of the system under study. The proposed methodology allows the efficient capture of statistical properties. Thus, it allows a good compromise between the proposed robustness criteria, the resolution and precision sought and a considerable time saving.

Key words : Reliability analysis; Reduced-order model; FORM and SORM; Monte Carlo; cyclic symmetry ; FEM ; modal analysis.

1. INTRODUCTION

Reliability analysis in mechanics has been developed for the study of mechanical structures in an uncertain context. Uncertainties can be directly related to manufacturing tolerances, unpredictable environment and other factors largely involved in engineering [1], [2], [3]. Although, conventional design rarely relies on absolute risk-based decision making, so the uncertainties are assessed in a bounded manner using a so-called safety factor. However, it is preferable to conduct structural analysis in a more judicious and realistic framework called probabilistic (also called Mechanical Reliability) to assess the reliability of structures [4].

Reliability assessment methods aim at evaluating the probability of a limit state violation by comparing probabilistic models of active loads and structural strength [4]. A limit state is a condition beyond which a structure exceeds a specified design requirement expressed in mathematical form by a limit state function G (X) = 0. The probability of failure (P_f) is then defined as the probability of occurrence of the failure event G (X) $\leq$ 0, where X is a vector of random variables representing the uncertainties in the loads, as well as in the material and geometric properties of the structure. The objective of these methods is to assess the probability that a system may be in a configuration considered to be faulty by taking into account the uncertain behaviour of the various variables acting on the system.

However, reliability formulation requires a mechanical reliability coupling, involving the coupling between reliability analysis algorithms and the mechanical model to estimate the probability of failure [5]. Among the reliability analysis methods is the Monte-carlo method, which is represented as the reference method [6]. Also, first and second order methods (FORM and SORM). Both of these methods are based on Taylor's development around the most likely point. Although these methods are accurate and simple to implement, they can become very expensive as the number of random parameters increases. Still, the complexity of the mechanical reliability coupling depends on the approximation mean of the performance function.

Reliability is defined as the probability that the performance function G is greater than zero [8],[9]. Negative values of the limit state function indicate the failure area. This allows the probability of failure to be estimated by [P_f = {G(E, ρ) < 0]. Typically, the complexity of the performance functions and the large number of variables in the model make it impossible or impractical to calculate the probability of failure directly. However, several techniques have been developed for reliability analysis. Among these methods is Monte Carlo simulation which can be used for problems related to explicit or implicit limit state functions [7]. Monte Carlo consists of generating a large set of random values according to known statistical laws, and the statistics of favorable cases are counted. The probability of default is calculated as the number of favorable cases divided by the total number of cases.

$$P_f = \frac{1}{N}\sum_{i=1}^{N} G(i) \quad (1)$$

Despite its simplicity, an inherent disadvantage of the method is the enormous computational effort required for the analysis. As a result, calculation costs increased considerably with each sampling cycle, sometimes reaching unachievable levels.

This study presents a mechanical reliability analysis of an aircraft wing. The model studied taking into account the uncertainties in the properties of the materials. The implemented technique combines modal finite element analysis with probabilistic methods MCS to analyze the system performance function. Implicitly, the ANSYS software is used to evaluate the limit state function. The calculation time depends on the number of calls to the calculation code. However, the study describes that the direct coupling between a finite element code and a reliability assessment technique remains too weak for real problems with a large number of degrees of freedom. To deal with this, an efficient method has been proposed that requires a single call to the finite element code. Simulations are presented to demonstrate the validity of the reduced model. The results of the proposed method represent a precision comparable to that of the numerical model, thus allowing a good compromise between precision and calculation time.

2. RELIABILITY ANALYSIS

The study presents an aircraft wing reliability analysis. Figure 2 shows the finite element model built using ANSYS software. It consists of a total number of nodes of 3990 and 3192 elements. The profile surface of the wing is meshed by the PLANE182 elements, which use quadratic elements. The mesh of the solid consists of SOLID185 elements. The wing is modeled from a material, considered isotropic and homogeneous (Young's modulus of 7.3E+10 GPa, density of 2698 kg / m3 and Poisson's ratio of 0.33). The wing has a constant transverse aerofoil profile (Figure 1). The model is embedded in the cross section relative to the origin of the geometry by resetting all degrees of freedom to zero. The dynamic response of the system under study can be evaluated by [4]:

$$(K - \lambda_i M)\phi_i = \{0\} \quad (2)$$

With M and K are the mass and stiffness matrices, while the roots of ($\lambda=\omega 2$) in the problem represent the eigenvalues, and Φ_i the corresponding eigenvector. The vibration modes are revealed after solving the problem of the eigenvalue equation (2). The finite element analysis was performed using the Lanczos Block Method [10], using a computer equipped with an Intel (R) Core (TM) i5-4310U processor with 2.00GHZ 2.60 GHZ and 8 GB RAM. Table 1 and Figure (2) show the modes and Eigen frequencies respectively.

Table 1: Natural frequencies

Mode Shapes	Natural Frequencies (Hz)
1	1.1687
2	5.7145
3	7.2963
4	15.0182
5	20.3657
6	33.2535

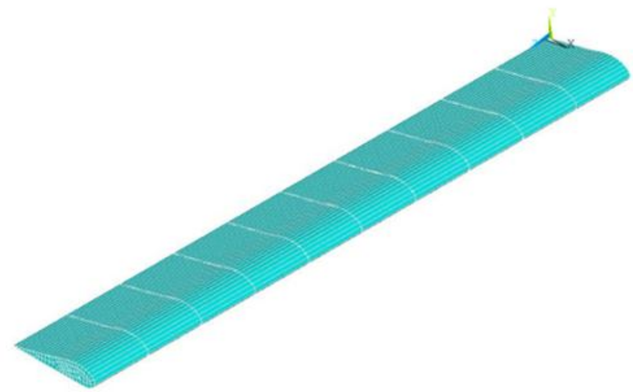

Figure 1: Wing Model.

In reality, any form of analysis model has to cope with a large dispersion [11]. However, a probabilistic study is used to determine the effect of dispersion on the results of the analysis. For the sake of simplicity, only sources of uncertainty related to material properties are discussed in this paper. Just as the variables follow mutually independent normal distributions, their statistics are presented in Table 2.

Table 2: Statistical of Parameters

Parameters	Distribution	Means	SD
Youg's Modulus (GPa)	Normal	7.3E+10	2.433E+10
Density (kg / m3)	Normal	2698	674

Reliability analysis focused on the probability of structural failure, not on the phenomena that cause failure, but on how often it occurs. It is therefore not a physical theory, but a theory of probability and statistics [12]. Structural Reliability provides methods to quantify the probability of structural failure, by setting a performance or limit state function G(E, ρ)=0;(a condition outside of a specified design requirement expressed in mathematical form by a limit state function ; equation (3) [13], [14], which depends on the first natural frequency of the wing f_c, and limited by the frequency (f_v=2.5 Hz).

$$G(E,\rho) = f_c - f_v \quad (3)$$

With (E, ρ) are called basic variables that represent uncertainties about the physical properties of the wing (Young's modulus, density) [13].

Figure (3) shows the reliability algorithm used in this work, which link the MONTE-CARLO reliability analysis (MCS), developed under MATLAB with the finite element code (ANSYS). The algorithm represents a procedure that draws samples directly from the probability distributions of the random variables. This procedure is very robust and relatively easy to apply. However, it requires a large number of evaluations of the performance of the random variables in order to estimate reliable results, as shown in Table 3.

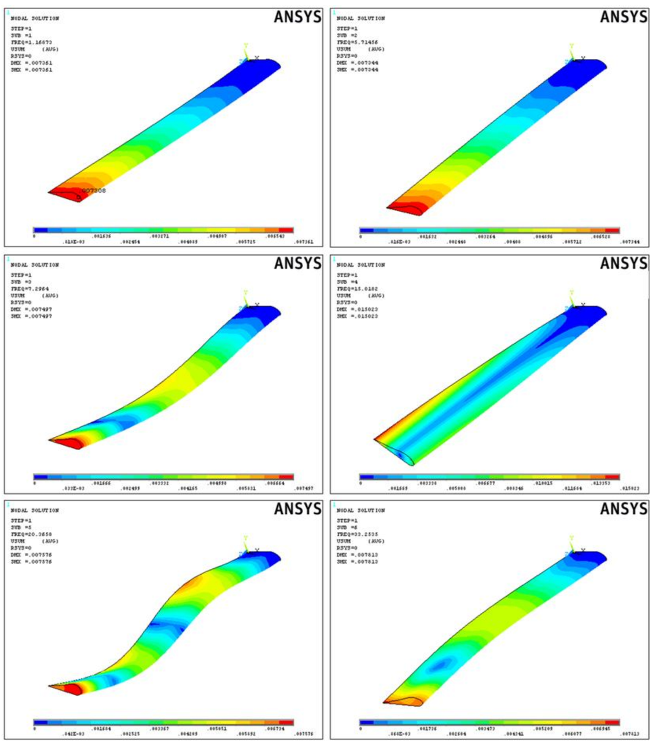

Figure 2: The firsts mode Shape of the Wing.

Table 3 shows the disadvantages inherent in probability calculus - the enormous computational effort for problems involving a low probability of failure or problems that require a considerable amount of computation in each sampling cycle.

In order to solve this disadvantage, we propose an efficient method, if we keep in mind that the measure of efficiency of the method is quantified by the number of calls to the finite element calculation code [15].

3. PROPOSED METHOD

In order to overcome the above-mentioned disadvantage and to reduce to an acceptable level the computational effort required for reliability analysis in dynamic structures, several techniques can be used in conjunction with reliability methods. This paper presents a procedure based on generalized matrices applied to establish a relationship between input variables and output parameters so that the performance function can be evaluated. To achieve this, a condensation method has been developed that reduces the number of calls to the FEM code in a single call.

As a result, the wing model is determined using ANSYS. The elementary mass and stiffness matrices that characterize the model are extracted from the FEM code (see Figure 4). The second part allows the calculation of generalized matrices. Based on the orthogonality properties of modal analysis.

These properties allow the decoupling of equilibrium equations, and are used in analytical resolutions, but especially today in finite element calculation codes. Equation (4) is written using the associated eigenmodes ($\overline{Y_i}$)

$$\overline{K}\,\overline{Y_i} = \omega_i^2\,\overline{M}\,\overline{Y_i} \quad (4)$$

The physical meaning of the orthogonality relations is that the virtual work of the inertial forces of mode i during a displacement according to mode j is zero, as well as the virtual work of the elastic forces of mode i during a displacement according to mode j is zero. It is possible to define the generalized mass of the mode under consideration and the corresponding generalized stiffness which are related by [16].

$$k = \omega^2 m \quad (5)$$

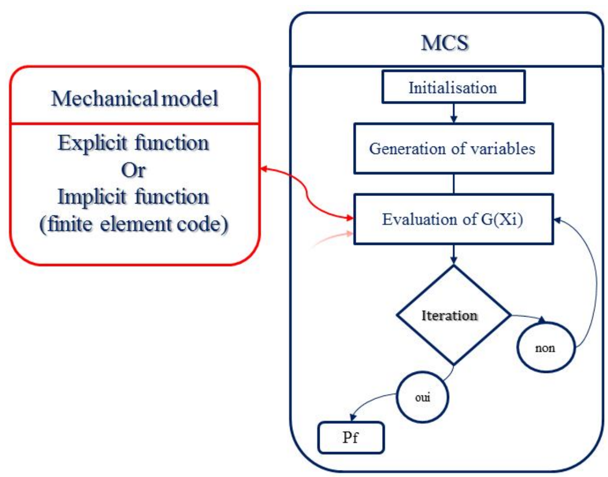

Figure 3: The mechano-reliability coupling algorithm.

Note that these two quantities are set to a pre constant. The amplitude can be raised by choosing a standard. We set the generalized mass m to unity. The relations between mass and stiffness become:

$$\frac{\overline{Y}^t\,\overline{K}\,\overline{Y}}{\overline{Y}^t\,\overline{M}\,\overline{Y}} = k / m = \omega^2 \Rightarrow \overline{Y}^t\,\overline{K}\,\overline{Y} = \omega^2 \quad (6)$$

Table 3: Wing's Reliability Study Results

Parameters	EFM coupled model			
	MCS	MCS	MCS	MCS
Reliability index β	3.7190	3.2905	3.0902	3,0029
Reliability %	99.99	99.95	99.90	99,866
Failure Probability	0.0001	0.0005	0.001	0,0013
Call to FEM	10E+2	10E+3	10E+4	10E+5
Computing time (s)	2782.7	27836.1	269370	276894

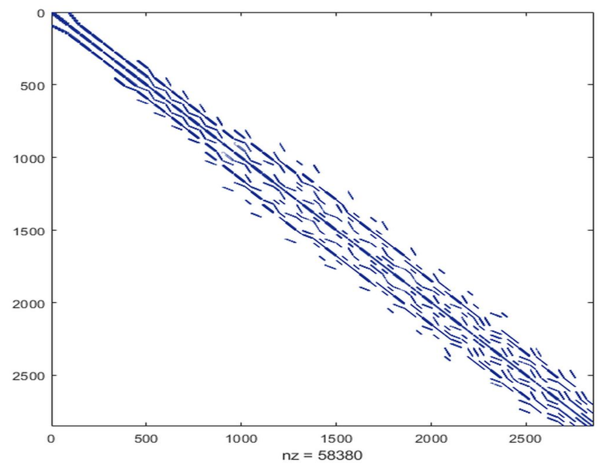

Figure 4: The mechano-reliability coupling algorithm.

ω^2 is multiple roots; there will be as many linearly independent eigenvectors as the degree of a multiplicity of that root. Therefore, the orthogonality relations remain valid. Finally, it should be noted that only the relationship between stiffness and generalized mass is important since both quantities are defined with a constant pre.

$$\overline{Y}^t\,\overline{K}\,\overline{Y} = \omega_i^2 \delta_{ij} \quad (7)$$

$$\overline{Y}^t\,\overline{M}\,\overline{Y} = \delta_{ij} \quad (8)$$

δ_{ij} is the symbol for Kronecker [16]. Probabilistic modelling consists in modelling the mass and stiffness matrix by random variable matrices, by modifying the mechanical properties of the material (Young's modulus and density). The reduced model allows the calculation of the first natural frequency fc defined in the equation (1). Admitting that it is a solution that allows reducing the dynamic amplification levels of the wings, by modifying the properties of materials [1], [14], [16],[17].

We define the generalized mass of the considered mode and the corresponding generalized stiffness which are connected by:

$$\sqrt{\frac{k_1}{m_1}} = \omega_1 = 2\pi f_c \quad (9)$$

It has defined the limit state function (19), where E, ρ, are the input variables that depend on Young's modulus and density respectively.

$$G(E,\rho) = \frac{\omega_1}{2\pi} - f_v \quad (10)$$

The mass random variable and the stiffness random variable defined by:

$$m = \rho . X_1 ; k = E . X_2 \quad (11)$$

Knowing that X1 and X2 are two constants, It is assumed that these two variables are normally distributed with a mean μ_i and a standard deviation σ_i or (i = (E, ρ)). We can show directly that the variables m, k follow a normal distribution with a mean (m_v, k_v) and a standard deviation (σ_m, σ_k) defined by :

$$\begin{cases} k_v = \mu_E . X_2 \\ m_v = \mu_\rho . X_1 \\ \sigma_k = \sigma_E . X_2 \\ \sigma_M = \sigma_\rho . X_1 \end{cases} \quad (12)$$

When calculating the random variables. The reduced model is linked to Monte Carlo simulation to predict the probability of failure (see Figure 5).

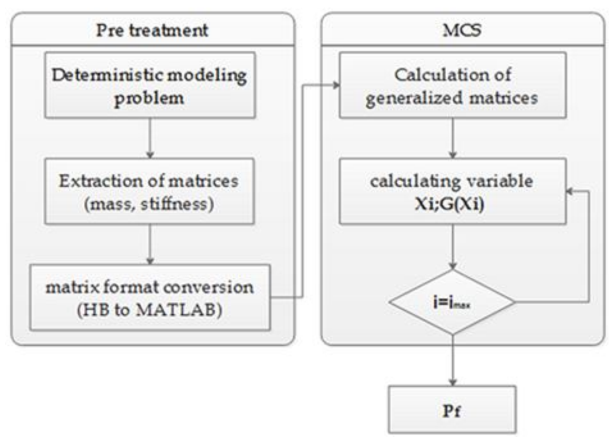

Figure 5: Proposed Algorithm.

Table 4: The wing Natural frequencies.

Mode Shapes	Natural Frequencies (Hz)		
	FEM model	Reduced model	Relative error
1	1.1687	1.1629	0.0049
2	5.7145	5.7692	0.0095
3	7.2963	7.9734	0.0928
4	15.0182	14.9694	0.0032
5	20.3657	20.2869	0.0038
6	33.2535	33.2523	0.00003

The proposed method has a high quality in terms of accuracy and a considerable time saving compared to FEM methods. Table (4) shows the first six Eigen frequencies of the aircraft's wing, calculated by the FEM model and the scale model respectively. The results show a strong agreement between the two methods. Such that the modal frequency of the scale model represents an error of 0.0058 with respect to the FEM code based model.

After reliability study is carried out by coupling the MCS method with the two deterministic models (FEM model and reduced). The number of iterations of the MCS method is fixed at 10^5. Figure 6, shows the density distribution of the first frequency calculated using the MCS coupled to the FEM code method (MCS_{FEM}) and the corresponding MCS coupled to the proposed model (MCS_{MG}). The standard distribution is used to generate stiffness and mass disturbances, using Young's modulus and density statistics. As shown in the figure, the vibration frequency increases around the midpoint equal to 1.1641 Hz, with a maximum limit of 2.5 Hz. Both distributions represent a very good agreement with a relative error around 0.0058 on average.

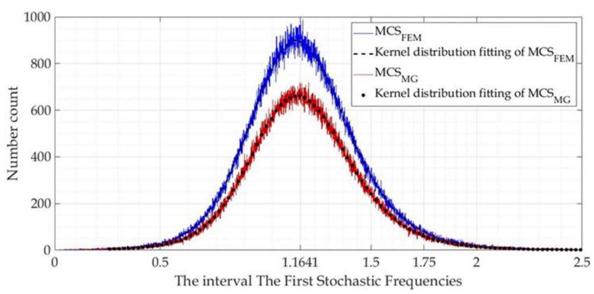

Figure 6: The first frequency density distribution.

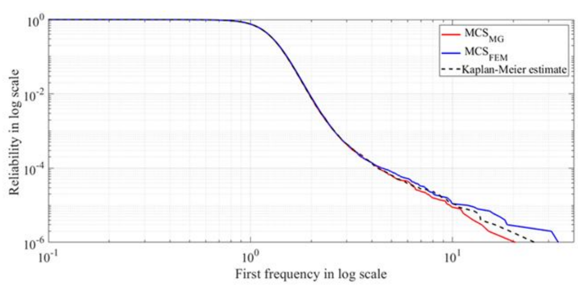

Figure 7: Reliability distribution

Table 4. The results of reliability analyze.

Parameters	EFM coupled model	Reduced coupled model
Reliability index β	3,0029	3,0124
Reliability	99,866	99,871
Failure Probability	0,0013	0,0013
Nombre d'exécution FEM	1E+6	1
Computing time (s)	276894	15,767

Figure 7 illustrates the assignment of reliability frequency responses using the empirical cumulative distribution function CDF. Reliability is assessed from the first frequency distribution, using MCS generation data coupled to the two methods. Taking this into account, the failure range gradually increases with decreasing reliability. The proposed method regularly converges towards an accurate result. The variation in reliability is qualitatively similar to that reported by the MCSFEM, confirming the applicability of the reduced model to structural reliability studies.

Table (4) presents a comparison of the results obtained by the two methods. We note small differences between the probabilistic reliability approximations for the reliability index estimation β and the probability of failure the reference [18], reports that the number of calls to the FEM code can measure the effectiveness of a probabilistic method. The proposed method has reasonable effectiveness compared to the EFM-based method.

Figure 8 shows that the reduced model represents a gain in computing time compared to the FEM model. The Monte-Carlo simulation acquired a little 3 days of calculation with the FEM method knowing that the reduced model took only 15 seconds of calculation. The reduced model allows a good compromise between the criteria of robustness offered, efficiency and desired precision and a considerable time saving.

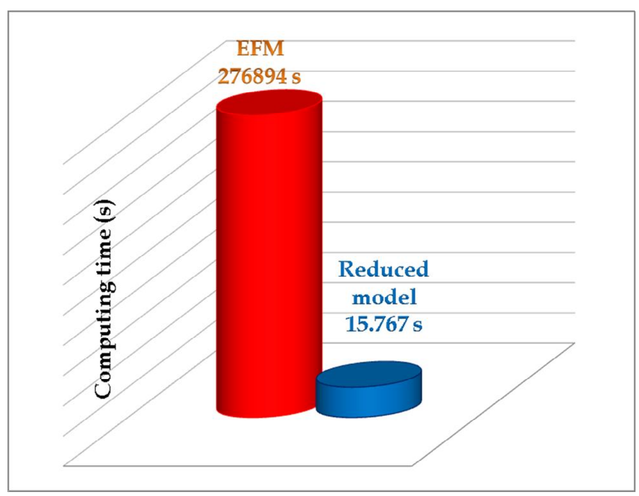

Figure 8: Calculation time comparison

4. CONCLUSION

In this study, a probabilistic analysis was conducted for an aircraft wing. Two reliability approaches were developed to assess the reliability index. The first is based on the direct coupling of the probabilistic MCS method with the deterministic model based on the finite element method. The second is based on an efficient condensation method that requires only one call to the finite element code. The developed method is then linked to MCS to predict the probability of failure. A quantitative reliability study was conducted to evaluate the efficiency and accuracy of the proposed models. With regard to the results of the analysis, it can be concluded that the proposed scaled-down model can effectively capture statistical properties in less time. The proposed method is of high quality in terms of accuracy and considerable time savings.

REFERENCES

1. N. Salvat, A. Batailly, and M. Legrand, “Modeling of Abradable Coating Removal in Aircraft Engines Through Delay Differential Equations,” Journal of Engineering for Gas Turbines and Power, vol. 135, no. 10, p. 102102, Aug. 2013. https://doi.org/10.1115/1.4024959
2. Rohit Bharat Patil et al, “Analysis Of Four Wheeler Front Automotive Axle Through Finite Element Analysis” , International Journal of Emerging Trends in Engineering Research, vol. 4, no. 4, April. 2016.
3. T. Kondaiah et al, “Shape And Material Optimization Of A Two Wheeler Front Suspension Frame For Pipe Type And Rectangular Cross Sections” , International Journal of Emerging Trends in Engineering Research, vol. 4, no. 6, June. 2016.
4. V. Papadopoulos and M. Papadrakakis, “The effect of material and thickness variability on the buckling load of shells with random initial imperfections,” Computer Methods in Applied Mechanics and Engineering, vol. 194, no. 12–16, pp. 1405–1426, 2005.
5. H. Chebli and C. Soize, “Experimental validation of a nonparametric probabilistic model of nonhomogeneous uncertainties for dynamical systems.,” The Journal of the Acoustical Society of America, vol. 115, no. 2, pp. 697–705, 2004.
6. F. Deheeger, “Couplage mécano-fiabiliste : 2 SMART - méthodologie d’apprentissage stochastique en fiabilité,” Jan. 2008.
7. A. Der Kiureghian, T. Haukaas, and K. Fujimura, “Structural reliability software at the University of California, Berkeley,” Structural Safety, vol. 28, no. 1–2, pp. 44–67, Jan. 2006.
8. J. Bect, D. Ginsbourger, L. Li, V. Picheny, and E. Vazquez, “Sequential design of computer experiments for the estimation of a probability of failure,” Statistics and Computing, vol. 22, no. 3, pp. 773–793, May 2012. https://doi.org/10.1007/s11222-011-9241-4
9. J.-M. Bourinet, “Reliability analysis and optimal design under uncertainty - Focus on adaptive surrogate-based approaches,” Jan. 2018.
10. P. Kohnke, “ANSYS Theory Reference - Release 5.6.” p. 1286, 1999.
11. K. Shanmugam and P. Balaban, “A Modified Monte-Carlo Simulation Technique for the Evaluation of Error Rate in Digital Communication Systems,” IEEE Transactions on Communications, vol. 28, no. 11, pp. 1916–1924, Nov. 1980.
12. R. El Maani, “Vibratory Reliability Analysis of an Aircraft ’ s Wing via Fluid – Structure Interactions,” no. August, 2017.
13. R. El Maani, A. Makhloufi, B. Radi, and A. El Hami, “Reliability-based design optimization with frequency constraints using a new safest point approach,” Engineering Optimization, vol. 50, no. 10, pp. 1715–1732, 2018.
14. A. H. Elhewy, E. Mesbahi, and Y. Pu, “Reliability analysis of structures using neural network method,” vol. 21, pp. 44–53, 2006.
15. T. Gmür, Dynamique des structures : analyse modale numérique. Lausanne: Presses polytechniques et universitaires romandes, 1997.
16. J. H. Suh, S. W. Hong, and C. W. Lee, “A generalized modal analysis of asymmetrical rotor system using modulated coordinates,” p. N1072, 2003.
17. B. Weiler, “Finite element method based analysis and modeling in rotordynamics.”
18. A. Alaimo, A. Esposito, A. Messineo, C. Orlando, and D. Tumino, “3D CFD analysis of a vertical axis wind turbine,” Energies, vol. 8, no. 4, pp. 3013–3033, 2015. https://doi.org/10.3390/en8043013