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Results with Matlab coding of Middle Graph of Cycle and its related graphs in context of Sum Divisor Cordial

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ABSTRACT

In this paper we have investigated that middle graph of cycle related graphs are sum divisor cordial. Middle graph is obtained by adding a vertex between two vertices. A sum divisor cordial labelling of graph G with V as the set of vertices and function f is bijective from V to { 1,2,…, |v|}, to assing each edge *uv*, we label 1 if 2 divides *(f(u)+f(v))* and if not we label 0 and according to definition of cordial labelling $|e_f(0) - e_f(1)| \leq 1$, A graph which satisfy the above conditions is called sum divisor cordial graph. In the present work we have given the MATLAB coding of our results too which is useful for checking the pattern for huge number of *n* .

Key words : Divisor cordial labelling, Sum divisor cordial labelling, Middle graph, cycle, cycle with one chord, cycle with twin chord, MATLAB coding.

1. INTRODUCTION

A vast literature has produced in graph labelling in the last decades. It gives wide range of applications. The graph labelling divided in to two parts qualitative and quantitative labelling. This can be understood from N V V N J Sri Lakshmi et al. [11]. The qualitative labelling of graph used in multifarious in social psychology, energy crises etc. and quantitative labelling of graph used in missile guidance codes, radar location codes, coding theory introduced in B. Vishnu Priya et al. [8] and also x-ray crystallography, circuit design, networking etc. described in Aaron Don M. Africa et. al. [9]. For standard terminology and notations related to graph theory we refer to Harary[5] and Gross[1].
A. Lourdusamy and F. Patrick [3] introduces the concept of sum divisor cordial graph and the divisor cordial graph introduced by Varatharajan et al. [4].
The concept of middle graph we obtain from Ghodasara [7].

Definition 1.1 (Varatharajan et al. [4]) Let $G = (V, E)$ be a graph and $f : V(G) \to \{1,2,\ldots,|V(G)|\}$ be a bijection. Consider the induced edge function $f*\ (uv): E(G) \to \{0,1\}$ defined as

$$f*(uv) = \begin{cases} 1; & if \frac{f(u)}{f(v)} \quad or \frac{f(v)}{f(u)} \\ 0; & otherwise \end{cases}$$

The function f is called a divisor cordial labeling if $|e_f(0) - e_f(1)| \leq 1$. A graph which admits a divisor cordial labeling is called a divisor cordial graph.

Definition 1.2 (A. Lourdusamy and F. Patrick [3]) Let G = (V,E) be a graph and f : V(G) → {1,2,3,...,|V(G)|} be a bijection. Consider the induced function f∗: E(G)→ {0,1} defined as

$$f*(uv) = \begin{cases} 1; & if \frac{2}{f(u)+f(v)} \\ 0; & otherwise \end{cases}$$

The function f is called a sum divisor cordial labeling if $|e_f(0) - e_f(1) = 0| \leq 1$. A graph which admits a sum divisor cordial labeling is called a sum divisor cordial graph . The definition is also explained by A.Sugumaran and K.Rajesh [6].

Definition 1.3 (J. A. Gallian [2]): A chord of a cycle $C_n\ (n \geq 4)$ is an edge joining two non-adjacent vertices.

Definition 1.4 (J. A. Gallian [2]) Two chords of a cycle $C_n\ (n \geq 5)$ are said to be twin chords if they form a triangle with an edge of C_n . For positive integers n and p with $5 \leq p + 2 \leq n$, C_n ,p denotes cycle C_n with twin chords where chords form cycles C_p , C_3 and C_{n+1-p} without chords with the edges of C_n .
For MATLAB coding we refer our own paper Sangeeta [10].

2. RESULTS

Theorem 2.1. The middle graph of the cycle is sum divisor cordial except $n \equiv 1, 3(mod\ 4)$ **or** $m \equiv 2(mod\ 4)$**.**

Proof : Let C_n be the *n* vertices of cycle, the middle graph of the cycle obtained by adding a vertex between each vertices of the cycle .Let $V(C_n)=\{v_1,v_2,\ldots,v_n\}$then $|V(M_m)|= 2n$, the vertices of middle graph of cycle will be always even since we have doubled it, and $|E(M_m)|= 2n$.

For

$\boldsymbol{C_n, n \equiv 1(mod\ 4) \rightarrow M_m\ , m \equiv 2(mod\ 4)}$

$\boldsymbol{C_n, n \equiv 0(mod\ 4) \rightarrow M_m\ , m \equiv 0(mod\ 4)}$

$\boldsymbol{C_n, n \equiv 2(mod\ 4) \rightarrow M_m\ , m \equiv 0(mod\ 4)}$

$\boldsymbol{C_n, n \equiv 3(mod\ 4) \rightarrow M_m\ , m \equiv 2(mod\ 4)}$

Thus labelling pattern exist on only two cases. We define vertex labelling function f : $V(M_m) \rightarrow \{1,2,3,...,2n\}$ as follows:

Case 1: For $\boldsymbol{n \equiv 0, 2(mod\ 4)}$ **or** $\boldsymbol{m \equiv 0(mod\ 4)}$**, For 1≤ i ≤ m:**

$$f(v_i) = \begin{cases} i; & i = 0,1(mod4) \\ i+1; & i = 2(mod4) \\ i-1; & i = 3(mod4) \end{cases}$$

Case 2: For $\boldsymbol{n \equiv 1, 3(mod\ 4)}$ **or** $\boldsymbol{m \equiv 2(mod\ 4)}$**, For 1≤ i ≤ m,**

Here M_m is not sum divisor cordial graph. In order to meet the definition of sum divisor cordial it must assure the edge condition $|e_f(0) - e_f(1)| \leq 1$, which is violating here.

Table 1: Edge Condition

Cases for n	Cases for m	Edge condition
$\boldsymbol{0, 2(mod\ 4)}$	$\boldsymbol{0(mod\ 4)}$	$\lvert e_f(0) - e_f(1)\rvert = 0$
$\boldsymbol{1, 3(mod\ 4)}$	$\boldsymbol{2(mod\ 4)}$	$\lvert e_f(0) - e_f(1) = 0\rvert \nleq 1$

Thus M_m, middle graph of the cycle is sum divisor cordial except $\boldsymbol{n \equiv 1, 3(mod\ 4)}$ **or** $\boldsymbol{m \equiv 2(mod\ 4)}$**.**

2.1.1 MATLAB coding of the above result

```
a=[];v=[];e=[];w=[];M=[];D=[];n=6;
if(rem(n,4)==0)|(rem(n,4)==2)
i=1;x=1;
n=2*n;
while i<=n-3
      a(x,:)=[i,i+2,i+1,i+3]; x=x+1;i=i+4;
   continue;
end
y=reshape(a.',1,[]);j=1;
for i=1:2:n
   v(j)=y(i);e(j)=y(i+1); j=j+1;
end
G=graph(v,e);
for i=2:length(v)
   G=addedge(G,v(i),e(i-1));
end
G=addedge(G,y(1),y(n));
if rem((y(1)+y(n)),2)==0
   w(length(y))=1;
else
  w(length(y))=0;
end
for i=2:length(y)
if rem((y(i-1)+y(i)),2)==0
   w(i-1)=1;
else
   w(i-1)=0;
end
end
h=plot(G);camroll(90)
title([' Cycle for n = ' num2str(n)])
disp('The lebling partten of cycle')
disp(y)
disp('edge lebling')
disp(w)
else(rem(n,4)==1)|(rem(n,4)==3);
   disp('Here M_m is not sum divisor cordial graph')
end
```

Example 2.1. Middle graph of $\boldsymbol{C_6}$, $\boldsymbol{n = 2(mod\ 4)}$ is $\boldsymbol{C_{12}}$, $\boldsymbol{n = 0(mod\ 4)}$ shown in Figure 1.

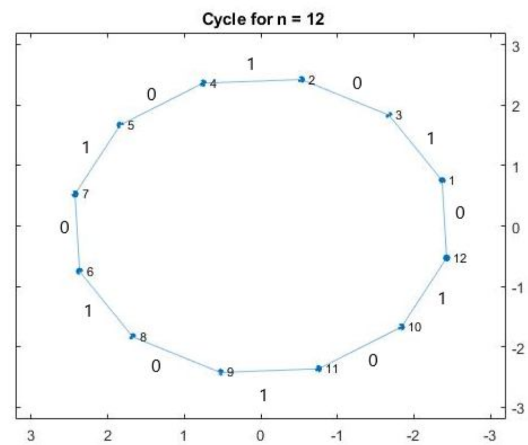

Figure 1: Middle graph of C_6

Theorem 2.2. The middle graph of the cycle with one chord is sum divisor cordial.

Proof : Let C_n be the *n* vertices of cycle, the middle graph of the cycle obtained by adding a vertex between each vertices of the cycle .Let $V(C_n)=\{v_1,v_2,...,v_n\}$then $|V(M_m)|= 2n$, the vertices of middle graph of cycle will be always even since we have doubled it, and $|E(M_m)|= 2n$. The chord of a cycle e = v_1v_n. Thus after middle graph of cycle with one chord are as follows

For

$\boldsymbol{C_{n,1}, n \equiv 1(mod\ 4) \rightarrow M_m\ , m \equiv 2(mod\ 4)}$

$\boldsymbol{C_{n,1}, n \equiv 0(mod\ 4) \rightarrow M_m\ , m \equiv 0(mod\ 4)}$

$\boldsymbol{C_{n,1}, n \equiv 2(mod\ 4) \rightarrow M_m\ , m \equiv 0(mod\ 4)}$

$\boldsymbol{C_{n,1}, n \equiv 3(mod\ 4) \rightarrow M_m\ , m \equiv 2(mod\ 4)}$

We define vertex labelling function f : $V(M_m) \rightarrow \{1,2,3,...,2n\}$ as follows:

Case 1: For $\boldsymbol{n \equiv 0, 2(mod\ 4)}$ **or** $\boldsymbol{m \equiv 0(mod\ 4)}$**, For 1≤ i ≤ m:**

$$f(v_i) = \begin{cases} i; & i = 1,2(mod4) \\ i+1; & i = 3(mod4) \\ i-1; & i = 0(mod4) \end{cases}$$

Case 2: For $\boldsymbol{n \equiv 1, 3(mod\ 4)}$ **or** $\boldsymbol{m \equiv 2(mod\ 4)}$**, For 1≤ i ≤ m:**

$$f(v_i) = \begin{cases} i; & i = 1,2(mod4) \\ i+1; & i = 3(mod4) \\ i-1; & i = 0(mod4) \end{cases}$$

From the above labelling Pattern, we obtain the following results

Table 2: Edge Labeling

Cases for n	Cases for m	Edge condition
$0,2(mod\ 4)$	$0(mod\ 4)$	$\lvert e_f(0) - e_f(1)\rvert = 1$
$1,3(mod\ 4)$	$2(mod\ 4)$	$\lvert e_f(0) - e_f(1) = 1$

Hence M_m , Middle graph of cycle with one chord is a sum divisor cordial graph.

2.2.1 MATLAB coding of the above result

```
a=[];v=[];e=[];w=[];M=[];D=[];n=6;
if(rem(n,4)==0)|(rem(n,4)==2)
i=1;x=1;
n=2*n;
while i<=n-3
      a(x,:)=[i,i+1,i+3,i+2]; x=x+1;i=i+4;
   continue;
end
y=reshape(a.',1,[]);j=1;
for i=1:2:n
   v(j)=y(i);e(j)=y(i+1); j=j+1;
end
G=graph(v,e);
for i=2:length(v)
   G=addedge(G,v(i),e(i-1));
end
G=addedge(G,y(1),y(n));
G=addedge(G,y(1),y(5));
if rem((y(1)+y(n)),2)==0
   w(length(y))=1;
else
  w(length(y))=0;
end
for i=2:length(y)
if rem((y(i-1)+y(i)),2)==0
   w(i-1)=1;
else
   w(i-1)=0;
end
end
h=plot(G);camroll(90)
title([' Cycle with one chord for n = ' num2str(n)])
disp('The lebling partten of cycle with one chord')
disp(y)
disp('edge lebling')
disp(w)
disp('The vertex 1 and 5 are joined by an edge according to defination the lebling of an edge be always 1')
else(rem(n,4)==1)|(rem(n,4)==3);
   i=1;x=1;
n=2*n;
while i<=n-3
      a(x,:)=[i,i+1,i+3,i+2]; x=x+1;i=i+4;
   continue;
end
y=reshape(a.',1,[]);j=1;
y(n-1)=n-1;
y(n)=n;
for i=1:2:n
   v(j)=y(i);e(j)=y(i+1); j=j+1;
end
G=graph(v,e);
for i=2:length(v)
   G=addedge(G,v(i),e(i-1));
end
G=addedge(G,y(1),y(n));
G=addedge(G,y(1),y(5));
if rem((y(1)+y(n)),2)==0
   w(length(y))=1;
else
  w(length(y))=0;
end
for i=2:length(y)
if rem((y(i-1)+y(i)),2)==0
   w(i-1)=1;
else
   w(i-1)=0;
end
end
h=plot(G);camroll(90)
title([' Cycle with one chord for n = ' num2str(n)])
end
disp('The lebling partten of cycle with one chord')
disp(y)
disp('edge lebling')
disp(w)
disp('The vertex 1 and 5 are joined by an edge according to defination the lebling of an edge be always 1')
```

Example 2.2. Middle graph of C_6 with one chord, $n = 2(mod\ 4)$ is C_{12} with one chord, $n = 0(mod\ 4)$ shown in Figure 2.

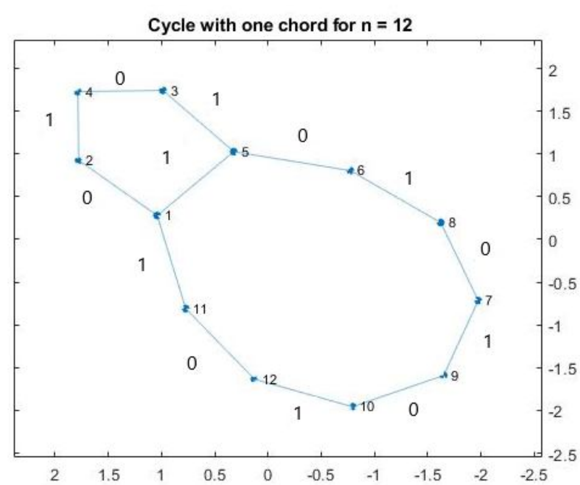

Figure 2: Middle graph of C_6 with one chord

Example 2.3. Middle graph of C_7 with one chord, $n = 3(mod\ 4)$ is C_{14} with one chord, $n = 2(mod\ 4)$ shown in Figure 3.

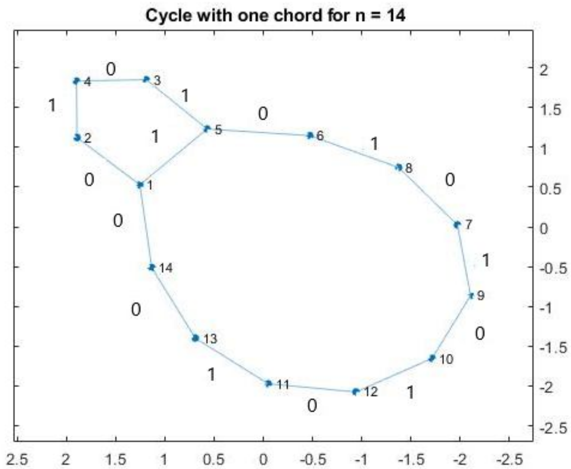

Figure 3: Middle graph of C_7 with one chord

3.CONCLUSION

In this paper we have investigated the graph labeling of middle graph and did the coding of the same. To investigate analogous results for different graphs and graphs using graph operations is an open area of research. It is also interesting to find the divisor cordial graphs which are sum divisor cordial and vice versa.

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