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# Determination of Parameters of High Density Pedestrian Flow upon Formed Traffic in Certain Direction

**Natalya Alexandrovna Naumova**
Kuban State Technological University, Krasnodar, Russia

## ABSTRACT

Simulation of pedestrian flows is an urgent subject for investigations since it has been applied in various fields during recent decades. Development of mathematical model is important in order to determine parameters of high density pedestrian traffic to control it in real time. This work presents pedestrian flow as a random Palm flow. Distribution of time intervals in the flow is approximated by the Erlang law of the sixth order. Using the renewal theory for random processes, analytical tools were developed to predict the number of pedestrians arriving at fixed point during preset time interval. Using the methods of queue theory and theory of random processes, analytical tools were developed to determine average number of pedestrians in front of emergency exits and properties of queue length. Herewith, both the cases of single pedestrian flow and of convergence of several flows into one were considered. Probability functions were derived characterizing cumulative pedestrian flow. Determination of flow parameters by experimental data is described. The developed analytical tools, using minimum amount of initial data, allow to predict online the intensity and to select optimum redirection of pedestrian flows during evacuation.

**Key words:** pedestrian flow, mathematical model, random process, emergency scenario, traffic management.

## 1. INTRODUCTION

Simulation of pedestrian flows is an urgent subject for investigations since it has been applied in various fields during recent decades. The regularities of traffic of large human groups are especially important for simulation of emergency scenarios as well as for management of traffic along street and road network.

Mathematical models can describe flow dynamics at various levels: microscopic, mesoscopic, and macroscopic [1]. Microscopic models take into account behavior of each individual in the flow [2]. It is possible to highlight the Social force model [3] and the model of cellular automata [3, 4]. At macroscopic level, it was proposed to describe the flows using gas dynamic and kinetic model [5]. Software and algorithms are available based on both types of the models allowing to solve local and global tasks by exchange of results and data inside software [4, 6, 7]. The difficulty is that initial data for models of various levels are absolutely different. This results in significant difficulties, since acquisition of large arrays of initial data is a complicated task itself.

For a separate class of tasks, it is required to simulate flow at certain average level, when all traffic participants are taken into account aiming at formation of parameters of total flow. Such models are known as mesoscopic.

***An urgent task*** is the development of mathematical model to determine parameters of high density pedestrian traffic to control it in real time.

***This work is aimed at*** development of mesoscopic model of high density pedestrian flow, allowing to predict online the parameters of such flow during evacuation using minimum amount of initial data.

## 2. METHODS

While simulating pedestrian flows, researchers are often based on similar models of transport flows. It is acceptable not for all types of solved problems since human flow is less organized and exposed to numerous random factors [1, 8, 9]. According to established opinions, there exists the effect of crowd self-organization occurring without external impact over certain time [2]. In this work we attempt to describe high density flow formed during evacuation, moving in certain direction to exit. In this case, we can be based on the methods and approaches used during development of TIMeR_Mod transport flows [10].

## 3. RESULTS AND DISCUSSION

The hypothesis of normal distribution of pedestrian flow moving in certain direction was experimentally proved by numerous researchers. However, while describing pedestrian flow as a random flow of events, the normal law of distribution can hardly be applied. The Erlang law at $k \geq 5$ is close to normal and corresponds to high density flows of events [11]. Therefore, in the case of evacuation of people from places of mass gathering along narrow passages, we will approximate the pedestrian flow (as well as transport one) using the Erlang law.

The distribution density of the special Erlang law (or just *Erlang law*) is as follows [11]:

$$f^{(k)}(t) = \lambda(\lambda t)^{k-1} e^{-\lambda t} / (k-1), \quad (t > 0) \tag{1}$$

The Erlang distribution function of the $k$-th order is as follows:

$$F^{(k)}(t) = 1 - \sum_{n=0}^{k-1} \left( (\lambda t)^n e^{-\lambda t} \right) / n! = 1 - R(k-1, \lambda t), \quad (t > 0) \tag{2}$$

Mathematical expectation $M(T)$ and dispersion $D(T)$ in this case are, respectively:

$$M(T) = \frac{k}{\lambda}; \; D(T) = \frac{k}{\lambda^2} \tag{3}$$

### 3.1. Presentation of Pedestrian Flow as Palm Flow

Let us apply the renewal theory [12] for simulation of pedestrian flow as a random flow of events: the Palm flow. We assume that the flow is already organized and moves to certain direction. A random event is arrival of pedestrian to space point with fixed coordinates along the direction axis. The interval between random events is the time interval between two consecutive arrivals to the fixed space point of two consecutive pedestrians in the flow. Let us consider the renewal function $H(t) = M(N_t)$: the mathematical expectance of the number of events in the time $t$ in the case of (special) Erlang distribution. The image of distribution function of time intervals in the case of the Erlang distribution is as follows [12]:

$$f^*(s) = \frac{\lambda^k}{(\lambda + s)^k} \tag{4}$$

The Laplace transform for this function is:

$$H^*(s) = \frac{\lambda^k}{s\left((\lambda + s)^k - \lambda^k\right)} \tag{5}$$

$H^*(s)$ can be expanded into simple fractions containing the following terms:

1) from the pole $s = 0$;
2) from the nonzero poles in the roots of the equation $f^*(s) = 1$.

Let us determine the roots of this equation:

$$\frac{\lambda^k}{(\lambda + s)^k} = 1; \; (\lambda + s)^k = \lambda^k \tag{6}$$

The nonzero roots are as follows ($i$ is the imaginary unit):

$$s_p = \lambda \cdot \left( e^{2\pi p i / k} - 1 \right), \quad p = 1, 2, ..., k-1 \tag{7}$$

Each simple nonzero root $s_p$ in the expansion $H^*(s)$ corresponds to the fraction:

$$\frac{-1}{s_p \cdot \left(f^*(s_p)\right)' \cdot (s - s_p)} = \frac{-1}{\left( s_p \left( -\frac{k\lambda^k}{(\lambda + s_p)^{k+1}} \right) \cdot (s - s_p) \right)} = \frac{\lambda + s_p}{k \cdot s_p \cdot (s - s_p)} \tag{8}$$

That is:

$$H^*(s) = \frac{\lambda}{ks^2} + \frac{1}{s} \cdot \frac{1-k}{2k} - \sum_{p=1}^{k-1} \frac{1}{s_p\left(f^*(s_p)\right)'(s - s_p)} = \frac{\lambda}{ks^2} + \frac{1}{s} \cdot \frac{1-k}{2k} + \sum_{p=1}^{k-1} \frac{\lambda + s_p}{k \cdot s_p \cdot (s - s_p)} \tag{9}$$

From this, using the tables, we determine the original, that is, the renewal function $H(t)$: the number of events occurring during the time interval (*0; t*). From the theory of operator calculus, it is known that each fraction $\frac{1}{s - s_p}$ corresponds to the original $e^{s_p t}$.

In the case of heavy density pedestrian flows, the distribution law is close to normal and it can be approximated by the (special) Erlang law of at least the fifth order.

In order to obtain the accurate analytical assignment of the renewal function, let us consider $k = 6$.

The roots of $(\lambda + s)^6 = \lambda^6$ in this case are as follows:

$$s_p = \lambda \cdot \left( e^{\frac{2\pi p}{6} i} - 1 \right) = \lambda \cdot \left( e^{\frac{\pi p}{3} i} - 1 \right), \; p = 1, 2, ..., k-1 \tag{10}$$

$$s_1 = \lambda \cdot \left( e^{\frac{\pi}{3} i} - 1 \right) = \lambda \cdot \left( -\frac{1}{2} + i \frac{\sqrt{3}}{2} \right)$$

$$s_2 = \lambda \cdot \left( e^{\frac{2\pi}{3} i} - 1 \right) = \lambda \cdot \left( -\frac{3}{2} + i \frac{\sqrt{3}}{2} \right)$$

$$s_3 = \lambda \cdot \left( e^{\frac{3\pi}{3} i} - 1 \right) = \lambda \cdot (-2)$$

$$s_4 = \lambda \cdot \left( e^{\frac{4\pi}{3} i} - 1 \right) = \lambda \cdot \left( -\frac{3}{2} - i \frac{\sqrt{3}}{2} \right)$$

$$s_5 = \lambda \cdot \left( e^{\frac{5\pi}{3} i} - 1 \right) = \lambda \cdot \left( -\frac{1}{2} - i \frac{\sqrt{3}}{2} \right)$$

The image of renewal function at $k = 6$ is as follows:

$$H^*(s) = \frac{\lambda}{6s^2} + \frac{1}{s} \cdot \frac{-5}{12} - \sum_{p=1}^{5} \frac{1}{s_p \left( f^*(s_p) \right)^{/} (s - s_p)} =$$
$$= \frac{\lambda}{6s^2} + \frac{1}{s} \cdot \frac{-5}{12} + \sum_{p=1}^{5} \frac{\lambda + s_p}{6 \cdot s_p \cdot (s - s_p)} = \frac{\lambda}{6s^2} + \frac{1}{s} \cdot \frac{-5}{12} + R^*(s) \qquad (11)$$

Let us express $R^*(s)$:

$$R^*(s) = \frac{\lambda + \lambda\left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)}{6\lambda \cdot \left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)\left(s - \lambda\left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)\right)} + \frac{\lambda + \lambda\left(-\frac{1}{2} - i\frac{\sqrt{3}}{2}\right)}{6\lambda \cdot \left(-\frac{1}{2} - i\frac{\sqrt{3}}{2}\right)\left(s - \lambda\left(-\frac{1}{2} - i\frac{\sqrt{3}}{2}\right)\right)} +$$
$$+ \frac{\lambda + \lambda(-2)}{6\lambda \cdot (-2)\left(s - \lambda(-2)\right)} + \frac{\lambda + \lambda\left(-\frac{3}{2} + i\frac{\sqrt{3}}{2}\right)}{6\lambda \cdot \left(-\frac{3}{2} + i\frac{\sqrt{3}}{2}\right)\left(s - \lambda\left(-\frac{3}{2} + i\frac{\sqrt{3}}{2}\right)\right)} +$$
$$+ \frac{\lambda + \lambda\left(-\frac{3}{2} - i\frac{\sqrt{3}}{2}\right)}{6\lambda \cdot \left(-\frac{3}{2} - i\frac{\sqrt{3}}{2}\right)\left(s - \lambda\left(-\frac{3}{2} - i\frac{\sqrt{3}}{2}\right)\right)}$$

(12)

After simplifying and rewriting it in more convenient form for determination of the original, we obtain the following:

$$R^{*}(s)=\frac{1}{6}\frac{1+i\sqrt{3}}{-1+i\sqrt{3}}\frac{1}{\left(s-\lambda\left(-\frac{1}{2}+i\frac{\sqrt{3}}{2}\right)\right)}+\frac{1}{6}\frac{1-i\sqrt{3}}{-1-i\sqrt{3}}\frac{1}{\left(s-\lambda\left(-\frac{1}{2}-i\frac{\sqrt{3}}{2}\right)\right)}+$$
$$+\frac{1}{6}\frac{1}{2}\frac{1}{\left(s-\lambda(-2)\right)}+\frac{1}{6}\frac{-1+i\sqrt{3}}{-3+i\sqrt{3}}\frac{1}{\left(s-\lambda\left(-\frac{3}{2}+i\frac{\sqrt{3}}{2}\right)\right)}+\frac{1}{6}\frac{-1-i\sqrt{3}}{-3-i\sqrt{3}}\frac{1}{\left(s-\lambda\left(-\frac{3}{2}-i\frac{\sqrt{3}}{2}\right)\right)} \quad (13)$$

Let us determine the original by its image:

$$R(t)=\frac{1}{6}\frac{1+i\sqrt{3}}{-1+i\sqrt{3}}e^{\lambda\left(-\frac{1}{2}+i\frac{\sqrt{3}}{2}\right)t}+\frac{1}{6}\frac{1-i\sqrt{3}}{-1-i\sqrt{3}}e^{\lambda\left(-\frac{1}{2}-i\frac{\sqrt{3}}{2}\right)t}+\frac{1}{6}\frac{1}{2}e^{-2\lambda t}+$$
$$+\frac{1}{6}\frac{-1+i\sqrt{3}}{-3+i\sqrt{3}}e^{\lambda\left(-\frac{3}{2}+i\frac{\sqrt{3}}{2}\right)t}+\frac{1}{6}\frac{-1-i\sqrt{3}}{-3-i\sqrt{3}}e^{\lambda\left(-\frac{3}{2}-i\frac{\sqrt{3}}{2}\right)t} \quad (14)$$

Then we obtain:

$$R(t)==\frac{1}{6}\cdot\frac{1}{-4}e^{\lambda\left(-\frac{1}{2}\right)t}\left(\left(1+i\sqrt{3}\right)^{2}\left(\cos\left(\frac{\sqrt{3}}{2}\lambda t\right)+i\sin\left(\frac{\sqrt{3}}{2}\lambda t\right)\right)+\left(1-i\sqrt{3}\right)^{2}\left(\cos\left(\frac{\sqrt{3}}{2}\lambda t\right)-i\sin\left(\frac{\sqrt{3}}{2}\lambda t\right)\right)\right)+$$
$$+\frac{1}{6}\frac{1}{2}e^{-2\lambda t}+$$
$$+\frac{1}{6}\cdot\frac{1}{12}e^{\lambda\left(-\frac{3}{2}\right)t}\left(\left(6-i\cdot 2\sqrt{3}\right)\left(\cos\left(\frac{\sqrt{3}}{2}\lambda\right)t+i\sin\left(\frac{\sqrt{3}}{2}\lambda t\right)\right)+\left(6+i\cdot 2\sqrt{3}\right)\left(\cos\left(\frac{\sqrt{3}}{2}\lambda t\right)-i\sin\left(\frac{\sqrt{3}}{2}\lambda t\right)\right)\right)$$

After simplification and reduction of similar terms, $R(t)$ is as follows:

$$R(t)=\frac{1}{12}e^{-2\lambda t}-\frac{1}{24}e^{\lambda\left(-\frac{1}{2}\right)t}\cdot\left[-4\cos\left(\frac{\sqrt{3}}{2}\lambda t\right)-4\sqrt{3}\sin\left(\frac{\sqrt{3}}{2}\lambda t\right)\right]+$$
$$+\frac{1}{6}\cdot\frac{1}{12}e^{\lambda\left(-\frac{3}{2}\right)t}\cdot\left[12\cos\left(\frac{\sqrt{3}}{2}\lambda t\right)+4\sqrt{3}\sin\left(\frac{\sqrt{3}}{2}\lambda t\right)\right] \quad (15)$$

Then, the renewal function at $k = 6$ is as follows:

$$H(t)=\frac{\lambda}{6}t-\frac{5}{12}+\frac{1}{12}e^{-2\lambda t}+\frac{1}{6}e^{\lambda\left(-\frac{1}{2}\right)t}\cdot\left[\cos\left(\frac{\sqrt{3}}{2}\lambda t\right)+\sqrt{3}\sin\left(\frac{\sqrt{3}}{2}\lambda t\right)\right]+$$
$$+\frac{1}{18}e^{\lambda\left(-\frac{3}{2}\right)t}\cdot\left[3\cos\left(\frac{\sqrt{3}}{2}\lambda t\right)+\sqrt{3}\sin\left(\frac{\sqrt{3}}{2}\lambda t\right)\right] \quad (16)$$

Equation (16) presets $H(t)$ equaling to the number of pedestrians arriving at fixed point in $t$ s. In the case of emergency scenarios, $H(t)$ equals numerically to the number of people arriving at closed emergency exit in $t$ s. In the case of simulation of street and road traffic, $H(t)$ determines the number of pedestrians arriving at crosswalk during active red traffic light [13, 14].

### 3.2. Convergence of Several Pedestrian Flows in Front of Closed Exit

Let us consider the case when in front of the emergency exit $s$ pedestrian flows are converged. According to the renewal theory [12], for average number of renewals in the interval $(0;t)$ in the converged process the following is valid:

$$H(t)=\sum_{i=1}^{s}H_i(t) \quad (17)$$

where $s$ is the number of converged processes, $H_i(t)$ is the renewal function of each of them.

That is, if in a certain point of plain $s$ pedestrian flows are converged, then the number of pedestrians crossing this point in the interval $(0;t)$ is calculated as follows:

$$H(t) = \sum_{i=1}^{s} H_i(t) = \sum_{i=1}^{s} \left( \frac{\lambda_i}{6} t - \frac{5}{12} + \frac{1}{12} e^{-2\lambda_i t} + \frac{1}{6} e^{\lambda_i\left(-\frac{1}{2}\right)t} \cdot \left[ \cos\left( \frac{\sqrt{3}}{2} \lambda_i t \right) + \sqrt{3} \sin\left( \frac{\sqrt{3}}{2} \lambda_i t \right) \right] + \right.$$
$$\left. + \frac{1}{18} e^{\lambda_i\left(-\frac{3}{2}\right)t} \cdot \left[ 3\cos\left( \frac{\sqrt{3}}{2} \lambda_i t \right) + \sqrt{3} \sin\left( \frac{\sqrt{3}}{2} \lambda_i t \right) \right] \right) \quad (18)$$

where $\lambda_i$ is the Erlang parameter for the $i-th$ converged flow.

### 3.3. Simulation of Pedestrian Flow as a Mass Service System

In order to determine the evacuation efficiency of pedestrian flows, let us apply the methods of mass service theory and the theory of random processes [11, 15]. Let the service time is distributed according to the exponential law with the parameter μ. The arrivals are distributed according to the generalized Erlang law with the parameters $\lambda_0, \lambda_1, \ldots, \lambda_k$ of the $(k + 1)$ order. Not more than $n$ requests can be serviced simultaneously. As is known, the Erlang flow of the $(k+1)$ order is obtained from the simplest if each $(k+1)$ event is considered and the intermediate $k$ are rejected. Thus, in order to determine the probability that $m$ $(n > m)$ requests are serviced, let us apply the method of pseudostates (Fig. 1):

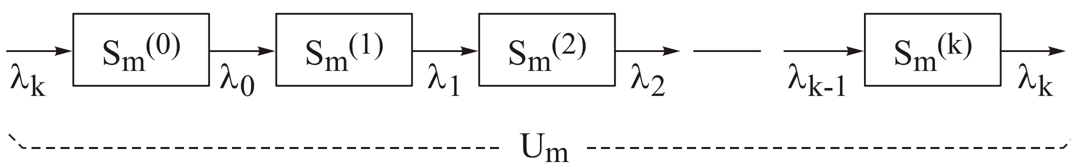


**Figure 1: Pseudostates of Erlang distribution of the *k* + *1* order.**

While considering transport flows in [10], the equations of probability of system existence in this pseudostate were obtained. Therefore, let us apply the ready result.

Let us introduce the following notations:

$p_n(t)$, $p_{n+1}(t)$, …, $p_{n+i}(t)$ are the existence of system in the states $s_0$, $s_1$, …, $s_i$, respectively;

$U_{n+i}$ is the system state, when all $n$ service channels are busy in the queue of $i$ requests;

$p_{n+i}^{(j)}(t)$ is the probability of system existence in transitive state $s_{n+i}^{(j)}(t)\,(j = 1, 2, 3, \ldots, k)$;

$p_{n+i}(t)$ is the probability of existing in queue of $i$ requests at the time $t$.

Differential equations of probability of system existence in transitive states $U_{n+i}$ are as follows:

$$\left(p_{n+i}^{(j)}(t)\right)_t^{/} = -\lambda_j p_{n+i}^{(j)}(t) + \lambda_{j-1} p_{n+i}^{(j-1)}(t) \qquad (j = 1, 2, 3, \ldots, k) \quad (19)$$

The set of differential equations of probabilities of $i$ requests in queue is as follows:

$$p_{n+i}^{/}(t) = -p_{n+i}(t) \cdot (\lambda_0 + n\mu) + p_{n+i-1}^{k}(t) \cdot \lambda_k + n\mu p_{n+i+1}$$
$$(i = 1; 2; 3; \ldots) \quad (20)$$

The differential equation for the interval without queue, and when $m$ $(m \le n)$ service channels are busy (state $U_m$), is as follows:

$$\left(p_m(t)\right)_t^{/} = -(\lambda_0 + m\mu)\, p_m(t) + (m+1)\mu \cdot p_{m+1} + \lambda_k p_{m-1}^{(k)}(t)$$
$$(m = 1, 2, \ldots, n). \quad (21)$$

For the time when the system is completely free, the differential equation is as follows:

$$\left(p_0(t)\right)_t^{/} = -\lambda_0 p_0(t) + \mu\, p_1(t) \quad (22)$$

Let us denote $r_m(t) = P(U_m)$, that is, $r_m(t)$ is the probability of system existence in the state $U_m$. According to the laws of probability theory:

$$r_m(t) = p_m(t) + \sum_{j=1}^{k} p_m^{(j)} \quad (23)$$

Then, the average length of queue at the time $t$ is as follows:

$$M(l(t)) = \sum_{i=1}^{\infty} i \cdot r_{n+i}(t) \quad (24)$$

Solution to this set of differential equations for stationary process is given in [13], hence, we use the ready results:

$$p_0 = \frac{1}{b\left( \sum_{j=0}^{n} \frac{\alpha^j}{j!} + \frac{\alpha^n}{n!} \frac{\alpha / n}{1 - \frac{\alpha}{n}} \right)} \quad (25)$$

$$p_m = \frac{\alpha^m}{m! \cdot b\left( \sum_{j=0}^{n} \frac{\alpha^j}{j!} + \frac{\alpha^n}{n!} \frac{\alpha / n}{1 - \frac{\alpha}{n}} \right)},$$

$$m \in \{0, 1, 2, ..., n-1\} \quad (26)$$

$$p_{n+i} = \frac{\alpha^{n+i}}{n! \cdot n^i \cdot b \left( \sum_{j=0}^{n} \frac{\alpha^j}{j!} + \frac{\alpha^n}{n!} \frac{\alpha / n}{1 - \frac{\alpha}{n}} \right)}, \; i \in \{0, 1, 2, ...\} \quad (27)$$

The efficiency performances of the system at $(\alpha/n) < 1$ are as follows:

1) probability of not more than $s$ requests in queue is:

$$P_{n+s} = \sum_{m=0}^{n} r_m + \sum_{j=1}^{s} r_{n+j} = bp_0 \left( \sum_{m=0}^{n} \frac{\alpha^m}{m!} + \frac{\alpha^n}{n!} \sum_{j=1}^{s} \left( \frac{\alpha}{n} \right)^j \right) \quad (28)$$

2) mathematical expectance of requests in queue is:

$$M(l) = \sum_{j=1}^{\infty} j r_{n+j} = bp_0 \frac{\alpha^{n+1}}{n! \cdot n \cdot \left(1 - \alpha/n\right)^2}. \quad (29)$$

During predictions of efficiency the method of pseudostates was applied, when the time axis was expanded in $(k+1)$ times. Thus, to match the results, let us assume:

$$b = \sum_{i=0}^{k} \frac{\lambda_0}{\lambda_i} > 1, \quad \alpha = \frac{\lambda_0}{\mu}, \quad \mu = (k+1) / (m_z) \quad (30)$$

**3.4. Determination of Parameters of Pedestrian Flows in Front of Emergency Exit**

In Eq. (30), $m_z$ is the average service time of one customer. While considering a pedestrian flow as Palm flow, where the time intervals have (special) Erlang distribution law with the parameters $k = 6$ and $\lambda$, we obtain the following values:

$$b = \sum_{i=0}^{k-1} \frac{\lambda}{\lambda} = k = 6, \quad \alpha = \frac{\lambda}{\mu}, \quad \mu = 6 / (m_z) \quad (31)$$

In order to determine average number of pedestrians in queue in front of emergency exit and probability of not more than $s$ pedestrians in queue, it is necessary to substitute into Eqs. (28) and (29) the values determined by Eq. (31).

Cumulative flow obtained upon convergence of $s$ high density pedestrian flows can be also approximated by the (special) Erlang law of the $k = 6$ order. The second parameter of distribution is determined as the sum of initial terms:

$$\lambda = \sum_{i-1}^{s} \lambda_i. \quad (32)$$

If in this case the average time of evacuation via each $n$ exit is $m_z$, then the average number of people in front of an exit is predicted by the same equation:

$$M(l) = bp_0 \frac{\alpha^{n+1}}{n! \cdot n \cdot \left(1 - \alpha/n\right)^2}, \quad (33)$$

where

$$b = \sum_{i=0}^{k-1} \frac{\lambda}{\lambda} = k = 6,$$

$$\alpha = \frac{\lambda_0}{\mu}, \quad \mu = 6 / (m_z).$$

**3.5. Determination of Distribution Function of Cumulative Flow Obtained upon Convergence of $s$ High Density Pedestrian Flows**

In Section 4, in order to determine the number of pedestrians in queue, we assumed that the cumulative flow also had the Erlang distribution. If it is required to determine the probability that in the time $T_0$ the space point with fixed coordinate in cumulative flow will not be arrived by any pedestrian, then it is required to determine more accurately the distribution function of cumulative flow.

In order to derive the distribution function of time intervals in cumulative flow, let us apply the method described in [11]. Let $\tau^*$ is an arbitrary point in the cumulative flow $\Pi^{(s)}$, not corresponding to any event in independent flows $\Pi_1, \Pi_2, ..., \Pi_s$. Then the time before next event in cumulative flow is $R^{(s)} = \min\{R_1, R_2, ..., R_s\}$.

If the hypothesis about distribution of intervals in flow by the Erlang law is valid, then, according to the theory of random processes, the probabilities that the time $Q$ after the last arrival of vehicle and the time $R$ before the next arrival are lower than certain preset $T_0$ are expressed as follows:

$$P(Q<T_0)=P(R<T_0)=\int_{-\infty}^{T_0}\frac{1-F^{(k)}(t)}{m_t^{(k)}}dt=\int_{-\infty}^{T_0}\frac{\lambda}{k}R(k-1,\lambda t)dt=$$
$$=1-\sum_{n=0}^{k-1}\frac{\lambda}{k}\int_{T_0}^{\infty}P(n,\lambda t)dt=1-\frac{1}{k}\sum_{n=0}^{k-1}R(n,T_0\lambda) \qquad (34)$$

The following notations are used:

$$R(n,a)=\int_a^{\infty}\frac{x^n}{n}e^{-x}dx=\sum_{K=0}^{n}P(k,a);$$
$$P(k,\alpha)=\left(\alpha^k e^{-a}\right)\Big/k! \qquad (35)$$

Hence, the distribution function of time before next event in the Erlang flow of the $k_i$ order is as follows:

$$F_{r_i}(t)=1-\frac{1}{k_i}\sum_{n=0}^{k_i-1}R(n,\lambda_i t) \qquad (36)$$

The distribution function of the minimum of $s$ random values is determined as follows:

$$F_{r^{(s)}}(t)=1-\prod_{i=1}^{s}\left(1-F_{r_i}(t)\right)=1-\prod_{i=1}^{s}\left(\frac{1}{k_i}\sum_{n=0}^{k_i-1}R(n,\lambda_i t)\right) \qquad (37)$$

The distribution function of time intervals $T^{(s)}$ in cumulative flow $\Pi^{(s)}$ is determined as follows:

$$F^{(s)}(t)=1-m^{(s)}(t)\cdot f_{r^{(s)}}(t) \qquad (38)$$

Mathematical expectance of the interval between events in the flow $\Pi_i$ is $m_i=\dfrac{k_i}{\lambda_i}$. Then:

$$m^{(s)}=\frac{1}{\lambda^{(s)}}=\frac{1}{\sum_{i=1}^{s}\lambda_i\big/k_i} \qquad (39)$$

Let us determine the distribution density $f_{r^{(s)}}(t)$:

$$f_{r^{(s)}}(t)=\left(F_{r^{(s)}}(t)\right)_t'=\left(1-\prod_{i=1}^{s}\left(\frac{1}{k_i}\sum_{n=0}^{k_i-1}R(n,\lambda_i t)\right)\right)_t'=$$
$$=-\left(\prod_{i=1}^{s}\left(\frac{1}{k_i}\sum_{n=0}^{k_i-1}\left((\lambda_i t)^n e^{-\lambda_i t}\right)\Big/k_i!\right)\right)_t'=-\left(\prod_{i=1}^{s}\left(\frac{1}{k_i\cdot k_i!}\sum_{n=0}^{k_i-1}(\lambda_i t)^n e^{-\lambda_i t}\right)\right)_t' \qquad (40)$$

*In particular*, if two Palm flows are converged, where the time intervals are distributed according to the (special) Erlang law of the $k=6$ order with the parameters $\lambda_1$ and $\lambda_2$, then:

$$f_{r^{(2)}}(t)=-\left(\prod_{i=1}^{2}\left(\frac{1}{6\cdot 6!}\sum_{n=0}^{5}(\lambda_i t)^n e^{-\lambda_i t}\right)\right)_t'=$$
$$=-\left(\frac{1}{6\cdot 6!}\right)^2\cdot\left[\left(\sum_{n=0}^{5}(\lambda_1 t)^n e^{-\lambda_1 t}\right)_t'\cdot\sum_{n=0}^{5}(\lambda_2 t)^n e^{-\lambda_2 t}+\left(\sum_{n=0}^{5}(\lambda_2 t)^n e^{-\lambda_2 t}\right)_t'\cdot\sum_{n=0}^{5}(\lambda_1 t)^n e^{-\lambda_1 t}\right] \qquad (23)(41)$$

Let us determine the derivative, substitute it into Eq. (23), and simplify:

$$f_{r^{(2)}}(t)=-\left(\frac{1}{6\cdot 6!}\right)^2\cdot e^{-\lambda_1 t}\cdot e^{-\lambda_2 t}\cdot\left[\lambda_1\left(\sum_{n=1}^{5}n\cdot(\lambda_1 t)^{n-1}-\sum_{n=1}^{5}(\lambda_1 t)^n\right)\cdot\sum_{n=0}^{5}(\lambda_2 t)^n+\right.$$
$$\left.+\lambda_2\left(\sum_{n=1}^{5}n\cdot(\lambda_2 t)^{n-1}-\sum_{n=1}^{5}(\lambda_2 t)^n\right)\cdot\sum_{n=0}^{5}(\lambda_1 t)^n\right] \qquad (42)$$

Therefore, the distribution function of time intervals in flow obtained by convergence of two pedestrian flows distributed by the Erlang law of the $k=6$ order with the parameters $\lambda_1$

and $\lambda_2$, is as follows:

$$F^{(2)}(t) = 1 - m^{(2)}(t) \cdot f_{r^{(2)}}(t) \tag{25)(43}$$

where the function $f_{r^{(2)}}(t)$ is determined by Eq. (42), and the mathematical expectance of interval between events is:

$$m^{(2)} = \frac{1}{\lambda^{(2)}} = \frac{6}{\lambda_1 + \lambda_2} \tag{44}$$

The probability that in the time $T_0$ the space point with fixed coordinates is not arrived by any pedestrian is:

$$P(T > T_o) = 1 - F^{(2)}(t) \tag{45}$$

and the probability of reverse event, that is, that at least one pedestrian is arrived, is:

$$P(T < T_o) = F^{(2)}(t) \tag{46}$$

### 3.6. Determination of Parameters of the Erlang Distribution by Experimental Data

Using video detectors, we determine the time intervals $T_i$ (in seconds) between two consecutive passages of pedestrians across the point with fixed coordinates. We calculate selective average of random value $T$ :

$$\overline{T} = \frac{\sum_{i=1}^{m} T_i}{m} \tag{47}$$

The parameter is $k = 6$ , then, the parameter $\lambda$ of the Erlang distribution is as follows:

$$\lambda = \frac{k}{\overline{T}} \tag{48}$$

## 4. CONCLUSION

While planning emergency scenarios or controlling pedestrian flows in emergency situations in online mode, it is required to determine workload of emergency exits. Simulation using macromodels in this case cannot be applied since it is time consuming and requires for large amount of initial data. The proposed in this work analytical approach allows to predict online the workload using minimum amount of initial data, thus, to select optimum variant of redirection of pedestrian flows during evacuation. In addition, it can be applied for management of street and road traffic aiming at determination of crosswalk holdups of vehicles and pedestrians. The advantage of this model in comparison with micromodels is in the rate of computations due to analytical tools.

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