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# Mathematical Simulations and Experiments on the Characterization of Stress-Strain State of Elastic Thin-layer Flexible Joints under Non-stationary Thermal Mechanical Loading

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## ABSTRACT

A mathematical model and a computational algorithm have been developed for studying the stress-strain state (SSS) parameters of thin-layer rubber-metal elements (TRME) used as elastic elements in pipeline compensators. As a part of the computational experiment, estimates were obtained for the distribution of the safety factor minimum values of the rubber-metal package. Tests for the TRME cyclic strength under mechanical tension have been carried out. For verification purposes, a comparative analysis of test data and the results of mathematical modeling by the finite element method (FEM) has been carried out.

**Key words :** mathematic simulation, stress-strain state, finite element method, thin-layer rubber - metal element, pipeline compensator, safety factor, thermal elastic behavior.

## 1. INTRODUCTION

Thin-layer rubber-metal elements (TRME) of various designs are used in the elastic flexible joints, e.g. , - in bridge bearings, helicopter industry (bearings), shipbuilding (elastic elements of compensator in high pressure pipelines and missile technology (elastic bearing joints). With the use of substantial anisotropy of the TRME rigidity characteristics along and across the rubber layers, it is possible to create unique designs with new qualities, for example - extremely high load carrying capacity across the layers and extremely low rigidity along the rubber layers. The pipeline compensators based on flat annular TRME may serve as an example. They have a transient vibration rigidity two orders of magnitude less than conventional pipeline compensators based on bellows, rubber-fabric hoses, rubber-cord shells, in a wide frequency range from zero to hundreds and thousands of hertz ([1] and Figure 1). Accordingly, they provide a much higher vibration isolation through the pipelines construction.

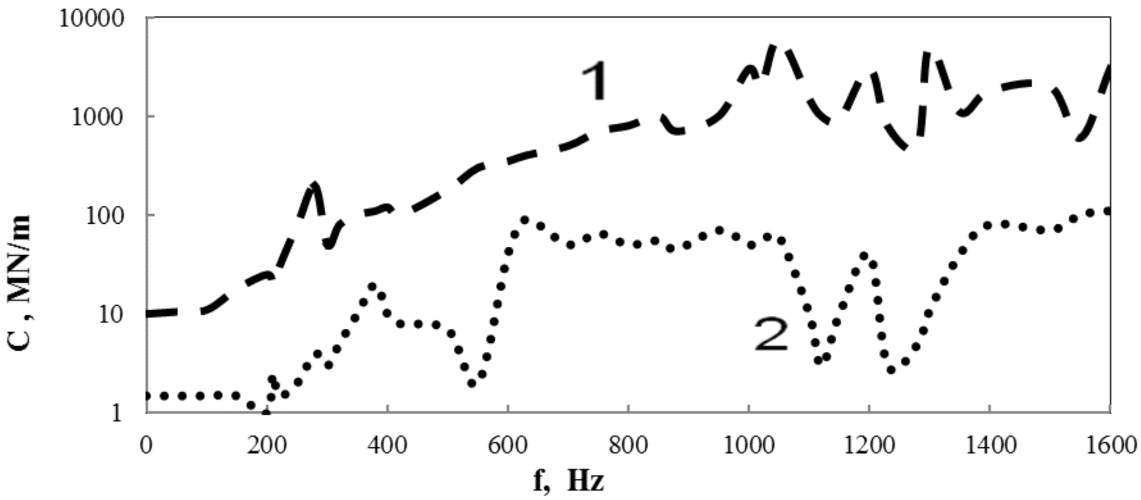



**Figure 1:** Experimentally measured vibrational transient rigidity C of sleeve-type compensators DU 80 mm (curve 1) and a compensator with TRME (curve 2) in the frequency range 0 - 1600 Hz at the water pressure of 10 MPa

When designing constructions based on TRME, it is necessary to ensure their strength in conditions of non-uniform temperature fields with varying loads, including the cyclic ones with significant amplitudes. The problem is solved by the finite element method (FEM) using the thermoelasticity theory. Thermoelasticity is an important branch of solid mechanics that generalizes the theory of elasticity for non-isothermal deformation. It has recently been developed in connection with the problems arising in the creation of gas turbines and rocket engines new designs. The stress condition problems during uneven heating are of great importance for the strength and reliability analysis of functioning designs, operating under conditions of alternating temperatures and strains.

The purpose of the current research is the development of a computational algorithm, which allows simulating thermoelastic behavior and the parameters stress-strain state (SSS) parameters of thin layer rubber -metal elements ( TRME ) under complex thermomechanical loading. At the first stage, the problem of the temperature field distribution under given boundary conditions has been solved, at the second one, the problem of determining the SSS parameters

## 2. DESIGN OF TRME

The design of the flat annular TRME, the main geometrical parameters and the loading scheme as part of the compensator are shown in Fig. 2. TRME consists of supporting rings 1 and a rubber-metal package 2. The package 2 consists of a set of steel plates separated with rubber mixture layers, plasticized with low molecular raw rubber with ring functional groups. The rubber modulus of elasticity during displacement, on which the vibration rigidity most depends, which determines the vibration-insulating properties of the compensator as a whole, is provided at the level of 0.22-0.26 MPa. The rubber mixture is attached by vulcanizing to the plates and the support rings with the system of adhesives: "Chemosil" of the German production or " Permlok " of the Russian production, providing adhesive strength during the breaking-off and displacement at 5.5 and 7.5 MPa, respectively.

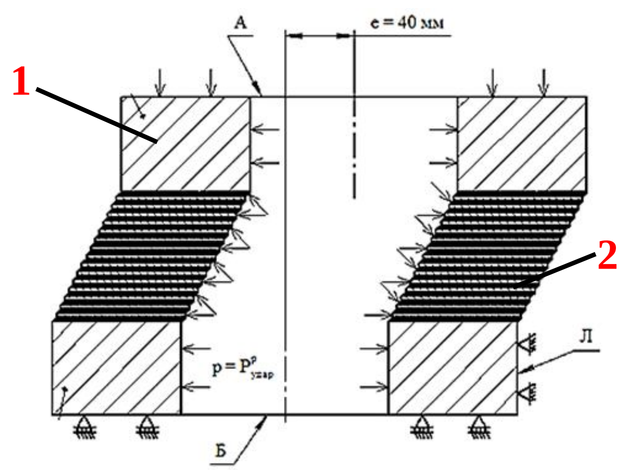



**Figure 2:** Structural design and loading scheme of the TRME with a displacement of the support rings by the value e

This design provides high axial bearing capacity and extremely low transverse rigidity along the rubber-metal layers. Considering the model of a flexible joint with the displaced axes of the support rings (Figure 2), it should be taken into account that the pressure P acts on the inner surface of the product.

## 3. MATHEMATICAL MODELING FOR THE STRESS-STRAIN STATE OF ELASTIC SUPPORT ELEMENTS UNDER COMPLEX THERMOMECHANICAL LOADING

A mathematical model for solving the problem using the finite element method (FEM) in the bulk formulation has been developed. When carrying out the thermal calculation, the equation of non-stationary thermal conductivity is used [2, 3]:

$$\rho C \frac{\partial T}{\partial t} = \lambda_{эф} \Delta T \quad (1)$$

In equation (1), the following designations are introduced: $t$ - time, $\rho$ - density of the thermal protection material, $T$ - temperature field, $C$ - heat capacity, $\Delta = \frac{\partial}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial}{\partial z}$ - Laplace operator of a cylindrical coordinate system, $\lambda_{эф}$ - effective thermal diffusivity. The initial condition of equation (1) is the uniform temperature distribution over the thickness of the multilayer wall:

$$T(0,r,z) = T_н \quad (2)$$

The boundary condition is the energy balance due to heat transfer from the side of the inner wall :

$$-\lambda \left(\frac{\partial T}{\partial r}\right)_{r=r_w} = \frac{\alpha}{C_p}(I_e - I_w) + \xi_{эф}\sigma_{SB}(T_\infty^4 - T_w^4) + Q\dot{m} \quad (3)$$

In expression (3 ), the following designations are introduced: $\alpha$ - coefficient of convective heat transfer, $C_p$ - specific heat capacity, m - mass rate of chemical entrainment, $I_e, I_w$ - enthalpy of the working medium , $T_\infty$ - thermodynamic temperature of the medium , $T_w$ - temperature of the medium at the wall , $T_w$ - effective coefficient of emissivity, $\sigma_{SB}$ - Stefan's exponent -Boltzmann, $Q$ - thermal effect of heat supply

At the junctions of the layers, the conjugation conditions are met:

$$T_{i+0} = T_{i-0}; \quad -\lambda_{i-0}\left(\frac{\partial T}{\partial r}\right)_{i-0} = -\lambda_{i+0}\left(\frac{\partial T}{\partial r}\right)_{i+0} \quad (4)$$

The boundary condition on the outer surface of the last layer is the condition of heat exchange with the environment:

$$-\lambda\left(\frac{\partial T}{\partial r}\right)_{r=r_k} = \alpha_н (T_н - T_{wн}) + \xi_{эфн}\sigma_{SBo}(T_{\infty н}^4 - T_{wн}^4) \quad (5)$$

In operation water flows at speeds up to 10 m/s inside TRME, water temperature ranges from -2 $^0$C and 40 $^0$C. The outer surface of TRME is surrounded by air the temperature of which ranges from 0 $^0$C to 55 $^0$C. In an emergency (in case of an external fire), the outer surface of the TRME is exposed to combustion products with a temperature of 180 $^0$C (boundary condition of the 1st kind) .

To solve the elastoplastic boundary value problem, the FEM was used based on the variational Lagrange approach [4]. Among the permissible displacement values, the actually existing ones are determined by the variational equation [5], which for the functional $\Lambda$ is written in the following form:

$$\Lambda = \frac{2E}{(1+\mu)(1-2\mu)}\sum_{i=1}^{n}\int_V \{(1-\mu)U + 2\mu H + \lambda_0 J\}dV - \int_S (P_r u_r + P_z u_z)dS \quad (6)$$

In formula (6), the following designations are used: $n$ - quantity, $V$ - volume of a finite element, $S$ - area of a finite element $u_r$ , $u_z$ - displacements in the radial and axial directions, $P_n, P_z$ - surface forces in the radial and axial directions.

Elastic deformations and stresses are calculated using the well-known relations of the elasticity theory [5, 6] :

$$\begin{cases} \varepsilon_r = \dfrac{\partial u_r}{\partial r} \\ \varepsilon_\theta = \dfrac{u_r}{r} \\ \varepsilon_z = \dfrac{\partial u_z}{\partial z} \\ \varepsilon_{rz} = \dfrac{\partial u_r}{\partial z} + \dfrac{\partial u_z}{\partial r} \end{cases} \quad (7),$$

$$\begin{cases} \sigma_r = \dfrac{E}{(1+\mu_r)(1-2\mu_r)}((1-\mu)\varepsilon_r + \mu\varepsilon_\theta + \mu\varepsilon_z) \\ \sigma_\theta = \dfrac{E}{(1+\mu)(1-2\mu)}((1-\mu)\varepsilon_\theta + \mu\varepsilon_r + \mu\varepsilon_z) \\ \sigma_z = \dfrac{E}{(1+\mu)(1-2\mu)}((1-\mu)\varepsilon_z + \mu\varepsilon_r + \mu\varepsilon_\theta) \\ \sigma_{rz} = \dfrac{E}{2(1+\mu)}\varepsilon_{rz} \end{cases} \quad (7)$$

In relations (6) and (7), the following designations are introduced:

$E$ - elasticity modulus $\mu$ - Poisson's ratio, $\varepsilon_r$ , $\varepsilon_\theta$ , $\varepsilon_z$ , $\varepsilon_{rz}$ - radial, circumferential, axial and deformations, $\sigma_r$ , $\sigma_\theta$ , $\sigma_z$ , $\sigma_{rz}$ - radial, circumferential, axial and tangential stresses. The equilibrium equation for a system of finite elements is:

$$[K]\{\delta\} = \{R\} \tag{8}$$

In *equation* (8), the following designations are adopted: $[K]$ - structure rigidity matrix, $\{\delta\}$ - vector of the unknowns, $\{R\}$ - load vector. Finite element rigidity matrices are determined using the following relationship:

$$[k]^e = 2\pi \int_S [B]^T [D][B] r dS \tag{9}$$

In relation (9), $[B]$ is a geometric matrix used to relate deformations and displacements in a finite element, depending on the type of a finite element, $[D]$ is an elasticity matrix, depending on the type of stress state. Taking into account the properties of anisotropy of materials [7] and the symmetry property of the matrix , we obtain:

$$[D] = \begin{vmatrix} \frac{E_r}{\nu}\left(1-\mu_r^2\frac{E_z}{E_\theta}\right) & \frac{E_r}{\nu}\mu_r(1+\mu_r) & \frac{E_r}{\nu}\mu_r\left(1+\mu_r\frac{E_z}{E_\theta}\right) & 0 \\ \frac{E_r}{\nu}\mu_r(1+\mu) & \frac{E_\theta}{\nu}\left(1-\mu_\theta^2\frac{E_r}{E_z}\right) & \frac{E_r}{\nu}\mu_z\left(1+\mu_z\frac{E_z}{E_\theta}\right) & 0 \\ \frac{E_r}{\nu}\mu_r\left(1+\mu_r\frac{E_z}{E_\theta}\right) & \frac{E_r}{\nu}\mu_r\left(1+\mu_r\frac{E_r}{E_z}\right) & \frac{E_z}{\nu}\left(1-\mu_z^2\frac{E_r}{E_\theta}\right) & 0 \\ 0 & 0 & 0 & G_{rz} \end{vmatrix} \tag{10}$$

In matrix (10): $E_r, E_\theta, E_z, \mu_r, \mu_\theta, \mu_z$ - elastic moduli and Poisson's ratios in the radial, annular and axial directions, respectively, $G_{rz}$ - displacement modulus,

$$\nu = 1-\mu_r^2\left(\frac{E_r}{E_z}+\frac{E_z}{E_\theta}\right)-\mu_r^2\frac{E_r}{E_\theta}(1+2\mu_r)$$

The temperature stresses and the deformed shape of the structure under nonuniform heating were calculated using the FEM [5]. Rigidity matrices of elements in thermoelastic problems have the same form as in case of force action on the system . It is thus required to take into account the additional thermal deformations arising under the action of thermal stress [8, 9]. As part of the numerical modeling of the hyperelastic behavior of the rubber mixture, the two-parameter Mooney- Rivlin model was used, according to which the total specific energy of deformation is determined by the ratio:

$$W = \frac{1}{d}(J-1)^2 + C_{10}(I_1-3) + C_{01}(I_2-3) \tag{11}$$

In expression (11): $I_i, J, C_{10}, C_{01}$ - $i$ -e invariants of the deformation deviator, determinant of the deformation gradient matrix, material constants of the model, respectively, $d = \dfrac{1-2\mu}{C_{10}+C_{01}}$ is the coefficient of rubber incompressibility, $\mu(T,t)$ is Poisson's ratio.

## 4. NUMERICAL SIMULATION RESULTS

The numerical simulation results of the TRME thermal state are visualized in Fig. 3 for the case of maximum heating. The maximum heating temperature of the TRME structure is realized on the outer surface of the package of TRME rubber-metal plates and is 180 C, the minimum is on the inner surface of the TRME (40 C).

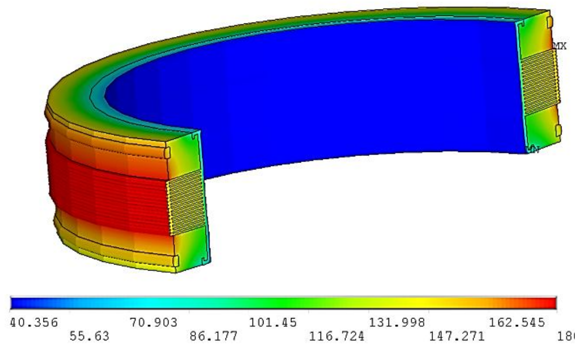



**Figure 3.** Temperature field distribution, $^0$C on the surface of the TRME

The distribution map of the heat-stressed state of the element base, taking into account the orthotropic dependence of the elastic modulus on the heating temperature, is shown in Fig. 4.

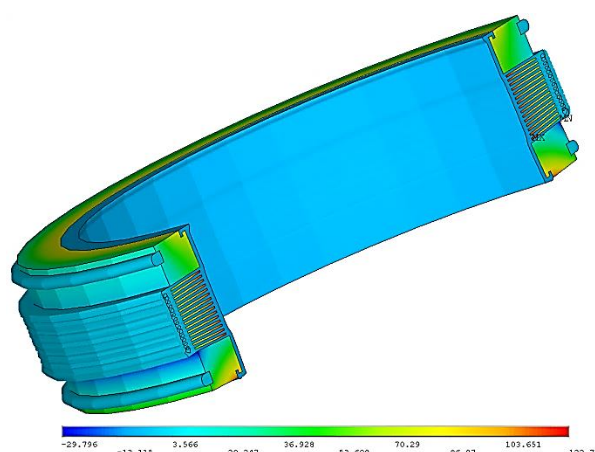



**Figure 4.** Distribution of the ring strain in TRME

The results of the computational experiment on the distribution of the SSS parameters of the thermoelastic boundary value problem for the TRME showed that the maximum intensity of Mises equivalent stresses is realized on the surfaces of metal plates and is $\sigma_{eqv\_Me\_тарел.}^{\max} = 130.82\ МПа$ (Fig. 5). The maxima of thermal and mechanical deformation of the TRME rubber layers are determined in a ratio

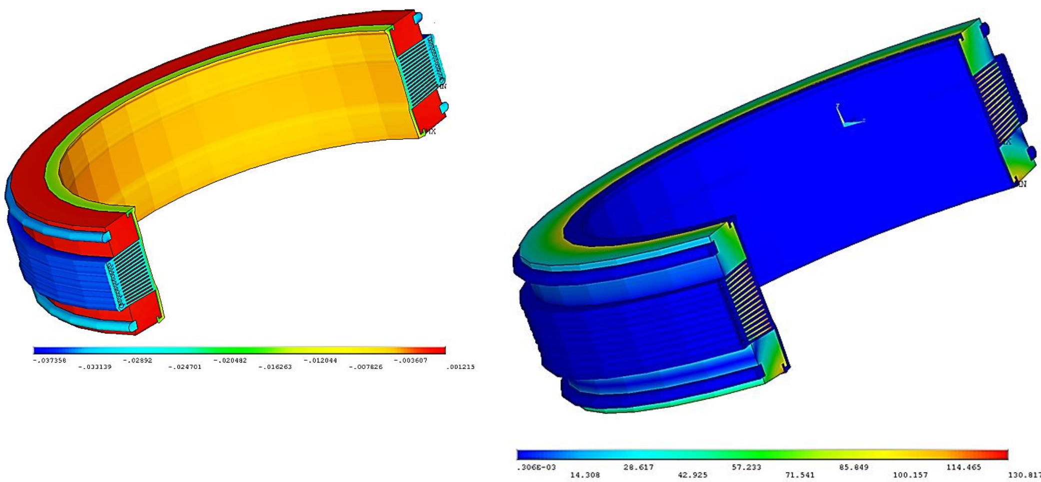
23.6:1.

The distribution of the minimum structural strength factor of the TRME construction is determined according to the classical strength theory :

$$\eta_{\min} = \frac{\sigma_B(T)}{\sigma_{\max}} > 1,$$

where $\eta_{\min}$ is the minimum safety factor; $\sigma_B(T)$ - ultimate strength of construction materials depending on heating temperature ;

$\sigma_{\max} = \max\{\sigma_r, \sigma_\theta, \sigma_z, \sigma_{eqv}\}$ - maximum among the design stresses; $\sigma_r, \sigma_\theta, \sigma_z, \sigma_{eqv}$ - radial, annular, axial and equivalent stresses, respectively.



A B

**Figure 5.** Map of the SSS parameters distribution of the thermoelastic boundary problem : A. relative axial thermal deformation of the TRME; B. intensity of equivalent stresses, MPa

The minimum safety margin for TRME metal plates is:

$$\eta_{\min}^{тарел} = \frac{\sigma_B(T)}{\sigma_{eqv\_Me\_тарел.}^{\max}} = 2.58$$

$\sigma_{eqv\_Me\_тарел.}^{\max} = 130.82\ МПа$ is maximum intensity of the equivalent stress in plates, $\sigma_B(T) = 337.4\ МПа$ is the strength limit of the plates material at the temperature $T = 128\ {}^0C$, minimum coefficient of the support rings safety margin $\eta_{\min}^{опор\_колец} = \frac{\sigma_B(T)}{\sigma_{eqv\_опор\_колец.}^{\max}} = 4.97$ is , minimum s afety margin of the rubber layers displacement is determined by the relation:

$$\eta_{\min}^{сдв\_рез} = \frac{\tau_B^{сдв}(T)}{\tau_{rz}^{\max}} = \frac{0.74МПа}{0.55МПа} = 1.35 > 1$$

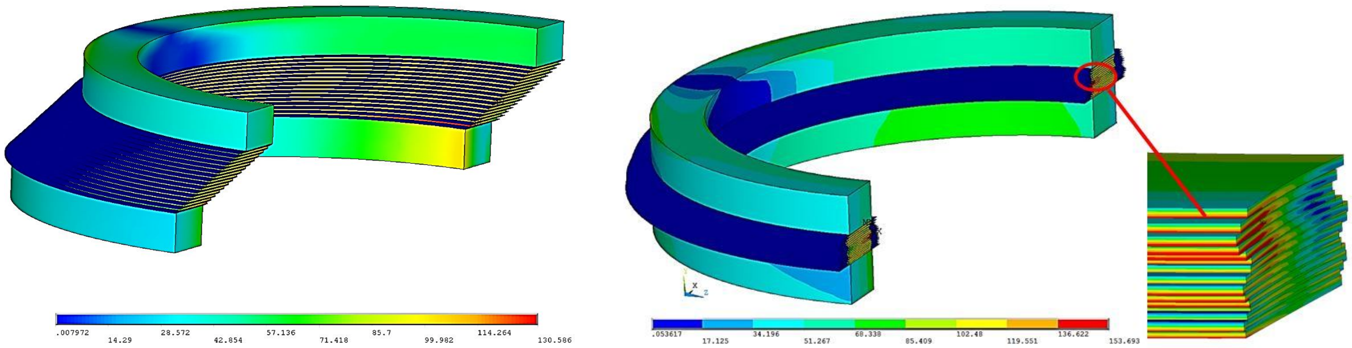
When the temperatures of the TRME mode of operation range from 40 C to 180 C TRME and rubber package strength is fully ensured.

## 5. CYCLIC STRENGTH OF TRME. NUMERICAL MODELING AND EXPERIMENT

TRME should provide dynamic strength under the action of short-term shock loads at a pressure of 10 MPa with an amplitude of ± 40 mm in the transverse direction with a deformation rate of 3.5 m/s without loss of performance. Under low-cycle loading with a frequency of less than 1 Hz and an amplitude of 40 mm, the maximum cyclic displacement stresses in TPME rubber are up to 0.38 MPa. Under high-cycle loading with an amplitude of 5 mm, they do not exceed 0.05 MPa. The distribution of displacement stresses is shown in Fig. 6A. Resource tests have shown that these values are in the range of permissible displacement stresses of 0.7 MPa - 0.14 MPa for each of the amplitudes, respectively (they provide the cyclic strength of the TRME for a given number of cycles). However, taking into consideration reliability requirements, the complexity of physical and mechanical transformations in the process of the rubber cyclic deformation, the effect of the dynamic component of the load, the scale factor, the frequency and rate of loading, the TRME cyclic strength requires experimental confirmation for the full-size models.

The tests revealed a loss of stability in the transverse direction of the rubber - metal mass during axial compression with buckling of the plates in their plane upon reaching a certain critical load. The calculated pattern of buckling under axial compression is shown in Fig. 6B.

The experiment showed that at higher deformation frequencies it is necessary to take into account the elastic-wave behavior of the elements of the TRME rubber-metal package in the transverse direction.




A B

**Figure 6.** Intensity of equivalent stresses, MPa . A cyclic loading with a transverse displacement of the upper support ring; B. static axial loading with loss of stability

Calculating experiment showed that at low-amplitude high frequency harmonic loading (A=0.25 mm, f=50 Hz) there occurs higher intensity of equivalent stress as compared with the static loading. An increase in the deformation frequency up to 70 Hz and its amplitude up to 3 mm leads to rubber extrusion from the metal plates of the TRME and the loss of stability of the package (see also Fig. 6B).

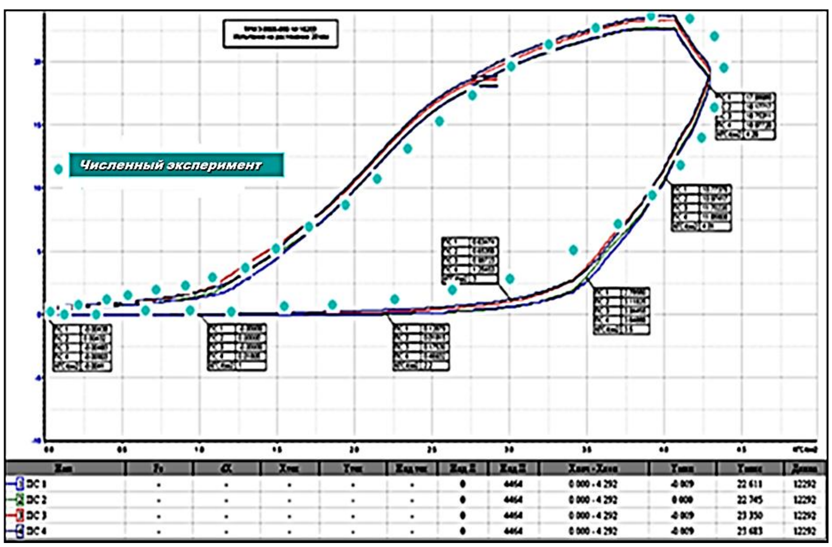



**Figure 7.** Calculated (points) and experimental (curves) study of TRME plates displacements under cyclic tension

The results of experimental and calculated analysis of TRME movements under axial cyclic tension are shown in Fig. 7. The results of calculations (points) and tests (curves) for 4-cycle quasi-static stretching showed good agreement. The material constants of the rubber plates were proved empirically. The area bounded by the hysteresis loop slightly increases with increasing loading cycles, which indicates the presence of a low level of the TRME residual deformations.

## 6. CONCLUSION

Computational algorithm and a number of programs have been developed allowing investigating the stress-strain state of thin-layer rubber-metal elastic supporting elements under nonstationary thermomechanical and cyclic loading. It allows desingning optimum structure based on TRME under complex thermal, power, and cyclic loading.

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