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A New Spectral Conjugate Gradient Method with Strong Wolfe-Powell Line Search

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ABSTRACT

The spectral conjugate gradient method is an efficient method for solving unconstrained optimization problems. In this paper, based on MMAR conjugate gradient method, a new spectral conjugate gradient method SMMAR is proposed with strong Wolfe-Powell line search. This method possesses sufficient descent and global convergence properties. Numerical results show that SMMAR method outperforms MMAR conjugate gradient method in terms of the number of iterations almost in all tested functions. But MMAR method outperforms SMMAR conjugate gradient method in terms of CPU time almost in all tested functions.

Key words: Conjugate gradient (CG) method; global convergence; spectral conjugate gradient method; strong Wolfe-Powell (SWP) line search; sufficient descent property.

1 INTRODUCTION

The conjugate gradient (CG) method are among the efficient methods for solving problems with large dimension unconstrained optimization problems of the form:

$$\min\{\ f\ (x): x \in \mathrm{R}^n\ \}. \quad (1)$$

where $f : \mathrm{R}^n \to \mathrm{R}$ is continuously differentiable. The CG method are iterative methods that computes it iterates and search direction d_k as follows:

$$x_{k+1} = x_k + \alpha_k d_k \quad , k = 0,1,2,3, \ldots. \quad (2)$$

$$d_k = \begin{cases} -g_k & , \ and\ k = 0 \\ -g_k + \beta_k d_{k-1} & , \ and\ k \geq 1. \end{cases} \quad (3)$$

where $\alpha_k > 0$ is the step size obtained through the line search approach, $g_k = g(x_k)$ is known as gradient and β_k is the CG coefficient such that β_k defines the different CG methods. Also, different spectral CG methods are generated according to different search directions [1]. The β_k for original MMAR conjugate gradient method [2] is defined by:

$$\beta_k^{MMAR} = \begin{cases} \frac{\|g_k\|^2 - \frac{\|g_k\|}{\|g_{k-1}\|}|g_k^T g_{k-1}|}{\|g_k\| + \|g_{k-1}\|^2}, & \|g_k\|^2 \geq \frac{\|g_k\|}{\|g_{k-1}\|}|g_k^T g_{k-1}| \\ 0 & , \text{and otherwise} \end{cases} \quad (4)$$

Some CG methods are computed such that the step size satisfy the exact line search defined by:

$$f\ (x_k + \alpha_k d_k) = \min f(x_k + \alpha\, d_k)\ ,\ \alpha \geq 0. \quad (5)$$

while other CG methods are said to satisfy the Strong Wolfe-Powell (SWP) defined by

$$f\ (x_k + \alpha_k d_k) - f(x_k) \leq \delta\, \alpha_k\ g_k^T d_k \quad (6)$$

$$|g\ (x_k + \alpha_k d_k)^T d_k| \leq \sigma |g_k^T g_k|. \quad (7)$$

where $0 < \delta < \sigma < 1$. The SWP line search is a modification of weak Wolfe-Powell (WWP) line search defined by (6) and

$$g\ (x_k + \alpha_k d_k)^T d_k \geq \sigma\, g_k^T g_k\ . \quad (8)$$

The CG method are applicable to real-life situation [3]. Some of the well-known formulas for β_k are: Fletcher-Reeves (FR) [4], Hestenes-Stiefel (HS) [5], Powell (PRP+) [6], Polak-Ribi`re-Polyak (PRP) [7], Liu-e Storey (LS) [8], Conjugate Descent (CD) [9], Wei et al. (WYL) [10], Dai-Yuan (DY) [11], which are given by:

$$\beta_K^{FR} = \frac{\|g_k\|^2}{\|g_{k-1}\|^2}, \quad (9)$$

$$\beta_K^{HS} = \frac{g_k^T y_{k-1}}{d_{k-1}^T y_{k-1}}, \quad (10)$$

$$\beta_K^{PRP+} = \max\{\beta_K^{PRP}, \quad 0\}, \quad (11)$$

$$\beta_K^{PRP} = \frac{g_k^T y_{k-1}}{\|g_{k-1}\|^2}, \quad (12)$$

$$\beta_K^{LS} = \frac{g_k^T y_{k-1}}{d_{k-1}^T g_{k-1}}, \tag{13}$$

$$\beta_K^{CD} = \frac{\|g_k\|^2}{d_{k-1}^T g_{k-1}}, \tag{14}$$

$$\beta_k^{WYL} = \frac{\|g_k\|^2 - \frac{\|g_k\|}{\|g_{k-1}\|} \; g_k^T g_{k-1}}{\|g_{k-1}\|^2}, \tag{15}$$

$$\beta_K^{DY} = \frac{\|g_k\|^2}{d_{k-1}^T(g_k - g_{k-1})}, \tag{16}$$

where $y_{k-1} = g_k - g_{k-1}$. Polak and Ribiere in [7] proved the global convergence of PRP method. In 2006, Wei et al. [10] presented a new CG method that is similar to the PRP method. Other modifications of the CG method are given in [12], [13], and defined as follows:

$$\beta_k^{DPRP} = \frac{\|g_k\|^2 - \frac{\|g_k\|}{\|g_{k-1}\|}|g_k^T g_{k-1}|}{w|g_k^T d_{k-1}| + \|g_{k-1}\|^2} \tag{17}$$

$$\frac{\|g_k\|^2 - \frac{\|g_k\|}{\|g_{k-1}\|} g_k^T g_{k-1}}{d_{k-1}^T y_{k-1}} . \tag{18}$$

where $w \geq 1$. Zhang [14] presented a simple modification of β_k^{WYL} as follows:

$$\beta_k^{NPRP} = \frac{\|g_k\|^2 - \frac{\|g_k\|}{\|g_{k-1}\|}|g_k^T g_{k-1}|}{\|g_{k-1}\|^2} \tag{19}$$

For more reference on the CG method, please refer to [15–16, 41-43]. The global convergence of MMAR conjugate gradient method is proved under the strong Wolfe-Powell (SWP):

$$f(x_k + \alpha_k d_k) - f(x_k) \leq \delta \alpha_k \, g_k^T d_k \tag{20}$$

$$|g(x_k + \alpha_k d_k)^T d_k| \leq \sigma | g_k^T g_k|. \tag{21}$$

where $0 < \delta < \sigma < 1$, with $\leq 3/4$, for all $k \geq 0$, and the relation

$$\frac{-1}{1-\sigma} < \frac{g_k^T d_k}{\|g_k\|^2} < \frac{2\sigma-1}{1-\sigma}.$$

Recently, a scaled CG algorithm was introduced by Birgin and Martinez [17] as follows:

$$\beta_K^{BM1} = \frac{g_{k+1}^T (\theta_k y_k - s_k)}{y_k^T s_k},$$

$$\beta_K^{BM2} = \frac{\theta_k y_k^T g_{k+1}}{\alpha_k \theta_{k-1} g_k^T g_k},$$

$$\beta_K^{BM3} = \frac{\theta_k g_{k+1}^T g_{k+1}}{\alpha_k \theta_{k-1} g_k^T g_k},$$

$$\theta_k = \frac{s_k s_k^T}{y_k s_k^T}.$$

where d_k is the direction and given by:

$$d_k = -\theta_k g_k + \beta_k d_{k-1}$$

and θ_k is a parameter and called the spectral gradient. The results got from experiments using Wolfe line search on the three CG coefficients above show that β_K^{BM1} has the best numerical performance. Under some reasonable assumptions, Birgin and Martinez [17] concluded that their spectral CG method is globally convergent. However, the spectral CG methods actually are not guaranteed to generate descent directions [18]. Hence, Andrei [19] proposed a descent scaled CG algorithm under Wolfe line search. Jiang et al. [20] designed a spectral CG method with sufficient descent feature based on the modified CG algorithm proposed by Zhang et al. [21]. The authors used β_K^{PRP} for the CG coefficient, where

$$\theta_k = \frac{y_{k-1} d_{k-1}^T}{\|g_{k-1}\|^2} - \frac{g_k^T g_k d_{k-1}^T g_{k-1}}{\|g_k\|^2 \|g_{k-1}\|^2},$$

$$\beta_K^{PRP} = \frac{g_k^T y_{k-1}}{\| g_{k-1} \|^2},$$

where $y_{k-1} = g_k - g_{k-1}$. The algorithm was implemented under modified Armijo-type line search and subsequently proven to globally convergent under some mild conditions. Liu and Jiang [22] proposed a spectral CG method, denoted SCD, using the CD method as the basis. The SCD algorithm possesses sufficient descent property under any line search used and is proven to be globally convergent under strong Wolfe line search. It is formulated by

$$\theta_k = 1 - \frac{g_k^T d_{k-1}}{g_{k-1}^T d_{k-1}},$$

$$\beta_K^{CD} = \frac{\| g_k \|^2}{d_{k-1}^T g_{k-1}}.$$

Liu et al. [23] proposed another spectral CG method. This method combined the CD method and DY method, where

$$\beta_k = \beta_K^{CD} + \min\{0, \psi_k, \beta_K^{CD}\},$$

$$\theta_k = 1 - \frac{g_{k-1}^T d_{k-1}}{g_k^T d_{k-1}},$$

$$\psi_k = -\frac{g_{k-1}^T d_{k-1}}{d_{k-1}(g_{k-1} - g_k)},$$

Zull et al. [24] extended the SCD method by using RMIL as the CG coefficient instead of CD, where the new method is called SRMIL. While its global convergence property is not truly established, the SRMIL method performed well numerically when compared to some of the classical CG methods. It is formulated by

$$d_k = -\theta_k g_k + \beta_K^{RMIL} d_{k-1},$$

$$\theta_k = 1 - \frac{g_k^T d_{k-1}}{g_{k-1}^T d_{k-1}},$$

$$\beta_K^{RMIL} = \frac{g_k^T (g_k - g_{k-1})}{\| d_{k-1} \|^2}$$

Khadijah et al. [25] proposed a spectral CG method. It is formulated by

$$d_k = -\theta_k g_k + \beta_K^{RMIL} d_{k-1},$$

$$\theta_k = 1 + \left(\frac{g_k^T (g_k - g_{k-1})}{\| d_{k-1} \|^2}\right)\left(\frac{g_k^T d_{k-1}}{\| g_k \|^2}\right),$$

Several scaled CG methods proposed by many authors can be referred to [26–35], and an application problem can be referred to [44]. In this paper, the spectral conjugate gradient method SMMAR with strong Wolfe-Powell line search is presented as follows: In the next section, the new spectral conjugate gradient formula and the algorithm. In section 3, the convergence analysis is presented. After that, the numerical results are presented in section 4. And finally, the conclusion is given in section 5.

2 THE NEW SPECTRAL CONJUGATE GRADIENT FORMULA AND ALGORITHM

In this study d_k is defined by:

$$d_k = \begin{cases} -g_k & , \text{ and } k = 0 \\ -\theta_k^{MMAR} g_k + \beta_k^{MMAR} d_{k-1} & , \text{ and } k \geq 1 \end{cases} \quad (23)$$

where β_k^{MMAR} and θ_k^{MMAR} are presented by:

$$\beta_k^{MMAR} = \begin{cases} \dfrac{\|g_k\|^2 - \frac{\|g_k\|}{\|g_{k-1}\|}|g_k^T g_{k-1}|}{\|g_k\| + \|g_{k-1}\|^2}, & \text{and} \|g_k\|^2 \geq \frac{\|g_k\|}{\|g_{k-1}\|}|g_k^T g_{k-1}| \\ 0 & , \text{ and otherwise} \end{cases} \quad (24)$$

$$\theta_k^{MMAR} = 1 + \frac{(g_k^T d_{k-1}) - \frac{|g_k^T g_{k-1}|(g_k^T d_{k-1})}{\|g_{k-1}\|\|g_k\|}}{\|g_k\| + \|g_{k-1}\|^2}. \quad (25)$$

$\| . \|$ represents the Euclidean norm. Note that

$$0 \leq \frac{\|g_k\|^2 - \frac{\|g_k\|}{\|g_{k-1}\|}|g_k^T g_{k-1}|}{\|g_k\| + \|g_{k-1}\|^2} < \frac{\|g_k\|^2}{\|g_k\| + \|g_{k-1}\|^2} < \frac{\|g_k\|^2}{\|g_{k-1}\|^2}. \quad (26)$$

Algorithm 1
Step 1: Given constants $0 < \delta < \sigma < 1$. Take a starting point x_0. Set $d_0 = -g_0$. Let $k = 0$, if $g_0 = 0$, then stop.
Step 2: Compute d_k based on (23), using (24) and (25).
Step 3: Find $\alpha_k > 0$ satisfying (6) and (7).
Step 4: Update x_{k+1} based on (2).
Step 5: If $\|g_k\| \leq 10^{-6}$ then stop; otherwise let $k = k + 1$ and go to Step 2.

3 THE CONVERGENCE ANALYSIS

The descent condition plays an important role in the convergence analysis of various CG methods. A CG method is said to be a descent method if the method possesses

$$g_k^T d_k \leq 0, \quad (27)$$

such that $f(x_{k+1}) < f(x_k)$. The algorithm is said to satisfy the following condition:

$$g_k^T d_k \leq -C\|g_k\|^2 \; For \; k \geq 0, C > 0, \quad (28)$$

then (28) is known as sufficient descent condition.
Before presenting the proof of the sufficient descent direction property (28) of spectral conjugate gradient method SMMAR, note that the coefficient β_k^{MMAR} satisfies

$$0 \leq \beta_k^{MMAR} \leq \frac{\|g_k\|^2}{\|g_{k-1}\|^2}$$

$$0 \leq \beta_{k+1}^{MMAR} \leq \frac{\|g_{k+1}\|^2}{\|g_k\|^2}. \quad (29)$$

Theorem 1. If $g_k \neq 0$, suppose the direction d_k is generated by Algorithm 1, then

$$g_k^T d_k = -\|g_k\|^2 \quad (30)$$

holds for any $k \geq 0$.

Proof. Clearly, the result is true for $k = 0$, $d_0 = -g_0$. Now for $k \geq 1$, from (3), (24) and (25), we have

$$d_k = -\theta_k^{MMAR} g_k + \beta_k^{MMAR} d_{k-1}. \quad (31)$$

Multiplying both sides by g_k^T, then we get

$$g_k^T d_k = -\theta_k^{MMAR}\|g_k\|^2 + \beta_k^{MMAR} g_k^T d_{k-1}.$$

$$= -\left(1 + \frac{(g_k^T d_{k-1}) - \frac{|g_k^T g_{k-1}|(g_k^T d_{k-1})}{\|g_{k-1}\|\|g_k\|}}{\|g_k\| + \|g_{k-1}\|^2}\right)\|g_k\|^2 + \left(\frac{\|g_k\|^2 - \frac{\|g_k\|}{\|g_{k-1}\|}|g_k^T g_{k-1}|}{\|g_k\| + \|g_{k-1}\|^2}\right) g_k^T d_{k-1}.$$

$$g_k^T d_k = -\left(1 + \frac{\left(1 - \frac{|g_k^T g_{k-1}|}{\|g_{k-1}\|\|g_k\|}\right)(g_k^T d_{k-1})}{\|g_k\| + \|g_{k-1}\|^2}\right)\|g_k\|^2 + \left(\frac{\|g_k\|^2 - \frac{\|g_k\|}{\|g_{k-1}\|}|g_k^T g_{k-1}|}{\|g_k\| + \|g_{k-1}\|^2}\right) g_k^T d_{k-1}.$$

$$= -\|g_k\|^2 - \left(\frac{\left(1 - \frac{|g_k^T g_{k-1}|}{\|g_{k-1}\|\|g_k\|}\right)(g_k^T d_{k-1})}{\|g_k\| + \|g_{k-1}\|^2}\right)\|g_k\|^2$$

$$+\left(\frac{\|g_k\|^2-\frac{\|g_k\|}{\|g_{k-1}\|}|g_k^Tg_{k-1}|}{\|g_k\|+\|g_{k-1}\|^2}\right)g_k^Td_{k-1}.$$

$$= -\|g_k\|^2-\left(\frac{\left(\|g_k\|^2-\frac{\|g_k\||g_k^Tg_{k-1}|}{\|g_{k-1}\|}\right)(g_k^Td_{k-1})}{\|g_k\|+\|g_{k-1}\|^2}\right)$$

$$+\left(\frac{\|g_k\|^2-\frac{\|g_k\|}{\|g_{k-1}\|}|g_k^Tg_{k-1}|}{\|g_k\|+\|g_{k-1}\|^2}\right)g_k^Td_{k-1}.$$

$$= -\|g_k\|^2.$$

So, we have $g_k^Td_k = -\|g_k\|^2$. This completes the proof.

Assumption 1. (i) The level set $L = \{x \in R^n: f(x) \leq f\,x0$ is bounded.
(ii) In some neighborhood N of L, f is continuously differentiable and its gradient is Lipschitz continuous, namely, there exists a constant $K > 0$ such that

$$\|g(x) - g(y)\| \leq K\|x-y\|,\ \forall\, x, y \in N.$$

Lemma 1. Let Assumption 1 hold x_k is given by Algorithm 1, then we have

$$\sum_{K=1}^{\infty}\frac{\|g_k\|^4}{\|d_k\|^2} < \infty.$$

See Zoutendijk [36] for the proof of Lemma 1.

We present the following Theorem which proves the global convergence of the spectral conjugate gradient method SMMAR, depending on the above Assumption and Lemma.

Theorem 1. Consider the sequence x_k is generated by Algorithm 1, and suppose that Assumption 1 holds. Then we obtain

$$\lim_{K\to\infty}\inf\|g_k\| = 0.$$

Proof. This prove is by contradiction. Suppose there exists a positive constant $\varepsilon > 0$ such that

$$\|g_k\| \geq \varepsilon, \text{ for all } k \geq 0, \tag{32}$$

which means

$$\frac{1}{\|g_k\|^2} \leq \frac{1}{\varepsilon^2} \text{ for all } k \geq 0 \text{ and } \|g_k\| \neq 0. \tag{33}$$

Rewriting (31) as $d_k + \theta_k^{MMAR} g_k = \beta_k^{MMAR} d_{k-1}$, and squaring both sides we obtain

$$\|d_k\|^2 + (\theta_k^{MMAR})^2\|g_k\|^2 + 2\theta_k^{MMAR}g_k^Td_k$$
$$= (\beta_k^{MMAR})^2\|d_{k-1}\|^2,$$

then

$$\|d_k\|^2 = -(\theta_k^{MMAR})^2\|g_k\|^2 - 2\theta_k^{MMAR}g_k^Td_k$$
$$+(\beta_k^{MMAR})^2\|d_{k-1}\|^2. \tag{34}$$

Dividing both sides of (34) by $(g_k^Td_k)^2$, and from (24), (25), (26), and (30), we have

$$\frac{\|d_k\|^2}{(g_k^Td_k)^2} = \frac{\|d_k\|^2}{\|g_k\|^4}$$
$$= -\frac{(\theta_k^{MMAR})^2}{\|g_k\|^2} - \frac{2\theta_k^{MMAR}}{\|g_k\|^2} + (\beta_k^{MMAR})^2\frac{\|d_{k-1}\|^2}{\|g_k\|^4}$$
$$= -\frac{1}{\|g_k\|^2}\left((\theta_k^{MMAR})^2 + 2\theta_k^{MMAR}\right) + (\beta_k^{MMAR})^2\frac{\|d_{k-1}\|^2}{\|g_k\|^4}$$
$$= -\frac{1}{\|g_k\|^2}\left((\theta_k^{MMAR})^2 + 2\theta_k^{MMAR} + 1 - 1\right) + (\beta_k^{MMAR})^2\frac{\|d_{k-1}\|^2}{\|g_k\|^4}$$
$$= -\frac{1}{\|g_k\|^2}\left((\theta_k^{MMAR}+1)^2 - 1\right) + (\beta_k^{MMAR})^2\frac{\|d_{k-1}\|^2}{\|g_k\|^4}$$
$$= -\frac{(\theta_k^{MMAR}+1)^2}{\|g_k\|^2} + \frac{1}{\|g_k\|^2} + (\beta_k^{MMAR})^2\frac{\|d_{k-1}\|^2}{\|g_k\|^4}$$
$$\leq \frac{1}{\|g_k\|^2} + (\beta_k^{MMAR})^2\frac{\|d_{k-1}\|^2}{\|g_k\|^4}$$
$$\leq \frac{1}{\|g_k\|^2} + \left(\frac{\|g_k\|^2}{\|g_{k-1}\|^2}\right)^2\frac{\|d_{k-1}\|^2}{\|g_k\|^4}$$
$$= \frac{1}{\|g_k\|^2} + \frac{\|d_{k-1}\|^2}{\|g_{k-1}\|^4} \tag{35}$$

Using (35) recursively, we get

$$\frac{\|d_k\|^2}{\|g_k\|^4} \leq \sum_{i=0}^{k-1}\frac{1}{\|g_i\|^2}. \tag{36}$$

Then from (33) and (36), we have

$$\frac{\|d_k\|^2}{\|g_k\|^4} \leq \frac{k}{\varepsilon^2},$$

which indicates

$$\sum_{K=1}^{\infty}\frac{\|g_k\|^4}{\|d_k\|^2} \geq \varepsilon^2\sum_{K=1}^{\infty}\frac{1}{k} = +\infty.$$

This contradicts Lemma 1. Therefore, the proof is completed.

4. NUMERICAL RESULTS

This section presents the numerial performance of the CG methods. Some test functions are selected for the efficiency analysis of the new method. These functions are considered from CUTEr [37], Andrei [38], and Adorio and Diliman [39]. A comparison is done between the new spectral conjugate gradient method SMMAR and MMAR CG method based on time of CPU and the number of iterations, using Strong (SWP) Wolfe-Powell line search. Let $\delta = 0.01, \sigma = 0.1$. ε is chosen to be 10^{-6} to check the speed of iteration methods towards the optimal. And here $\|g_k\| \leq 10^{-6}$ is taken to be the stopping criteria. The method is considered as failed if the number of iterations getting more than 1000 times. MATLAB R2017a subroutine program is used for all methods, on a computer with CPU processor Intel R Core TM, i5-2410M CPU and 3 GB RAM under strong (SWP) Wolfe line search. Figs. 1-2, respectively, show the performance results using a performance profile introduced by Dolan and More´ [40]. Generally, the best method is which with high values of $P_s(t)$ and appear in the upper right and left corners of the figure. The $P_s(t)$ axis of the figures shows the percentage of the test problems which were successfully solved by each method. The t axis of the figures shows which of the methods is the fastest. From Figure 1, it is clear that SMMAR method strongly outperforms the other tested method MMAR in terms of the number of iterations almost in all tested functions. In Figure 2, which presents CPU time, MMAR method strongly outperforms SMMAR method almost in all tested functions

Table 1: List of Test Functions

N Function	Dimension/s	Initial points
1 FLETCHCR	2	(0.5, 0.5), (5, 5)
2 QUARTC	500,1000,5000	(2, 2, . . ., 2), (5, 5, . . ., 5), (10, 10, . . ., 10), (15, 15, . . ., 15)
3 Extended Block Diagonal	2,5000, 10000	(0.1, 0.1), (1, 1, . . ., 1)
4 SINCOS B81	2	(3, 3)
5 Generalized quartic GQ1	2,500,1000,5000,10000	(1, 1, . . ., 1)
6 Three hump	2	(1, 1), (15, 15), (5, 5)
7 Generalized Quartic	2	(1, 1)
8 DENSCHNB	2,500,1000,5000,10000	(1, 1, . . ., 1)
9 Raydan 1	2	(1, 1)
10 Extended DENSCHNB	2,500,1000,5000,10000	(1, 1, . . ., 1), (5, 5, . . ., 5)
11 Shallow	2, 5000,10000	(-2, -2, . . ., -2)
12 Perturbed Quadratic	2	(0.5, 0.5)
13 Raydan 2	500,5000,10000	(1, 1, . . ., 1), (10, 10, . . .,10)
14 HIMMELBC	2,500,1000,5000,10000	(1, 1, . . ., 1)
15 DIXMAANA	6000,9000,12000	(2, 2, . . ., 2)
16 DIXMAANB	300,6000,9000,12000	(2, 2, . . ., 2)
17 Extended Himmelblau	2, 500,1000,5000,10000	(1, 1, . . ., 1)
18 EG2	2, 500,1000,5000,10000	(0.01, 0.01), (1, 1, . . ., 1) (2, 2, . . ., 2)
19 DENSCHNF	2	(1.5, 1.5)
20 HIMMELBH	2, 500	(1.5, 1.5, . . ., 1.5)
21 LIARWHD	2	(4, 4)
22 Extended quadratic penalty QP1	2	(1, 1)
23 Six hump	2	(1, 1), (10, 10), (5, 5)
24 EG3	2	(1, 1)
25 A Quadratic QF2	2	(0.5, 0.5)
26 Quadratic QF1	2	(1, 1)
27 Diagonal 1	2	(1, 1), (2, 2)
28 Hager	2	(1, 1)
29 DIXON3DQ	2	(-1, -1)
30 Generalized quartic GQ2	2,500,1000,5000,10000	(1, 1, . . ., 1)
31 Tridiagonal double Borded	2	(-1, -1)
32Raydan 1	2	(1, 1)
33 Extended Trigonometric	2	(0.2, 0.2)
34 ENGVAL8	2	(2, 2)
35 DENSCHANA	2,500,1000,5000,10000	(1, 1, . . ., 1)
36 VARDIM	2	(2, 2)

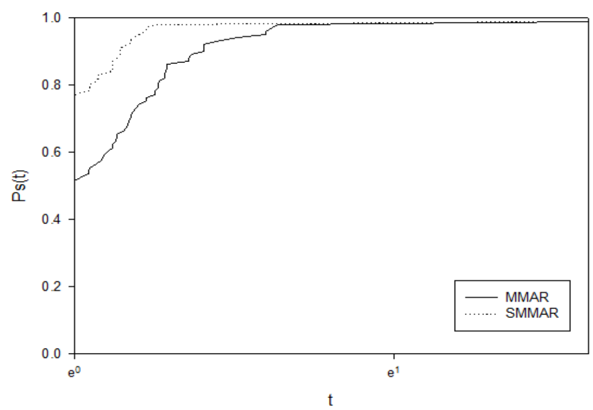

Figure 1: Performance profile based on number of Iteration.

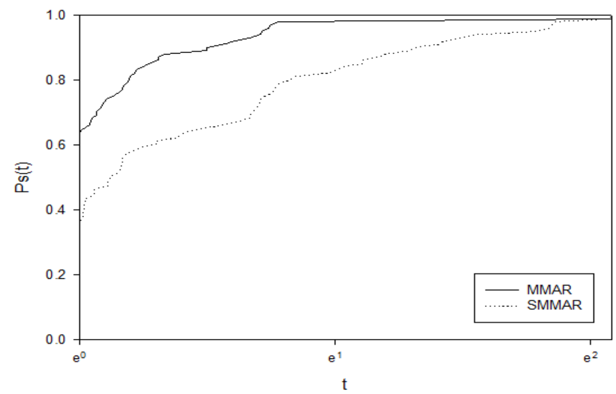

Figure 2: Performance profile based on CPU time.

5. CONCLUSION

In this study, we proved the global convergence and the sufficient descent property of the new spectral conjugate gradient method SMMAR with strong Wolfe line search. Numerical results show that SMMAR method outperforms MMAR conjugate gradient method in terms of the number of iterations almost in all tested functions. But MMAR method outperforms SMMAR conjugate gradient method in terms of CPU time almost in all tested functions.

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REFERENCES

1. Y. D. Fu, X. Y. Cheng and Y. H. Tang. Optimization theory and method (Chinese). Beijing: National Defense Industry Press, 2008.
2. M. Dawahdeh, M. Mamat, A. Alhawarat, and M. Rivaie. A New Modification of Conjugate Gradient Method with Global Convergence Properties. In *AIP Conference Proceedings*, 2013(1), 020038, 2018. https://doi.org/10.1063/1.5054237
3. J. Cai, Q. Li, L. Li, H. Peng, and Y. Yang. A fuzzy adaptive chaotic ant swarm optimization for economic dispatch. *International journal of electrical power and energy systems*, *34*(1), 154-160, 2012.
4. R. Fletcher and C. M. Reeves. Function minimization by conjugate gradients. *The computer journal*, *7*(2), 149-154, 1964. https://doi.org/10.1093/comjnl/7.2.149
5. M. R. Hestenes and E. Stiefel. Methods of Conjugate Gradients for Solving Linear Systems: *Journal of Research of The National Bureau of Standards*, 49, 409–435, 1952.
6. M. J. Powell, Nonconvex Minimization Calculations and The Conjugate Gradient Method. *Numerical Analysis*, 122-141, 1984. https://doi.org/10.1007/BFb0099521
7. E. Polak and G. Ribiere. Note on The Convergence of Methods of Conjugate Directions. *Revue Francaise d'Informatique Et De Recherche Operationnelle*, 3(16), 35-43, 1969.
8. Y. Liu and C. Storey. Efficient Generalized Conjugate Gradient Algorithms, Part 1: Theory. *Journal of Optimization Theory and Applications,* 69(1), 129-137, 1991. https://doi.org/10.1007/BF00940464
9. R. Fletcher. Practical methods of optimization, Wiley-Interscience John Wiley and Sons, New York, NY, USA, 2nd edition, 1987.
10. Z. Wei, S. Yao, and L. Liu. The Convergence Properties of Some New Conjugate Gradient Methods. *Applied Mathematics and computation,* 183(2), 1341-1350, 2006.
11. Y. H. Dai and Y. Yuan. A Nonlinear Conjugate Gradient Method with A Strong Global Convergence Property. *SIAM Journal on optimization,* 10(1), 177-182, 1999.
12. Z. Dai and F. Wen. Another Improved Wei–Yao–Liu Nonlinear Conjugate Gradient Method with Sufficient Descent Property. *Applied Mathematics and Computation*, 218(14), 7421-7430, 2012.
13. Y. Shengwei, Z. Wei, and H. Huang. A Note About WYL's Conjugate Gradient Method and Its Applications. *Applied Mathematics and Computation,* 191(2), 381-388, 2007.
14. L. Zhang. An Improved Wei–Yao–Liu Nonlinear Conjugate Gradient Method for Optimization Computation. *Applied Mathematics and Computation,* 215(6), 2269-2274, 2009.
15. M. Rivaie, M. Mamat, L. W. June, and I. Mohd. A new class of nonlinear conjugate gradient coefficients with global convergence properties. *Applied Mathematics and Computation*, 218(22), 11323–11332, 2012.
16. A. Alhawarat, M. Mamat, M. Rivaie, and I. Mohd. A new modification of nonlinear conjugate gradient

coefficients with global convergence properties. *International Journal of Mathematical, Computational, Statistical, Natural and Physical Engineering*, 8(1), 54–60, 2014.

17. E. Birgin and M. J. Martinez. A spectral conjugate gradient method for unconstrained optimization. *Appl. Math. Optim.* 43, 117-128, 2001.
18. L. Zhang, W. Zhou, and D. Li. Global Convergence of a Modified Fletcher–Reeves Conjugate Gradient Method with Armijo-Type Line Search. *Numerische Mathematik*, 104(4), 561-572, 2006.
19. N. Andrei. Scaled Conjugate Gradient Algorithms for Unconstrained Optimization. *Computational Optimization and Applications*, 38(3), 401-416, 2007.
20. H. Jiang, S. Deng, X. Zheng, and Z. Wan. Global Convergence of a Modified Spectral Conjugate Gradient Method. *Journal of Applied Mathematics, 2012*.
21. L. Zhang, W. Zhou, and D. Li. A Descent Modified Polak–Ribière–Polyak Conjugate Gradient Method and Its Global Convergence. *IMA Journal of Numerical Analysis*, 26(4), 629-640, 2006.
22. J. Liu, and Y. Jiang. Global Convergence of a Spectral Conjugate Gradient Method for Unconstrained Optimization, *Hindawi Publishing Corporation, Abstract and Applied Analysis*, Vol.12, Article ID 758287, 2012.
23. J. K. Liu, X. Du, and K. Wang. A Mixed Spectral CD-DY Conjugate Gradient Method. *J. Applied Mathematics*, 569795-1, 2012.
24. N. Zull, M. Rivaie, M. Mamat, Z. Salleh, and Z. Amani. Global Convergence of a New Spectral Conjugate Gradient by Using Strong Wolfe Line Search. *Applied Mathematical Sciences*, 9(63), 3105-3117, 2015.
25. W. Khadijah, M. Rivaie, M. Mamat, and I. Jusoh. A Spectral KRMI Conjugate Gradient Method Under the Strong-Wolfe Line Search. *AIP Conference Proceedings, AIP Publishing, 1739: 020072.* Doi: 10.1063/1.4952552, 2016.
26. N. Andrei. Scaled Memoryless BFGS Preconditioned Conjugate Gradient Algorithm for Unconstrained Optimization. *Optimization Methods and Software*, 22(4), 561-571, 2007.
27. N. Zull, W. Khadijah, U.K.M. Yusof, S. Shoid, M. Rivaie, and M. Mamat. A Comparative Study of the Spectral Conjugate Gradient Methods for Regression Analysis. *International Journal of Engineering and Technology*, 7, 316-320, 2018.
28. N. Andrei. Accelerated Scaled Memoryless BFGS Preconditioned Conjugate Gradient Algorithm for Unconstrained Optimization. *European Journal of Operational Research*, 204(3), 410-420, 2010.
29. C. Hu and Z. Wan. An Extended Spectral Conjugate Gradient Method for Unconstrained Optimization Problems. *British Journal of Mathematics and Computer Science*, *3*(2), 86-98, 2013. https://doi.org/10.9734/BJMCS/2013/2342
30. S. Babaie-Kafaki. Two Modified Scaled Nonlinear Conjugate Gradient Methods. *Journal of Computational and Applied Mathematics*, 261, 172-182, 2014.
31. Z. Wan, J. Guo, J. Liu, and W. Liu. A Modified Spectral Conjugate Gradient Projection Method for Signal Recovery. *Signal, Image and Video Processing*, 12(8), 1455-1462, 2018.
32. Z. Khoshgam and A. Ashrafi. A New Modified Scaled Conjugate Gradient Method for Large-Scale Unconstrained Optimization with Non-Convex Objective Function. *Optimization Methods and Software*, 34(4), 783-796, 2018.
33. G. Chandramouli and V. Narayanan. A Scaled Conjugate Gradient Based Direct Search Algorithm for High Dimensional Box Constrained Derivative Free Optimization. *arXiv preprint arXiv:*1901.05215, 2019.
34. J. K. Liu, Y. M. Feng, and L. M. Zou. A Spectral Conjugate Gradient Method for Solving Large-Scale Unconstrained Optimization. *Computers and Mathematics with Applications*, 77(3), 731-739, 2019.
35. A. Y. Al-Bayati and S. M. Abbas. A Robust Spectral Three-Term Conjugate Gradient Algorithm for Solving Unconstrained Minimization. *AL-Rafidain Journal of Computer Sciences and Mathematics*, 13(1), 87-104, 2019.
36. G. Zoutendijk. Nonlinear Programming Computational Methods. In: Abadie J. (Ed.) Integer and Nonlinear Programming, North Holland. 37-86, 1970.
37. I. Bongartz, A. R. Conn, N. Gould, and P. L. Toint. ACM Transactions on Mathematical Software, 21, 123–160, 1995.
38. N. Andrei. Advanced Modeling and Optimization, 10, 147–161, 2008.
39. E. P. Adorio and U. P. Diliman. Mvf-multivariate test functions library in C for unconstrained global optimization, 2005.
40. E. D. Dolan and J. J. Mor´e. Mathematical Programming, 91, 201–213, 2002.
41. I.M. Sulaiman, M. Mamat, A. Abashar, M. Rivaie. The global convergence properties of an improved conjugate gradient method *Applied Mathematical Science*; Vol 9, No. 38, pp. 1857-1868. 2015.
42. N. Aini, M. Rivaie, M. Mamat, and I. M. Sulaiman. A Hybrid of Quasi-Newton Method with CG Method for Unconstrained Optimization. *J. Phys.: Conf. Ser*. 1366(012079), pp. 1-10. 2019.
43. I. M. Sulaiman, Sukono, S. Sudradjat, and M. Mamat. New class of hybrid conjugate gradient coefficients with guaranteed descent and efficient line search. *IOP Conf. Ser.: Mater. Sci. Eng.* 621(012021), 2019.
44. P. Madhuravani, K. Yamini, T. Nirmala. Large Scale Community Discovery of Complex Networks Based On Approximate Optimization. International journal of emerging trends in engineering research. 7(9), 306-310, 2019. https://doi.org/10.30534/ijeter/2019/13792019