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Study of the Response of PW Traffic Flow Model to a Bottleneck on a Circular Road and its Suitability for Heterogeneous Traffic Conditions

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ABSTRACT

The traffic flow conditions in developing countries are predominantly heterogeneous. The early developed traffic flow models have been derived from fluid flow to capture the behavior of the traffic. The very first two-equation model derived from fluid flow is known as the Payne-Whitham or PW Model. Along with the traffic flow, this model also captures the traffic acceleration. However, the PW model adopts a constant driver behavior which cannot be ignored, especially in the situation of heterogeneous traffic.This research focuses on testing the PW model and its suitability for heterogeneous traffic conditions by observing the model response to a bottleneck on a circular road. The PW model is mathematically approximated using the Roe Decomposition and then the performance of the model is observed using simulations.

Key words : Heterogeneous Traffic, Payne-Whitham Model, Roe decomposition, Simulation.

1. INTRODUCTION

With the increase in population and interest of people in urban life, complications related to traffic have become common. Focus of town planners and engineers these days, is on solving the issues of traffic congestion and environmental pollution. However, it is not that simple. Observing and recording the traffic behavior on-site and then studying it is very laborious, time consuming and uneconomical. For this reason, mathematical modelling of traffic flow is carried out for the analysis of traffic flow and hence, using numerical simulations, the problems are identified and solutions are presented.

The types of traffic flow conditions can be classified as; homogeneous traffic flow and heterogeneous traffic flow. Homogeneous traffic flow can be recognized by the vehicles closely following the lanes. Also, the vehicles are not very diverse as far as the physical size is concerned [1]. In this type of traffic, the vehicles are inclined to stay within the center of their respective lanes. Conversely, heterogeneous traffic flow can be acknowledged by the absence of lane-discipline. Moreover, it is composed of both the engine-driven as well as non-engine-driven vehicles and the vehicles differ significantly from each other in terms of the physical size. Because of this, different vehicle types enjoy different operating conditions on the road [2]. There is continuous movement both in longitudinal and cross directions within the heterogeneous traffic flow as opposed to the homogeneous conditions [3].Hence, the interactions between the vehicles in heterogeneous traffic flow are much more intricate. This poses a serious challenge to traffic engineers and transport planners.

Speed, traffic density and traffic flow are the three traffic parameters considered in macroscopic traffic flow modeling and this approach is applicable to large road networks because of less complications, mathematically [4].The first macroscopic traffic flow model is called Lighthill, Whitham and Richards (LWR) Model and was developed first by Lighthill and Whitham[5] and then separately, by Richards [6]. The LWR model is based on a single equation that mimics the conservation law of fluid flow. The LWR model is mathematically represented as follows:

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho v(\rho))}{\partial x} = 0 \qquad (1)$$

where ρ is the traffic density and $v(\rho)$ is the equilibrium velocity distribution. Physically, this equation can be defined as the number of vehicles leaving a road section is equal to the number of vehicles entering it. LWR model is an easy to apply model however, it does not seize the traffic acceleration. Also, it undertakes equilibrium velocity distribution at every instant of time which is idealistic. Then, firstly Payne [7] and Whitham [8] separately, presented a two-equation model famously known as the Payne-Whitham or PW model. It is represented mathematically as follows:

$$\frac{\partial\rho}{\partial t}+\frac{\partial(\rho v(\rho))}{\partial x}=0 \quad (2)$$

$$\frac{\partial v}{\partial t}+\frac{v\partial v}{\partial x}=-\frac{C_0^2}{\rho}\frac{\partial\rho}{\partial x}+(\frac{v(\rho)-v}{\tau}) \quad (3)$$

where C_0^2 is known as the driver's behavior constant while $\tau$is known as the relaxation time. The second equation models the traffic acceleration. The first term on the right-hand side of this equation is called the driver behavior term while the second term is known as the relaxation term. This second equation has been derived from Navier-Stokes equation of fluid flow. PW model is considered as one of the state-of-the-art traffic flow models. However, because of the constant driver behavior assumption, it has been criticized in detail by [9]. After that, numerous traffic flow models have been developed by different researchers which can be briefly reviewed in [4].

This research concentrates on studying and observing the response of the PW model to a bottleneck on a circular road and then on the basis of this behavior, its suitability for heterogeneous traffic flow conditions is discussed.

In section 2, we present the numerical solution of our model using the Roe decomposition. In section 3, we put forward the simulation conditions and then the results and discussions on it. Lastly, section 4 comprises of the research conclusions

2. NUMERICAL SOLUTION OF THE MODEL

In this section, the PW model is numerically approximated using the Roe decomposition [10] which is a finite volume method. The Entropy Fix [11] is also applied so as to remove any discontinuous approximations at the ends of the cells. The Roe's scheme involves writing the equations in conservation form given as follows:

$$u_t+f(u)_x=S \quad (4)$$

where u_t is a matrix of traffic flow parameters, $f(u)_x$ is a matrix of the function of traffic flow parameters and S is called the source term. Since (2) is already in conservation form, (3) can be written in conservation form as follow:

$$(\rho v)_t+(\rho v^2+C_0^2\rho)_x=\rho(\frac{v(\rho)-v}{\tau}) \quad (5)$$

(2) and (5) are summarized in conservation form inTable 1.

Table 1: PW Model Conservation Vectors

Vectors	PW Model Parameters
u	$\begin{bmatrix}\rho\\ \rho v\end{bmatrix}$
$\boldsymbol{f(u)}$	$\begin{bmatrix}\rho v\\ \rho v^2+C_0^2\rho\end{bmatrix}$
S	$\begin{bmatrix}0\\ \rho(\frac{v(\rho)-v}{\tau})\end{bmatrix}$

According to Roe's scheme, firstly, the source term is assumed to be zero and thus (4) can be re-written as:

$$\frac{\partial u}{\partial t}+A(u)\frac{\partial u}{\partial x}=0 \quad (6)$$

where $A(u)$ is known as the Jacobian matrix. The Roe's scheme then approximates the velocity at the edges of the finite volumes as follows:

$$v=\frac{\sqrt{\rho_2}v_2+\sqrt{\rho_1}v_1}{\sqrt{\rho_2}+\sqrt{\rho_1}} \quad (7)$$

while the average density at the limits of finite volumes is calculated as:

$$\rho=\sqrt{\rho_2\rho_1} \quad (8)$$

After that, the matrix of the function of traffic parameters at these limits of the finite volumes is approximated using the following equation:

$$\begin{aligned}f_{i+\frac{1}{2}}^t(u_i^t,u_{i+1}^t)&=\frac{1}{2}\left(f(u_i^t)+f(u_{i+1}^t)\right)\\&\quad-\frac{1}{2}A\left(u_{i+\frac{1}{2}}\right)(u_{i+1}^t-u_i^t)\end{aligned} \quad (9)$$

For all the edges or boundaries, this matrix is estimated using(9). Finally, for the next time step and all distance steps, the traffic variables are estimated using the following equation:

$$u_i^{t+1}=u_i^t-\frac{dt}{dx}\left(f_{i+\frac{1}{2}}^t+f_{i-\frac{1}{2}}^t\right)+dtS(u_i^t) \quad (10)$$

In this way, the traffic parameters are estimated for the whole road section and at all time instants.

3. SIMULATION OF THE MODEL ON CIRCULAR ROAD

In this section, the PW model is simulated in MATLAB to assess the performance. The road conditions are selected such that they represent a circular or a ring road. It means that no vehicle enters or exits the road section. Rather, the vehicles at the end of the section are the ones entering the start of the same road section. The total length of the road section is taken as 200 meters while the total simulation time is chosen as 15 seconds. The maximum velocity is 20 m/sec and the relaxation time is 2.5 sec. The distance step and time step are selected such that they satisfy CFL stability condition [11]. The input constraints for simulation are given in Table 2.

The initial density conditions are given as follows:

$$\rho = \begin{cases} 0.1, & x < 90m \\ 0.7, & 90m \leq x \leq 112m \\ 0.1, & x > 112m \end{cases}$$

Table 2: Simulation Inputs

Input Parameters	Values
Total road length	200 m
Total simulation time	15 s
Distance step	2 m
Time step	0.01 s
Equilibrium velocity distribution	Greenshield
Relaxation time	2.5 s
Driver behavior constant	20 m/s
Maximum velocity	20 m/s
Maximum density	1

This means that at the distance less than 90 meters, the density is very low. But beyond that, the density suddenly increases to 0.7 until 112 meters. This can be pondered as a bottleneck at a distance of 91-112 meters. The bottleneck diminishes after that and the density reduces to 0.1 again. The bottleneck has been introduced to observe the behavior of the model for extreme traffic conditions. The velocity distribution at t=0, is assumed to be Greenshield distribution.

Before beginning simulations, the important points to be kept in mind for observing the behavior of the model includes observing the values of the densities as well as velocities and seeing if they are within the upper and lower limits. Another key point in observations will be the response of the model to the bottleneck.

The density conduct of the PW model can be observed in Figure 1. Observing the density conduct in general, the model yields quite practical values. In other words, the density is within the upper and lower boundaries.

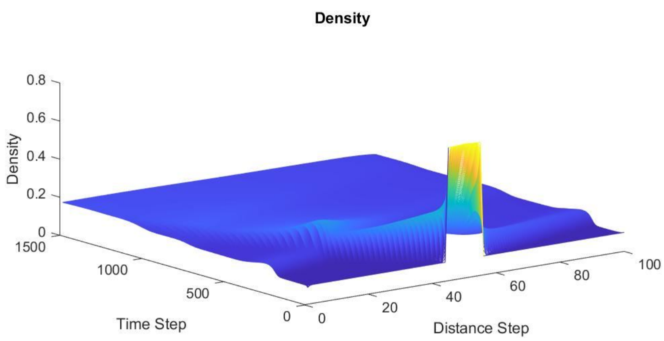

Figure 1: Density profile of the PW model

Despite this, aninadequacy in this behavior can be observed. The bottleneck is there but its influence cannot be seen at any other point throughout the simulation. The curve very quickly levels down. In reality, especially in case of heterogeneous traffic conditions, the impact of an obstruction like a bottleneck is definitely recorded. According to the projected reality, the effect of this bottleneck must travel backward and must result in amplified densities at other distances and moments. This abnormal conduct of the density can be more clearly observed in Figure 2. The initial bottleneck at time, t=0 is very clear. But no consequence of this bottleneck is observed at other time instants.

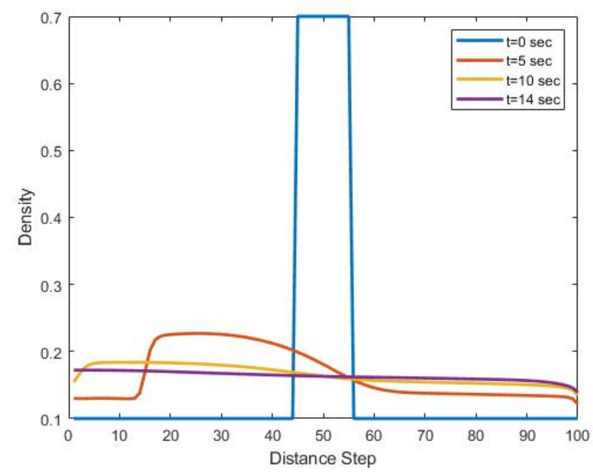

Figure 2: Density profile of the PW model in 2D

As far as the velocity performance of the PW model is concerned, it yields even more unrealistic values. As indicated earlier by [9], the PW model results in negative velocities which is not possible. In this simulation, it is also observed that the model exceeds the upper limit of velocity. This limitation of the model was also confirmed by [13], [14]. The velocity behavior of the PW model can be observed in Figure 3.

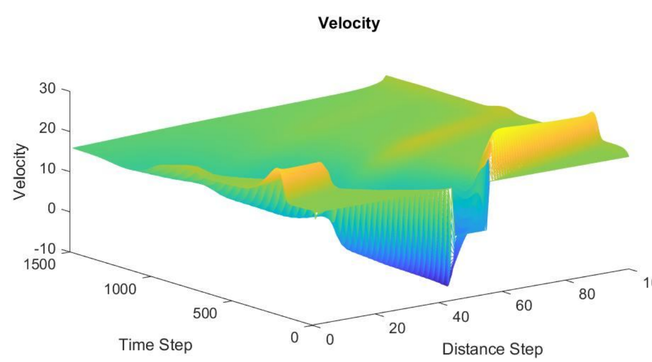

Figure 3: Velocity profile of the PW model

In detail, the velocity behavior can be observed in Figure 4. At time, t=0.43 seconds, the velocity reaches up to29.73 m/sec. While the maximum allowable velocity is 20 m/sec. This violation of the upper limit of the velocity can be recorded till time, t=4.3 seconds. Here, the velocity is just above 20 m/sec. The negative velocities can also be clearly seen. The velocities go as low as -6.73 m/sec at time, t=0.29 seconds. These negative velocities can be recorded till time, t=2.22 seconds, where the velocity is just below 0. The negative velocity displays right at the location where there is the bottleneck. This can be physical thought as when the vehicles reach the bottleneck, instead of just slowing down, they start moving in the reverse direction. This is one of the most criticized drawbacks of the PW model.

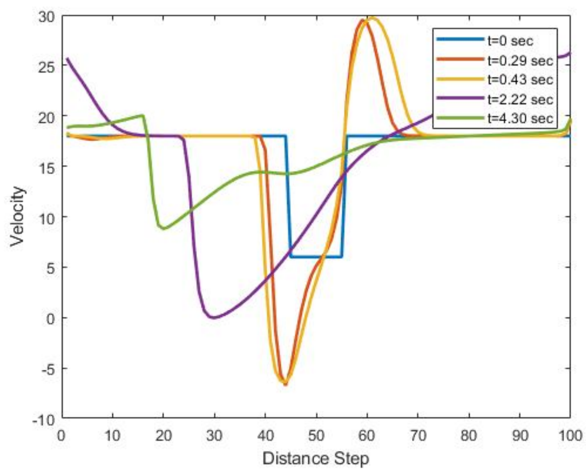

Figure 4: Velocity profile of the PW model in 2D

4. CONCLUSION

In this paper, the behavior of the PW model is studied by observing its response to a bottleneck on a circular road. On the basis of these results, its suitability for capturing and analyzing the heterogeneous traffic flow is determined.

The PW model was mathematically approximated using Roe decomposition along with the Entropy fix. The model was then numerically simulated for a circular or ring road.

The density behavior of the model indicated that there are no abnormal or impractical values. However, dropping of the densities very quickly to a uniform behavior indicated that the model did not respond to the bottleneck in a realistic manner.

In terms of velocity, the PW model violated both the upper as well as lower velocity limits. The PW model exhibited negative velocities right at the location of the bottleneck. Again, the response of the model to the bottleneck is nowhere near the reality.

In light of this, it can be concluded that the PW model responded idealistically to the bottleneck condition. Since, no consequence of this bottleneck was observed, it can also be concluded that the use of this model for studying heterogeneous traffic flow is not recommended.

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