



Fuzzy Max Plus Algebra Algorithm for Traffic Problems

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ABSTRACT

The traffic problem has a great importance due to its economic and environmental impact on delay life.

This paper introduces a new algorithm for solving the problem using Fuzzy Max Algebra Approach (FMAA). The proposed algorithm is relatively simple and efficient to give a suitable solution to this problem

Key words : Traffic Problems, Fuzzy programming technique, Max-Plus Algebra, Transportation.

1. INTRODUCTION

In recent years both industry and the academic world have become more and more interested in techniques to model, to analyze, and to control complex Discrete Event Systems (DES) such as flexible manufacturing systems, telecommunication networks, multiprocessor operating systems, railway networks, traffic control systems, logistic systems, intelligent transportation systems. The class of DES that can be described by a max-plus linear time-invariant model is only a subclass of the class of all DES. [1]

Cities and their transport systems are fully complementary. As defined by Rodrigue et al (2006), cities are locations with a high level of accumulation and concentration of economic activities, which form complex spatial structures that are supported by transport systems. The transportation systems according to Berry and Hurton (1970) are the veins and arteries of urban areas linking together social and functional zones[2].

Transportation or traffic problems are solved with the assumptions that the parameters of the transportation problem are specified in crisp environment. The parameters of the transportation problem are not always exactly known and stable. This imprecision may follow from the lack of exact information, uncertainty in judgment etc [3]. Today nearly all

cities in both developed and developing countries suffer from traffic congestion. It manifests itself predominantly in recurrent queues, delays and time wastage which commuters experience along major networks especially during rush hours. Due to incessant increase in population, increase in household incomes and its resultant increase in the level of car usage coupled with poor land-use planning, poor transport design and planning, traffic congestion has become an intractable problem in urban centers. (Rodrigue et al, 2006). [2]

Max-plus algebra dates from the early 1980's. Max-plus algebra in the field of discrete event systems appeared in Komenda, Lahaye, Boimond, and van den Boom . a class of ordinary 1 Petri nets, namely the Timed Event Graphs (TEGs) has been identified to capture the class of stationary max-plus linear systems (Cohen, Dubois, Quadrat, & Viot, 1985) and subsequent publications by Max Plus team. TEG which each place has a single input transition and a single output transition. A single output transition means that no conflict is considered for the tokens consumption in the place. For the large survey in (Baccelli et al., 1992; Butkovic, 2010; Gunawardena, 1998; Heidergott, Olsder, & van der Woude, 2006) . [4]

Zadeh introduced the notion of fuzziness, that was reinforced by Bellman and Zadeh. Hitchcock originally developed the basic transportation problem. Appa discussed several variations of the transportation problem. Zimmermann fuzzy linear programming has developed into seven fuzzy optimization methods for solving transportation problems. To find the fuzzy optimal solution of fuzzy transportation problems, where trapezoidal fuzzy numbers represent all the parameters, Pandian & Natrajan proposed a new algorithm, namely fuzzy zero point method. Gani et al obtained the fuzzy optimal solution of fuzzy transportation problems having parameters as trapezoidal fuzzy numbers[5]. In this paper using arithmetic operations triangular fuzzy number, for the treatment of uncertainty in traffic problems.

2. TRAFFIC PROBLEM

Traffic problem is one of the major problems in many metropolitan cities around the world. This traffic problem can

affect the economy, slow down the development, reduce the production, increase cost, and hamper people’s daily life. There are several causes that can create traffic problem in a big city. Among them increasing number of vehicles, shortage of sufficient roads and highways, and traditional traffic light system. All of these factors can create traffic congestion in the intersection but among them traditional traffic light system is one of the major factors. Traffic signals are common features of urban areas throughout the world, controlling number of vehicles. Their main goals are improving the traffic safety at the intersection, maximizing the capacity at the intersection and minimizing the delays[6].

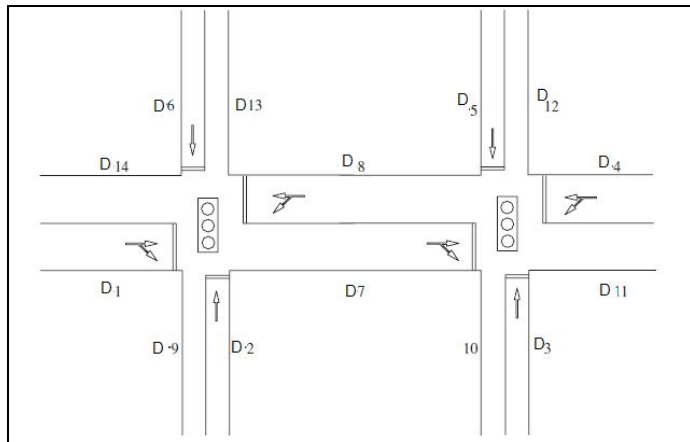


Figure 1: traffic area intersections.

3. AN OVERVIEW ON MAX-PLUS ALGEBRA

Max-plus algebra is a mathematical system in which the arithmetic operation of addition is replaced by determining the maximum, and the multiplication operation is replaced by the addition. This mathematical approach provides an interesting way suitable for modeling discrete event systems (DES) and optimization problems in production and transport. Moreover, there is a strong similarity with the classical linear algebra[7].

Max-plus algebra is one of many idempotent semi-rings which have been considered in various of mathematics. Max-plus algebra is becoming more popular not only because its operations are associative, commutative and distributive as in conventional algebra but because it takes systems that are non-linear in conventional algebra and makes them linear [8].

In this revolutionary era, the demands for transportation is increasing; there is a need to add new or alternative routes. The significant increase in the volume of transportation equipment that is not comparable to the expansion of roads will cause transportation problems such as traffic jams, delays, or even road accidents. increasing important as a means of public transportation, which is expected to be able to

overcome the occurring problems .Transportation movement issues, especially congestion problems in urban areas, often occur at crossroads. An intersection is a meeting place for road sections and places where traffic conflicts occur, functioning as a place for vehicles to change traffic direction movements. Intersections are a very important part of road network, this is due to their influence on the movement and safety of vehicle traffic flows. One of the methods used to overcome this congestion is by building underpasses. Kentungan-Gejayan underpass is one of the alternative traffic systems to facilitate the density that occurs in the area [9].

3.1 BASIC OPERATIONS OF THE MAX-PLUS ALGEBRA

The basic operations of the max-plus algebra are maximization and addition, which will be represented by \oplus and \otimes respectively where:

$$x \oplus y = \max(x,y) \text{ and } x \otimes y = x+y$$

The reason for using these symbols is that there is a remarkable analogy between \oplus and conventional addition, and between \otimes and conventional multiplication: many concepts and properties from linear algebra (such as the Cayley-Hamilton theorem, eigenvectors and eigenvalues, Cramer’s rule, . . .) can be translated to the max-plus algebra by replacing + by \oplus and \times by \otimes . Therefore, we also call \oplus the max-plus-algebraic addition, and \otimes the max-plus-algebraic multiplication.

The max-plus-algebra consists of two basic operators: maximization or the max-plus addition, \oplus and addition or the max-plus multiplication, \otimes . Compared to the conventional algebra, the max is replaced by \oplus and the + by \otimes .

$$A \oplus B = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \oplus \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \tag{1}$$

$$= \begin{bmatrix} a_{11} \oplus b_{11} & a_{12} \oplus b_{12} \\ a_{21} \oplus b_{21} & a_{22} \oplus b_{22} \end{bmatrix} \tag{2}$$

$$= \begin{bmatrix} \max(a_{11}, b_{11}) & \max(a_{12}, b_{12}) \\ \max(a_{21}, b_{21}) & \max(a_{22}, b_{22}) \end{bmatrix} \tag{3}$$

The max-plus addition of two matrices can be compared to the matrix addition in the conventional algebra. The same calculation rules apply here. It should be noted that, as in the conventional algebra, the max-plus addition of two matrices A and B is defined only if A and B have the same number of rows and the same number of columns, in other words, if A and B are of equal size.

The second operation is addition or the so called max-plus multiplication. This operation is represented by \otimes . Similar to the max-plus multiplication, the \otimes can be explained best with two examples

$$A \otimes B = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \otimes \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \tag{4}$$

$$= \begin{bmatrix} (a_{11} \otimes b_{11}) \oplus (a_{12} \otimes b_{21}) & (a_{11} \otimes b_{21}) \oplus (a_{12} \otimes b_{22}) \\ (a_{21} \otimes b_{11}) \oplus (a_{22} \otimes b_{21}) & (a_{21} \otimes b_{21}) \oplus (a_{22} \otimes b_{22}) \end{bmatrix} \tag{5}$$

$$= \begin{bmatrix} \max(a_{11} + b_{11}, a_{12} + b_{21}) & \max(a_{11} + b_{21}, a_{12} + b_{22}) \\ \max(a_{21} + b_{11}, a_{22} + b_{21}) & \max(a_{21} + b_{21}, a_{22} + b_{22}) \end{bmatrix} \tag{6}$$

Here a_{ij} and b_{ij} denote the elements in the i th row and the j th column of matrix A and B respectively.

Observe that, $A \otimes B \neq A + B$.

4. AN OVERVIEW ON FUZZY TECHNIQUES

Since its inception in 1965, the theory of fuzzy sets has advanced in a variety of ways and in many disciplines. Applications of this theory can be found, for example, in artificial intelligence, computer science, medicine, control engineering, decision theory, expert systems, logic, management science, operations research, pattern recognition, and robotics. Mathematical developments have advanced to a very high standard and are still forthcoming to day. Since 1992 fuzzy set theory, the theory of neural nets and the area of evolutionary programming have become known under the name of ‘computational intelligence’ or ‘soft computing’.

The relationship between these areas has naturally become particularly close. [9], [10], [11].

A research conducted recently has clearly stated that fuzzy logic based approaches are capable of improving the vehicle throughput and minimize unnecessary delays in highways (Kulkarni and Wainganka, 2007). It was done based on isolated junction with vehicle actuated controller. The vehicle-actuated controller operates based on the traffic demand as registered by the actuation of vehicle. Here the length of the green phase is adjusted between minimum and maximum length depending upon the traffic flow. Controlling the timing of a traffic signal indicates a constant evaluation to terminate the current phase and change to the next most appropriate phase or continue the current phase[9].

4.1 TRIANGULAR FUZZY NUMBER

Definition 1:

A fuzzy number \tilde{a} on R is said to be a triangular fuzzy number (TFN) or linear fuzzy number if its membership function $\tilde{a} : R \rightarrow [0, 1]$ has the following characteristics:

$$\tilde{a}(x) = \begin{cases} \frac{x - a_1}{a_2 - a_1}, & \text{for } a_1 \leq x \leq a_2 \\ \frac{a_3 - x}{a_3 - a_2}, & \text{for } a_2 \leq x \leq a_3 \\ 0, & \text{elsewhere} \end{cases} \tag{7}$$

We denote this triangular fuzzy number by $\tilde{a} = (a_1, a_2, a_3)$. We use F(R) to denote the set of all triangular fuzzy numbers.

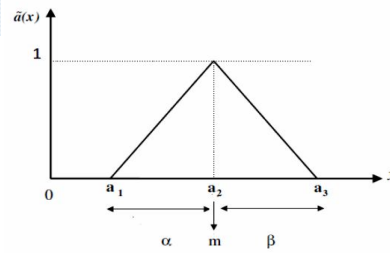


Figure 2: Triangular fuzzy number

5. PROPOSED APPROACH

In this section a proposed algorithm presented to solve traffic max algebra under fuzzy environment.

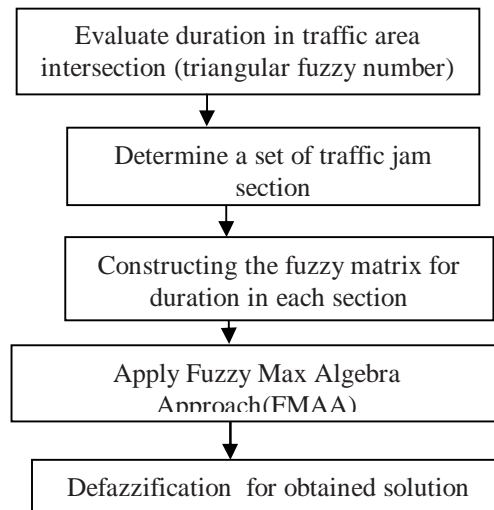


Figure 3: An approach for uncertain fuzzy max algebra traffic problem

5.1 MODELING AND SOLUTION OF THE FUZZY TRAFFIC PROBLEM (PROPOSED APPROACH)

A new method called (FMAA) in order to solve Fuzzy Max Algebra Approach for traffic problem. This method is very suitable for solve traffic max algebra problem under fuzzy environment. is shown in Figure 1.

i. EVALUATE DURATION IN TRAFFIC AREA INTERSECTION (TRIANGULAR FUZZY NUMBER)

The problem is described as a composition of the fuzzy traffic can be seen as continuation of the upstream the traffic. There exists association between traffic data of neighboring junctions or sections and that of target junctions or sections

ii. DETERMINE A SET OF TRAFFIC JAM SECTION

The problem at the rush hours there is Length of the vehicle queue, Frequency of vehicles arriving at the junction and Number of turning patterns in the junction

iii. CONSTRUCTING THE FUZZY MATRIX FOR DURATION IN EACH SECTION

Construct the fuzzy matrix of each row determine the durations of the of the selected area

$$\tilde{A} = \begin{bmatrix} \tilde{a}_{11} & \dots & \tilde{a}_{1n} \\ \vdots & \ddots & \vdots \\ \tilde{a}_{m1} & \dots & \tilde{a}_{mn} \end{bmatrix} \quad (8)$$

Where m – number of rows, n – number of columns , \tilde{a}_{ij} representing the performance fuzzy values of the i rows in terms of the j criterion , $\tilde{a}_{ij} = (a_{ij}^l, a_{ij}^m, a_{ij}^u)$ is defined as lower, medium and upper values respectively

iv. APPLY FUZZY MAX ALGEBRA APPROACH(FMAA)

a) Determining the fuzzy duration using max-plus addition \oplus in traffic problem:

$$\tilde{a} \oplus \tilde{b} = \begin{bmatrix} (a_{11}^l, a_{11}^m, a_{11}^u) & (a_{12}^l, a_{12}^m, a_{12}^u) \\ (a_{21}^l, a_{21}^m, a_{21}^u) & (a_{22}^l, a_{22}^m, a_{22}^u) \end{bmatrix} \oplus \begin{bmatrix} (b_{11}^l, b_{11}^m, b_{11}^u) & (b_{12}^l, b_{12}^m, b_{12}^u) \\ (b_{21}^l, b_{21}^m, b_{21}^u) & (b_{22}^l, b_{22}^m, b_{22}^u) \end{bmatrix} \quad (9)$$

$$= \begin{bmatrix} (a_{11}^l, a_{11}^m, a_{11}^u) \oplus (b_{11}^l, b_{11}^m, b_{11}^u) & (a_{12}^l, a_{12}^m, a_{12}^u) \oplus (b_{12}^l, b_{12}^m, b_{12}^u) \\ (a_{21}^l, a_{21}^m, a_{21}^u) \oplus (b_{21}^l, b_{21}^m, b_{21}^u) & (a_{22}^l, a_{22}^m, a_{22}^u) \oplus (b_{22}^l, b_{22}^m, b_{22}^u) \end{bmatrix} \quad (10)$$

$$= \begin{bmatrix} ((a_{11}^l \oplus b_{11}^l), (a_{11}^m \oplus b_{11}^m), (a_{11}^u \oplus b_{11}^u)) & ((a_{12}^l \oplus b_{12}^l), (a_{12}^m \oplus b_{12}^m), (a_{12}^u \oplus b_{12}^u)) \\ ((a_{21}^l \oplus b_{21}^l), (a_{21}^m \oplus b_{21}^m), (a_{21}^u \oplus b_{21}^u)) & ((a_{22}^l \oplus b_{22}^l), (a_{22}^m \oplus b_{22}^m), (a_{22}^u \oplus b_{22}^u)) \end{bmatrix} \quad (11)$$

$$= \begin{bmatrix} \max((a_{11}^l, a_{11}^m, a_{11}^u), (b_{11}^l, b_{11}^m, b_{11}^u)) & \max((a_{12}^l, a_{12}^m, a_{12}^u), (b_{12}^l, b_{12}^m, b_{12}^u)) \\ \max((a_{21}^l, a_{21}^m, a_{21}^u), (b_{21}^l, b_{21}^m, b_{21}^u)) & \max((a_{22}^l, a_{22}^m, a_{22}^u), (b_{22}^l, b_{22}^m, b_{22}^u)) \end{bmatrix} \quad (12)$$

$$\begin{bmatrix} (\max(a_{11}^l, b_{11}^l), \max(a_{11}^m, b_{11}^m), \max(a_{11}^u, b_{11}^u)) & (\max(a_{12}^l, b_{12}^l), \max(a_{12}^m, b_{12}^m), \max(a_{12}^u, b_{12}^u)) \\ (\max(a_{21}^l, b_{21}^l), \max(a_{21}^m, b_{21}^m), \max(a_{21}^u, b_{21}^u)) & (\max(a_{22}^l, b_{22}^l), \max(a_{22}^m, b_{22}^m), \max(a_{22}^u, b_{22}^u)) \end{bmatrix} \quad (13)$$

$$\begin{bmatrix} (\max(a_{11}^l, b_{11}^l), \max(a_{11}^m, b_{11}^m), \max(a_{11}^u, b_{11}^u)) & \dots & (\max(a_{1p}^l, b_{1p}^l), \max(a_{1p}^m, b_{1p}^m), \max(a_{1p}^u, b_{1p}^u)) & \dots & (\max(a_{1n}^l, b_{1n}^l), \max(a_{1n}^m, b_{1n}^m), \max(a_{1n}^u, b_{1n}^u)) \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ (\max(a_{m1}^l, b_{m1}^l), \max(a_{m1}^m, b_{m1}^m), \max(a_{m1}^u, b_{m1}^u)) & \dots & (\max(a_{mp}^l, b_{mp}^l), \max(a_{mp}^m, b_{mp}^m), \max(a_{mp}^u, b_{mp}^u)) & \dots & (\max(a_{mn}^l, b_{mn}^l), \max(a_{mn}^m, b_{mn}^m), \max(a_{mn}^u, b_{mn}^u)) \end{bmatrix} \quad (14)$$

b) Determining the fuzzy duration using max-plus multiplication, \otimes in traffic problem:

$$\tilde{a} \otimes \tilde{b} = \begin{bmatrix} (a_{11}^l, a_{11}^m, a_{11}^u) & (a_{12}^l, a_{12}^m, a_{12}^u) \\ (a_{21}^l, a_{21}^m, a_{21}^u) & (a_{22}^l, a_{22}^m, a_{22}^u) \end{bmatrix} \otimes \begin{bmatrix} (b_{11}^l, b_{11}^m, b_{11}^u) & (b_{12}^l, b_{12}^m, b_{12}^u) \\ (b_{21}^l, b_{21}^m, b_{21}^u) & (b_{22}^l, b_{22}^m, b_{22}^u) \end{bmatrix}$$

$$\begin{bmatrix} (b_{11}^l, b_{11}^m, b_{11}^u) & (b_{12}^l, b_{12}^m, b_{12}^u) \\ (b_{21}^l, b_{21}^m, b_{21}^u) & (b_{22}^l, b_{22}^m, b_{22}^u) \end{bmatrix} \quad (15)$$

$$= \begin{bmatrix} (a_{11}^l, a_{11}^m, a_{11}^u) \otimes (b_{11}^l, b_{11}^m, b_{11}^u) & (a_{12}^l, a_{12}^m, a_{12}^u) \otimes (b_{12}^l, b_{12}^m, b_{12}^u) \\ (a_{21}^l, a_{21}^m, a_{21}^u) \otimes (b_{21}^l, b_{21}^m, b_{21}^u) & (a_{22}^l, a_{22}^m, a_{22}^u) \otimes (b_{22}^l, b_{22}^m, b_{22}^u) \end{bmatrix} \quad (16)$$

$$= \begin{bmatrix} ((a_{11}^l \otimes b_{11}^l) \oplus (a_{11}^m \otimes b_{11}^m) \oplus (a_{11}^u \otimes b_{11}^u)) & ((a_{12}^l \otimes b_{12}^l) \oplus (a_{12}^m \otimes b_{12}^m) \oplus (a_{12}^u \otimes b_{12}^u)) \\ ((a_{21}^l \otimes b_{21}^l) \oplus (a_{21}^m \otimes b_{21}^m) \oplus (a_{21}^u \otimes b_{21}^u)) & ((a_{22}^l \otimes b_{22}^l) \oplus (a_{22}^m \otimes b_{22}^m) \oplus (a_{22}^u \otimes b_{22}^u)) \end{bmatrix} \quad (17)$$

$$= \begin{bmatrix} \max((a_{11}^l, a_{11}^m, a_{11}^u) \otimes (b_{11}^l, b_{11}^m, b_{11}^u)) & \max((a_{12}^l, a_{12}^m, a_{12}^u) \otimes (b_{12}^l, b_{12}^m, b_{12}^u)) \\ \max((a_{21}^l, a_{21}^m, a_{21}^u) \otimes (b_{21}^l, b_{21}^m, b_{21}^u)) & \max((a_{22}^l, a_{22}^m, a_{22}^u) \otimes (b_{22}^l, b_{22}^m, b_{22}^u)) \end{bmatrix} \quad (18)$$

v. DEFUZZIFICATION FOR OBTAINED SOLUTION

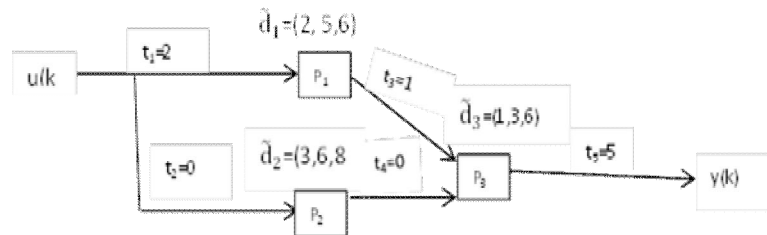
Convert the solution of traffic fuzzy max algebra problem output solution into suitable, real crisp variable.

$$Y = (Y_1, Y_2, Y_3) \\ Y_{out} = \frac{Y_1 + Y_2 + Y_3}{3} \quad (19)$$

Where y_{out} the final crisp results of FMAA.

vi. NUMERICAL EXAMPLE

This road consists of 3 streets: P₁, P₂, and P₃. The times for P₁, P₂, and P₃ are respectively. The fuzzy duration (\tilde{d}_1, \tilde{d}_2 and \tilde{d}_3), we assume that it takes $t_1 = 2$ time to get from the source to P₁ and that it takes $t_3 = 1$ time to reach P₁ to P₃. The other transportation times (t_2, t_4 , and t_5) are assumed to be negligible. A processing can only start working on new streets if it has finished the previous one. We Define



- u(k): time instant at which streets is fed to the system for the k_{th} time,
- xi(k): time instant at which the ith starts working for the k_{th} time,
- y(k): time instant at which the k_{th} finished vehicle leaves the system.

NUMERICAL EXAMPLE BY USING TRIANGULAR FUZZY

$$\tilde{A} = \begin{bmatrix} 2,5,6 & -\infty,-\infty,-\infty & -\infty,-\infty,-\infty \\ -\infty,-\infty,-\infty & 3,6,8 & -\infty,-\infty,-\infty \\ 9,11,13 & 10,12,15 & 1,3,6 \end{bmatrix}$$

6 RESULTS AND DISCUSSION

By applying the steps of FMAA, we obtain the results in table1, by Eqs. (9 to 18) and Defuzzification for obtained FMAA as listed in table 2 are calculated by Eqs. (19).

Table 1: Results of FMAA

| | x ₁ | x ₂ | x ₃ |
|----------------------|----------------|----------------|----------------|
| a₁ | 2 | 0 | 8 |
| | 2 | 0 | 8 |
| | 2 | 0 | 8 |
| a₂ | 5 | 4 | 12 |
| | 8 | 7 | 14 |
| | 9 | 9 | 16 |
| a₃ | 9 | 8 | 16 |
| | 14 | 4 | 20 |
| | 16 | 18 | 25 |
| a₄ | 15 | 14 | 22 |
| | 22 | 23 | 29 |
| | 25 | 29 | 36 |
| a₅ | 16 | 15 | 23 |
| | 25 | 27 | 33 |
| | 29 | 35 | 42 |

Table 2: Defuzzification for obtained FMAA

| | \tilde{y} | Y _{out} |
|----------------------|-------------|------------------|
| a₁ | 11 | 11 |
| | 11 | |
| | 11 | |
| a₂ | 15 | 17 |
| | 17 | |
| | 19 | |
| a₃ | 19 | 23.333 |
| | 23 | |
| | 28 | |
| a₄ | 25 | 32 |
| | 32 | |
| | 39 | |
| a₅ | 26 | 35.667 |
| | 36 | |
| | 45 | |

Table 3: Comparison of results obtained using proposed method and crisp method

| | Results by crisp max algebra | Results by FMAA |
|----------------------|------------------------------|-----------------|
| a₁ | 11 | 11 |
| a₂ | 17 | 17 |
| a₃ | 23 | 23.333 |
| a₄ | 32 | 32 |
| a₅ | 36 | 35.667 |

Then the final results of obtained with FMAA method the solution is better than the Crisp max algebra.

From above table 3 gives the results obtained by the proposed method which varies from crisp method because, the proposed method has the ability to handle fuzzy max algebra, while the other method can deal with the crisp data.

7. CONCLUSION

The main purpose of this paper is to solve the fuzzy max algebra in traffic problem. The problem has been described as a max algebra method under uncertainty. Anew fuzzy technique of the max algebra is applied to calculate the result.

The Fuzzy Technique expresses uncertainty, which is typical of real decision problems, with Triangular fuzzy number in the time matrix. Here, the Max-Plus Algebra technique using Triangular fuzzy number as the basis for evaluating values of The Traffic Problems

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