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Thermomechanical Solutions for Functionally Graded Beam subject to various Boundary Conditions

Manish Bhandari[1], Neeraj Sharma[2]
[1]MBM Engg. College, JNV University, Jodhpur, Rajasthan, India.
manishbha78@gmail.com
[2] Jodhpur Inst. of Engg. and Tech., Jodhpur, Rajasthan, India
Neeraj.itm91@gmail.com

ABSTRACT

In functionally graded materials one surface is generally a pure metal while the opposite surface is usually pure ceramic or a majority ceramic. The metal portion of the material acts in the role of a structural support while the ceramic provides thermal protection when subjected to harsh temperatures. The function describing the material variation throughout the material and more importantly the material property variation makes it possible to tailor the function to suit the needs of various applications. Effective properties of FGM are obtained by basic three laws i.e. Power Law (P-FGM), Exponential Law (E-FGM) and Sigmoid Law (S-FGM). In current work three types of beams are considered: cantilever, fixed and simply supported beam of uniform rectangular and I- cross-section composed of FGM and Metal individually. The beams are subjected to variable thermal environment under constant mechanical load. Corresponding to these loads, we can obtain deflection, stress and shear stress for both rectangular and I-cross section. Each beam is characterized by plotting the curve between deflection and temperature, stress and temperature and shear stress and temperature with the help of ANSYS Software.

Key words : Beam, rectangular, FGM, deflection, thermomechanical

1.INTRODUCTION

Functionally graded materials (FGMs) are heterogeneous materials with properties variation in a predefined and controlled manner following certain laws. Generally FGM is made of ceramic metal combination in which metal portion of the material acts in the role of a structural support while the ceramic provides thermal protection when subjected to harsh temperatures. Also the smooth transition through the various material properties reduces both thermal and residual stresses. The ceramic face of the material is generally exposed to a high temperature, while the metallic face is usually subjected to a relatively cooler temperature [1]. Praveen [2] examined the effects of the functional form of gradation including the presence and structural arrangement of monolithic Al_2O_3-Ni regions in combination with the graded region, on the thermal residual stresses, arising from the fabrication of a FGM system. Wang et.al. [3] explained that the most significant difference between the classical and shear deformation theories is the effect of including transverse shear deformation on the predicted deflections, frequencies, and buckling loads. Zero transverse shear stress condition of the upper and lower fibres of the cross-section is satisfied without a shear correction factor in TSDT. The relationship between the bending solution of TSDT and those of CBT and FSDT was presented. The governing equations were based on Reddy's [4] HSDT with a von Karman.Donnell-type of kinematic nonlinearity including thermal effects. Nirmala et. al. [5] derived an analytical expression to determine the thermoelastic stresses in a three layered composite beam system having an FGM as the middle layer. Guo et al [6] investigated the coupled thermoelastic vibration characteristics of axially moving beams using differential quadrature (DQ) method. Rout [7] presented Stability of Functionally Graded Timoshenko Beam. The effects of material composition, temperature dependent properties and slenderness ratio on thermal buckling and vibration of FG beams are investigated by Wattanasakulpong et al [8]. Aminbaghai et al. [9] carried out the modal analysis of second order shear deformable FGM-beams considering property variations in both transverse and longitudinal directions. Aldousari [10] analysed the thermal buckling behaviour under uniform and non-uniform temperature rise without considering time-dependent temperature rise. Samir et.al. [11] developed two symmetric and anti-symmetric functions and compared their effects on the static deflection and bending stresses with classical power-law distribution. They found that the symmetric power function is more reliable and can considerably reduce the stress than the other two functions. They presented a solution to predict the interfacial stresses of a FG graded beam reinforced by a prestressed CFRP plate under thermo-mechanical load. A finite element (FE) analysis is also employed to carry a parametric study to identify the effects of various material and geometrical properties on the magnitude of interfacial stresses. Brajesh and Gautam [12] obtained first non-linear differential equations of motion of a beam Timoshenko beam theory, converted into a set of non-linear algebraic equations using harmonic balance method. Subsequently an amplitude incremental iterative technique is imposed in order to obtain steady-state solution in frequency amplitude plane. It was observed that the domain of applicability of the method is enhanced up to a considerable extent as the stable and unstable solution can be captured. Bhandari [13] studied

FGM plate under thermomechanical loading. Zhang et.al. [14] studied thermoelastic behaviour of simply supported thick beams with temperature-dependent material properties under thermomechanical loads. The heat conduction analysis was based on the one-dimensional Fourier's law, and the displacement and stress analysis was based on the 2-D thermoelasticity theory. The solution of temperature field across the thickness has been obtained by dividing the beam into a series of thin slices, the temperature and the material properties in each slice were considered to be uniform. The state space method has been used to give the displacements and stresses for every slice. The transfer-matrix method was utilized to give the displacements and stresses for the beam. Finally, an example is conducted to analyze the temperature, displacement, and stress fields in a carbon steel beam. The example reveals that the temperature not only produces displacements and stresses itself but also affects the displacements and stresses induced by the mechanical load.
A thermo-mechanical analysis of functionally graded beams has been carried out in this paper. Several one-dimensional displacements-based beam models have been derived by means of a unified formulation, via this formulation, higher-order theories as well as classical Euler–Bernoulli's a Timoshenko's models can be formulated straightforwardly. Beams made of a single FGM layer as well as a sandwich configurations have been studied. Slender and thick beams have been investigated in terms of temperature, displacements and stresses. Results have been validated through comparison with three-dimensional FEM solutions obtained via the commercial code ANSYS. It has been shown that the considered thermo-mechanical problems, although presenting a global bending deformation, are governed by three-dimensional stress fields that call for very accurate models. Through an appropriate choice of the approximation order over the cross-section, the proposed formulation yields accurate results with reduced computational costs.

2. METHODOLOGY

2.1 Governing Equation

The FGM can be produced by continuously varying the constituents of materials in a predetermined profile. The most distinct features of an FGM are the non-uniform structures with continuously graded properties. An FGM can be defined by the variation in the volume fractions. Most researchers use the power-law function (P-FGM), exponential function (E-FGM), or sigmoid function (S-FGM) to describe the volume fractions. In this study, properties distribution is defined by power law function and method of problem solution can be extended for other type of distributions.
Let us consider an Exponential-FGB, with different exponential variations for distributions of thermal expansion, thermal conductivity and modulus of elasticity through the thickness direction of beam, respectively

$$\alpha_{(z)} = A_\alpha . e^{\beta . z},\ A_\alpha \sqrt{\alpha_1 \alpha_2}\ ,W=\frac{1}{h}\ln\left[\frac{\alpha 2}{\alpha 1}\right] \quad \textbf{(1)}$$

$$E_{(z)} = A_E . e^{\lambda . z},\ A_E \sqrt{\alpha_1 \alpha_2}\ ,W=\frac{1}{h}\ln\left[\frac{E2}{E1}\right] \textbf{(2)}$$

Where α and E are the material properties in the z=-h/2, z=h/2 surfaces, respectively and the constants A_α, A_β, A_λ, α, β and λ can be obtained by boundary conditions (BCs).

Then strain in the x-direction as follows,

$$\varepsilon_{xx}=\frac{\partial u_o}{\partial x}+z\frac{\partial^2 y}{\partial x^2}+z\frac{\partial \emptyset_x}{\partial y} \quad \textbf{(4)}$$

The strain in z-direction is because of temperature

$$\varepsilon_z = \alpha_{(z)} T_{(z)} \quad (5)$$

The strain considering Poisson's ratio

$$\varepsilon_{zz} = \left[\frac{\varepsilon_{xx}}{1+v} - \alpha_{(z)} T_{(z)}\right] \quad (6)$$

Based on the plane strain condition, stress-strain relation for an FGB that subjected to thermal load is [2];

$$\sigma_{xx} = \frac{E_z}{1-v}\left[\frac{\varepsilon_{xx}}{1+v} - \alpha_{(z)} T_{(z)}\right] \quad \textbf{(7)}$$

$$\sigma_{xx} = \frac{E_z}{1-v^2}\left[\frac{\partial u_o}{\partial x} - z\frac{\partial^2 w}{\partial x^2} + z\frac{\partial \emptyset_y}{\partial x} - \alpha_{(z)} T_{(Z)}(1+v)\right] \textbf{(8)}$$

Where σ_{xx} (z), v and T_z are the axial stress on the surface with distance z from mid surface, the Poisson's ratio and temperature distribution along z-direction of beam, respectively. The unknown terms may be obtained by using the equilibrium equations and boundary conditions.

The stress resultants per unit length of the middle surfaces are defined by integrating stresses along the thickness, assuming the thermal loading only with distribution in z-direction. When the beam is in equilibrium, the axial resultant forces in the x-direction must be zero, i.e.

$$\sum F_{x=0} \Rightarrow \int_{-h/2}^{h/2} \sigma_{xx(z)} b dz = 0,\ \ 0 \le x \le l \quad \textbf{(9)}$$

Where, b and l are the width and length of beam, respectively.

It is evidence that axial stress can be obtain from Eq. (8) by substituting for C_1, C_2, and C_3 which are constants obtained from boundary conditons.

$$\sigma_{xx} = \frac{E_z}{1-v^2}\left[C_1 - zC_2 + zC_3 - \alpha_{(z)} T_{(Z)}(1+v)\right] \textbf{(10)}$$

If the temperature distribution is assumed to be T=T (z), the thermal bending takes place in the X-Z plane and the shear stress is obtained by Eq. (9)

$$\tau_{xz} b dx + b\int_z^{h/2}\left(\sigma_{xx} + \frac{\partial \sigma_{xx}}{\partial x} dx\right) dz - b\int_z^{h/2} \sigma_{xx} dz = 0 \quad \textbf{(11)}$$

Where $-h/2 \le z \le h/2$

Where, τ_{xz} is the transverse shear stress. Rearranging the terms yields:

$$\tau_{xz} = -\int_z^{h/2} \frac{\partial}{\partial x}\left[\frac{E_z}{1-v^2} \times \left(C_1 - zC_2 + zC_{3} - \alpha_{(z)} T_{(Z)}(1+v)\right]\right) dz \quad \textbf{(12)}$$

2.2 Problem formulation

Two FGM beams made of Ni-Al_2O_3 are considered here. The properties of Ni and Al_2O_3 are listed in Table 1.

Table 1 : Numerical data of thermal and mechanical properties of Ni and Al_2O_3

Sr. No	Properties	Constituents	
		Ni	Al_2O_3
1.	Density (kg/m^3)	8900.0	3970.0
2.	Young's modulus (GPa)	199.5	393.0
3.	Poisson's ratio	0.3	0.25
4.	Specific heat (J/kg K)	444.0	775.0
5.	Thermal conductivity (W/m K)	90.7	30.1
6.	Thermal expansion coefficient (°Cx10^{-6})	13.3	8.8

One beam is of rectangular section and the other is of I section. The width and depth are 200mm and 300mm of rectangular section beam. The I section beam is having 200mm as flange width, 300 mm as overall depth, flange thickness is 20mm and web thickness is 10mm. The span of the beam is taken as 3000mm. Bottom surface is kept at room temperature (290K) while top surface is subjected to variable temperature. Furthermore, the condition of no temperature gradient is assigned at the left- and right-hand sides in order to exclude the boundary effect (i.e., for the x-invariant temperature field). With this FGM model, parametric studies have been conducted using FEM and ANSYS (Figure 1 and 2). Parameters such as deflection, stress and shear stress are calculated.

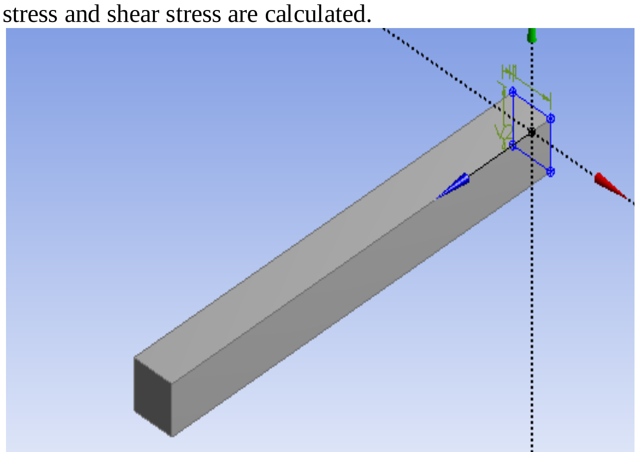

Figure 1: Rectangular solid models

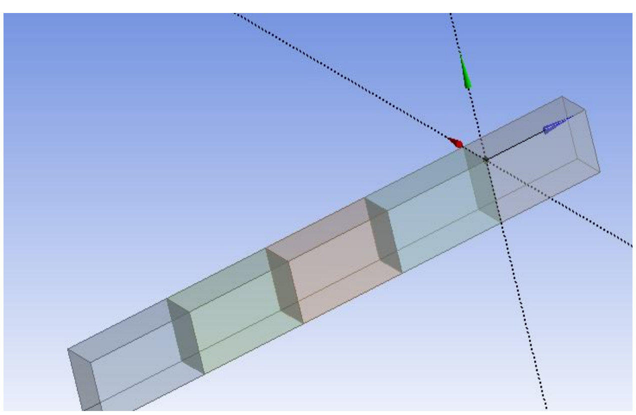

Figure 2: Rectangular solid model divided into elements

3. RESULTS

The deflection, stress and shear stress are computed under variable thermal environment and a constant mechanical load (0.8 and 1.2 MPa). The beam is subjected to various boundary conditions such as fixed beam and simply supported beam.

3.1 Deflection

Figures 3 and 4 show deflection for fixed beam under 0.8 MPa and 1.2 MPa respectively. Figures 5 and 6 show deflection for simply supported beam under 0.8 MPa and 1.2 MPa respectively. It can be observed from Figures 3-6 that Deflection of Metal beams of I and rectangular section observe greater value under different temperatures compared to corresponding FGM beams. Also the deflection increase with increase in temperature almost linearly. The value of maximum deflection in case of fixed beam is found to more than simply supported beam for the similar conditions. The strain is function of temperature increase as in Eq. (5), the linear increase of deflection may be explained.

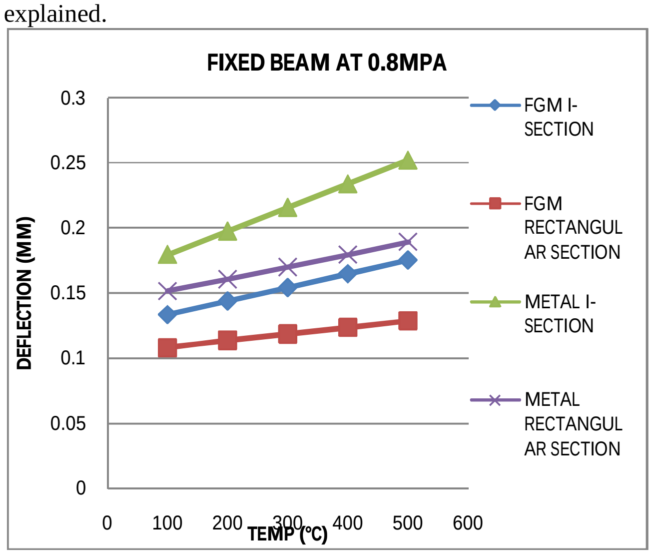

Figure 3: Deflection of FGM and metal for fixed beam with temp at load 0.8 MPa

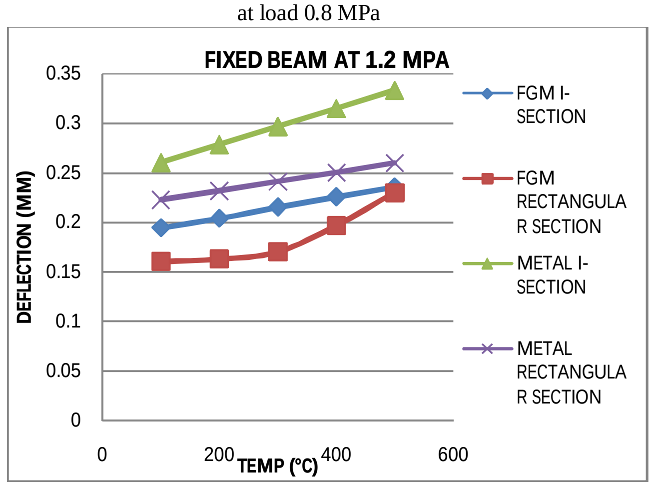

Figure 4: Deflection of FGM and Metal for Fixed Beam with Temp at Load 1.2 MPA

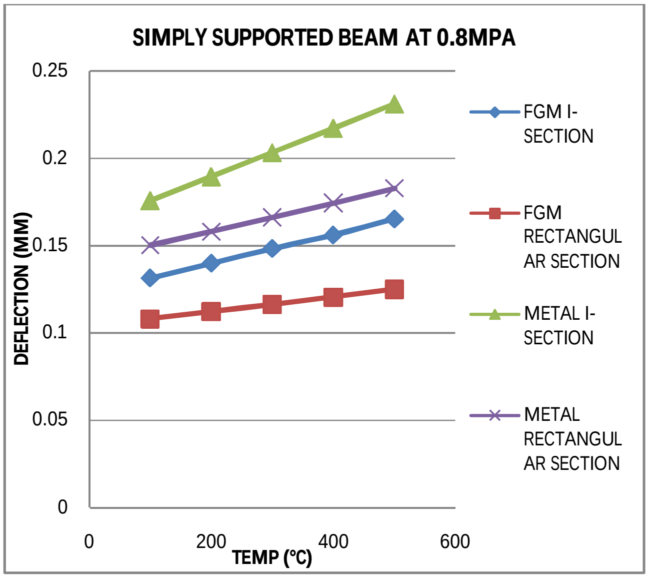

Figure 5: Deflection of FGM and metal for simply supported beam with temp at load 0.8 MPa

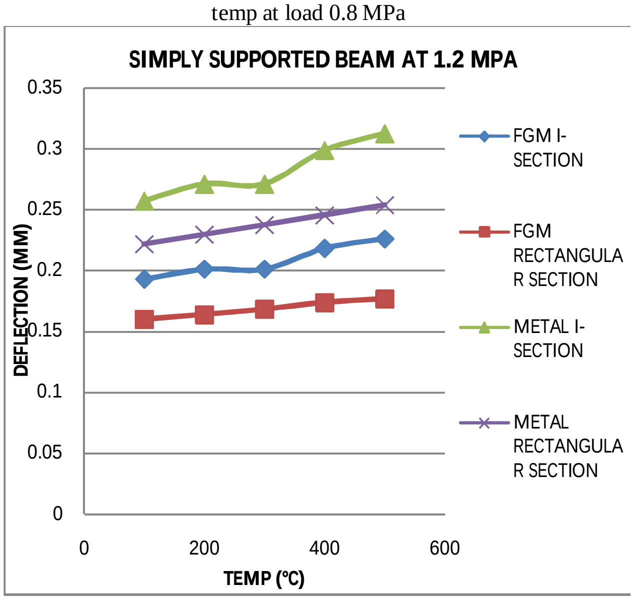

Figure 6: Deflection of FGM and metal for simply supported beam with temp at load 1.2 MPa

3.2 Stress Behaviour

Figures 7-9 show stress behaviour for Cantilever, fixed and simply supported beams under 1.2 MPa respectively. It can be observed that Metal rectangular section fixed beam has more stress than FGM rectangular section fixed beam under different temperature. FGM I-section fixed beam has lesser stress than Metal I-section fixed beam. Also temperature-stress curve for all beams under different temperature is observed to be linear. The value of the stress in fixed and simply supported beam s is found to be more than the stress in the cantilever beam for the similar operating conditions.

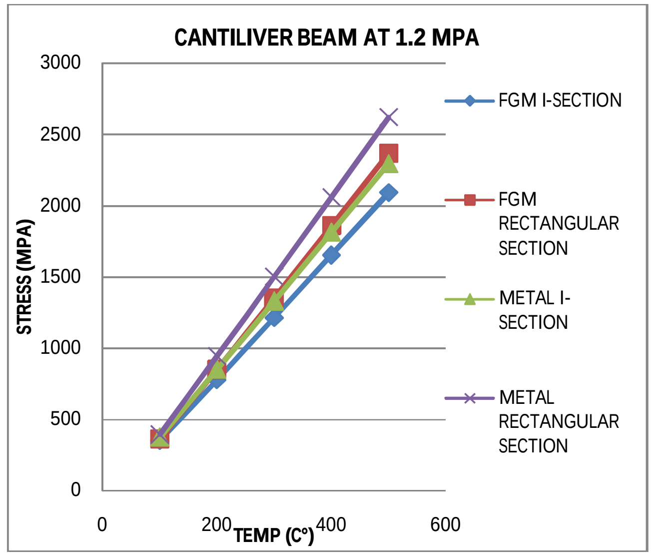

Figure 7: Stress of FGM and metal for cantilever beam with temp at load 1.2 MPa

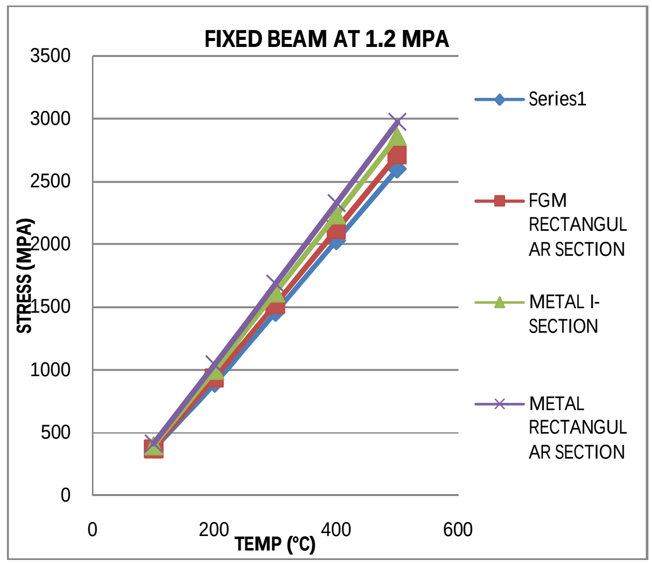

Figure 8: Stress of FGM and metal for fixed beam with temp at load 1.2 MPa

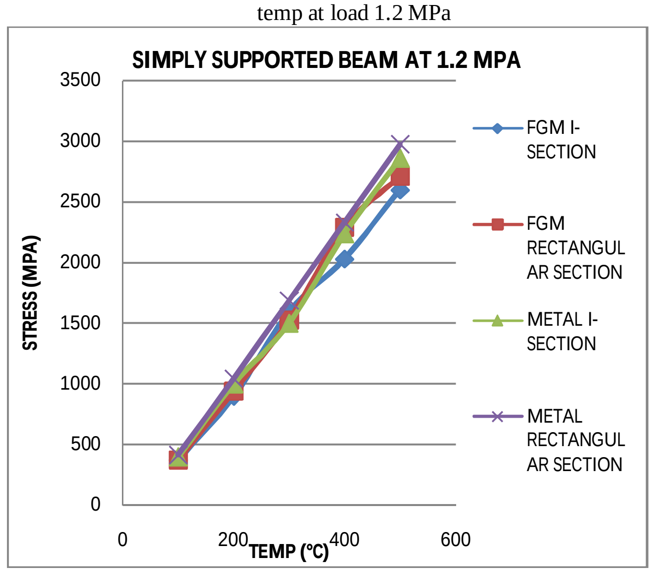

Figure 9: Stress of FGM and metal for simply supported beam with temp at load 1.2 MPa

3.3 Shear Stress

In shear stress behaviour we have plotted the curves against shear stress and temperature for different beam material and cross-section for comparable analysis between beams to find that which beam is preferred. Figures 10-12 show shear stress variation with temperature for cantilever, fixed and simply supported beam respectively. It can be depicted by closely observing the figures 3.8-3.10 that Metal I-section fixed beam has less shear stress than FGM I-section fixed beam under different temperature. The shear stress-temperature curve for all beams under different temperature is found to be linear.

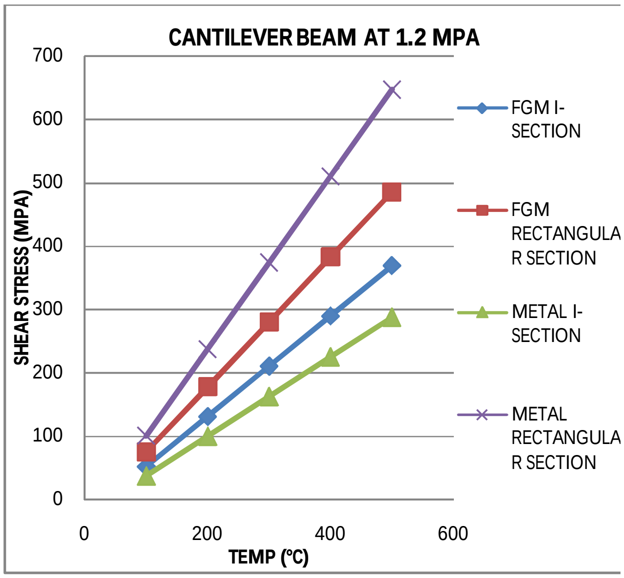

Figure 10: Shear stress of FGM and metal cantilever beam with temp at load 1.2 MPa

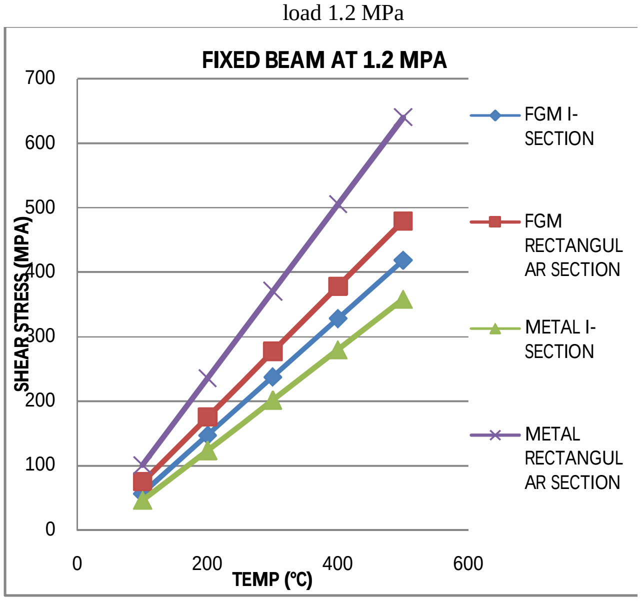

Figure 11: Shear stress of FGM and metal fixed beam with temp at load 1.2 MPa

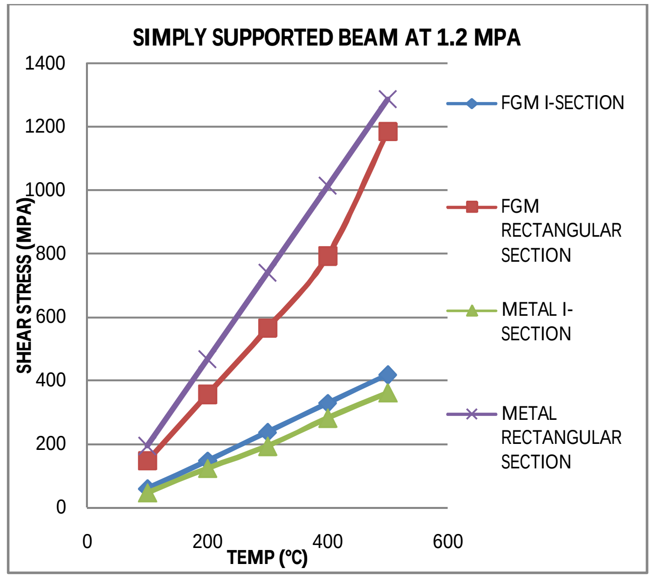

Figure 12: Shear stress of FGM and metal simply supported beam with temp at load 1.2 MPa

4.CONCLUSION

Functionally graded materials are good replacement of composite materials because they overcome the debonding type problems. The basic properties of FGM can be obtained by any of the three function laws, power law (P-FGM),sigmoid law (S-FGM) and exponential law (E-FGM). In the present work the deflection, stress and shear stress in FGM beam under different load and different temperature are calculated using ANSYS software. All curves follows linear pattern with shear stress and different loading conditions. FGM rectangular section cantilever beam has less deflection than Metal rectangular section cantilever beam under different temperature and constant load. The Deflection-temperature curve is linear under constant load. FGM I-section cantilever beam has lesser stress than Metal I-section cantilever beam under different temperature. A further investigation regarding the techniques for estimating effective material properties of FGM beam is desirable. In the graded layer of real FGMs ceramic and metal particle of arbitrary shapes are mixed up in arbitrary dispersion structures. Hence, the prediction of the thermo-mechanical properties is not a simple problem, but complicated due to the shape and orientation of particles, the dispersion structure, and the volume fraction. This situation implies that the reliability of FGM beam property.

FGMs are a relatively new material and the potential for future research is promising in this area of Thermal Engg. The specific FGM used in this experiment lends itself to further study in other areas to help characterize these beams and to determine the deflection, stress and shear stress of the beam in future applications.

Variations in temperature gradients applied to these beam should also be studied. FGMs beam are only beneficial in harsh temperature gradient environments, so it would be valuable to study how effective they are at providing thermal protection.

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