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Fractional Model Architecture for a Crude-gas Turbulent Flow

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ABSTRACT

Currently, petroleum is one of the primary economic resources for Mexico and other developing countries. The legal production begins at the wells and storage systems; once required, it is transported to the refinery or the ship exportation. The oil industry uses various mathematical models to know the pressure drop in a pipeline or reproduce different variables included; however, they require lengthy analysis times that can be reduced with solutions of analytical differential equations. In the present work, a model is developed to know the pressure drops in the pipeline but from the fractional calculation, reducing the analysis steps and developing predictions quickly. The obtained model allows the reproductionof the velocity profiles for the various Reynold values that characterize the exposed flow.

Key words: Fractional oil model, heavy crude oil industry, pipeline flow mathematical model.

1.INTRODUCTION

The transport of high viscosity fluids leads to high-pressure losses due to friction related to high costs associated with the pumping power required in these systems [1,2]. One of the solutions to mitigate these effects is based on the use of chemical products (flow improvers) to reduce the viscosity of the fluid or to change flow patterns[3,4], without significantly affecting the quality of the final product and the operations associated with obtaining this product, looking a stabilization of heavy compounds[5].When the fluid is very viscous, it generally flows in a laminar regime, such that the improver must be injected at a point before pumping, or it must be mixed with the fluid in equipment designed for this purpose. Suppose this equipment is not available and the improver injection occurs after pumping. In that case, a two-phase flow is generated, where pressure losses depend not only on fluids viscosityand composition but also on the form of improver injection [6].The most desirable effects are obtained when the improver is injected in contact with the walls of the tube. Thus, it is necessary to know the velocity profile established in the system and how it is temporarily generated when the enhancer injection occurs [7], and other possibilities that can change fluid flow [8].

In this context, the work's objective is to obtain the expressions of the transitory behavior of the velocity and flow profile for a specified pressure gradient when the viscous fluid has a non-Newtonian rheological behavior considering a single-phase system but with a fractional model [9].

Nowadays, transport in multiphase flow is an essential operation in many industrial processes like food, viscous liquid production, and petroleum pipelines.

This work aims to present a model to describe the pressure drop behavior of a flow system where there are two phases. For model obtaining, it was established:

1. the flow is moving between two parallel plates.
2. the distance H between both plates is smaller than their width Y so that the velocity v only depends on H.
3. Steady-state
4. Viscosity and density are constants.
5. the flow regime of each flow phase can be laminar or turbulent.

2.MODEL

Turbulent velocity profile of one phase flow

It is well known that it is obtained from continuity and change equations for rectangular coordinates and established suppositions.

$$0=\frac{\Delta P}{L}+\frac{\mu}{\rho}\frac{d^2v}{dx^2}$$
$$v(0)=0 \qquad (1)$$
$$v(H)=0$$

where v is the mass velocity kg.m^{-2}.s^{-1}, p is pressure drop Pa, u is the viscosity Pa.s, x m the distance from a plate, and L m the length of both plates (Figure 1).

As both plates come into direct contact with the same liquid, the velocity profile must be symmetrical. To consider this, defined as the dimensionless variables:

$$\phi = \frac{v}{V} \quad (2)$$
$$\gamma = \frac{(2x-H)}{H}$$
$$\Phi = \frac{1}{4}\frac{H^2}{LV}\frac{\rho}{\mu}\Delta P$$
$$\gamma \in (-1,1)$$

where V is average mass velocity, in such a way that equation (1) can be written as:

$$0 = \Phi + \frac{d\tau}{d\gamma} \quad (3)$$
$$\tau(0) = 0$$
$$\tau = \frac{d\phi}{d\gamma} \quad (4)$$
$$\phi(\pm 1) = 0$$

where τ represents the shear stress, which has a linear spatial behavior in the laminar regime. But, in a turbulent regime, thevelocity fluctuations influence the average velocity so that the shear stress shows a non-linear behavior. This fact can be taken into account if the differential equations (3) is considered as a fractional differential equation:

$$0 = \Phi + \frac{d^\alpha \tau}{d\gamma^\alpha}$$
$$\tau(0) = 0 \quad (5)$$
$$\alpha \geq 1$$

The fractional derivative in equation (5) directly relates to the turbulence level and equals 1 in the laminar regime. In this context, α is identified with the turbulence index, which can quantify the flow turbulence level.
The value of shear stress must be a number, and if it is considered that:

$$\tau(0) = 0$$
$$\left(\frac{d\tau}{d\gamma}\right)_{\gamma=0} = 0 \quad (6)$$

then the solution of a fractional differential equation (5) is:

$$\tau = -\Phi \frac{|\gamma|^\alpha}{\Gamma(\alpha+1)} \quad (7)$$

The velocity profile is obtained substituting equation (7) in equation (3) and solving its:

$$\phi = \frac{\Phi}{\alpha(1+\alpha)\Gamma(\alpha)}\left(1-|\gamma|^{\alpha+1}\right) \quad (8)$$

where the shear stress is given by:

$$\tau = -\frac{\Phi}{\alpha\Gamma(\alpha)}|\gamma|^\alpha \frac{|\gamma|}{\gamma} \quad (9)$$

It is unknown the value of α, but it can be estimated from the empirical relationship used frequently to calculate the flow or pressure drop in a turbulent flow. The average velocity is given by:

$$\langle \phi \rangle = \Phi \frac{\int_0^y \int_{-1}^1 \left(1-|\gamma|^{\alpha+1}\right) d\gamma dy}{\int_0^y \int_{-1}^1 d\gamma dy} \quad (10)$$

If it is considered the dimensionless variables (equation 2), then it is obtained:

$$\Delta P = \frac{2L\mu V\Gamma(\alpha+1)(\alpha+2)}{\rho H^2} \quad (11)$$

For this system, the equivalent diameter is given by:

$$D_e = 4\frac{HY}{2H+2Y} \quad (12)$$
$$D_e = 2H$$
$$Y >> H$$

and from the macroscopic balance:

$$\Delta P = \frac{2f}{\rho}V^2 \frac{L}{2H} \quad (13)$$

where f is the friction factor. By substituting equation (13) in equation (11), it is obtained:

$$f = \frac{2}{HV}\mu\Gamma(\alpha+1)(\alpha+2) \quad (14)$$
$$= \frac{4}{R_e}\Gamma(\alpha+1)(\alpha+2)$$

where Re is the Reynolds number given by:

$$R_e = \frac{2HV}{\mu} \quad (15)$$

Notice that If f is known, then α can be calculated using numeric methods of solution.
the friction factor can be estimated from empirical correlations, so:

$$f = \frac{0.0791}{R_e^{\frac{1}{4}}} \quad (16)$$
$$2.1\times10^3 < R_e < 10^5$$

$$\frac{1}{f^{\frac{1}{2}}} = -3.6\log\left(\frac{6.9}{R_e} + \left(\frac{r}{3.7}\right)^{\frac{10}{9}}\right)$$
$$4\times10^4 < R_e < 10^8 \quad (17)$$
$$0 < r < 0.05$$

Where $r = \frac{k}{2H}$

and k is the wall roughness.Notice that at laminar regime:

$$f = \frac{12}{R_e} \quad (18)$$

therefore:

$$\frac{12}{R_e} = \frac{4}{R_e}\alpha(\alpha+2)\Gamma(\alpha)$$
$$3 = \alpha(\alpha+2)\Gamma(\alpha)$$
$$\alpha = 1$$

which is the expected result.

The behavior of turbulence index αabout Reynolds number is given in Figure 2. As can be observed, it is predicted that the turbulent index increases with the Reynolds number, which is an expected result.

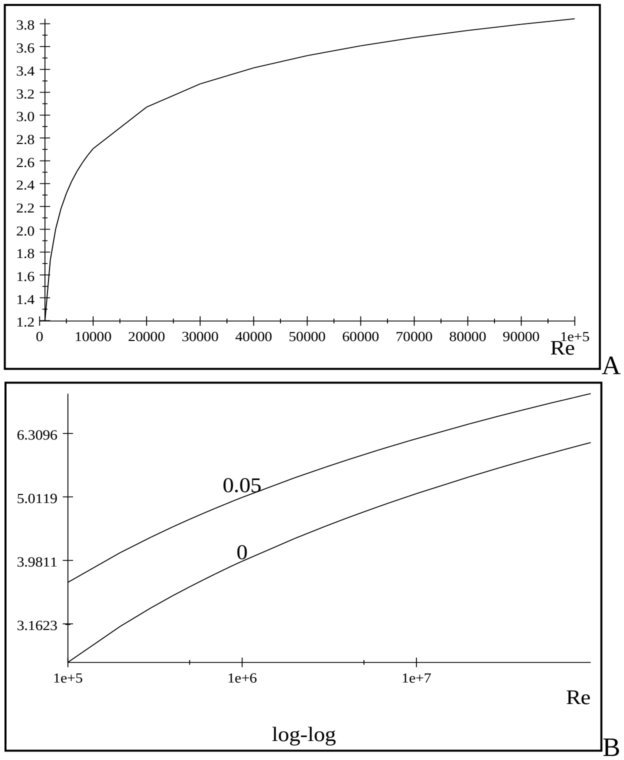

Figure 2. Turbulence Index vs Reynolds numbers. A: transition regime; B: turbulent regime

Both the predicted velocity profile and shear stress at different values of Reynolds are shown in Figure 3. In this case, the area corresponding to shear stress equal to zero, where the velocity is approximately constant, increases with Reynolds number, i.e., and, therefore, with the turbulence level quantified through turbulence index α .

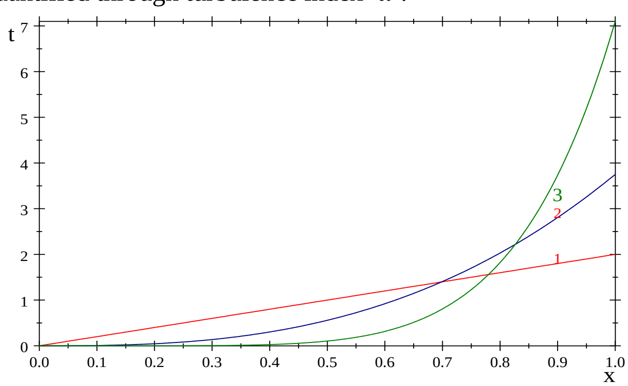

Figure 3. shear stress predicted at 1: re = 103 (laminar regime); 2: re = 104 (transition regimen); re = 108 (turbulent regime)

Turbulent velocity profile of two phases flow

The considered case corresponding to stratified two-phase flow, where the interface between both flows are well defined, and it is parallel to each plate (figure 4). the fractional differential equations system that describes the velocity profile is given by:

$$\begin{aligned}
&0=\Phi_1+\frac{d^{\alpha}\tau_1}{d^{\alpha}\gamma},-1\le\gamma\le b\\
&0=M\Phi_1+\frac{d^{\beta}\tau_2}{d^{\beta}\gamma},b\le\gamma\le 1\\
&\tau_1=\frac{d\phi_1}{d\gamma},-1\le\gamma\le b\\
&\tau_2=\frac{d\phi_2}{d\gamma},b\le\gamma\le 1\\
&\phi_1(-1)=0\\
&\phi_2(1)=0\\
&\phi_1(b)=\phi_2(b)\\
&\tau_1(b)=\tau_1(b)
\end{aligned}\tag{19}$$

where b represents the interface position and

$$\Phi_1=\frac{H^2\Delta P}{4VL}\frac{\rho_1}{\mu_1}\tag{20}$$

$$M=\frac{\mu_1}{\mu_2}\frac{\rho_2}{\rho_1}$$

the average mass velocity V depends on the composition of two-phase flow and the average volumetric velocity V_q, in such a way that:

$$V=\frac{\rho_1}{y+\frac{\rho_1}{\rho_2}(1-y)}V_q$$

where y is the composition expresses as a mass fraction concerning fluid 1.
The exact solution of equation (19) is.

$$\begin{aligned}
\phi_1(\gamma)=\;&\Phi_1\left(\frac{-M\left(|b|^{\beta+1}(\beta-\beta b+1)-b\right)}{2b\Gamma(\beta+2)}+\frac{-\left(|b|^{\alpha+1}(b\alpha-\alpha-1)-b\right)}{2b\Gamma(\alpha+2)}\right)\\
&+\Phi_1\left(\frac{-M\left(|b|^{\beta+1}(\beta-\beta b+1)-b\right)}{2b\Gamma(\beta+2)}+\frac{-\left(b+|b|^{\alpha+1}(b\alpha-\alpha-1)\right)}{2b\Gamma(\alpha+2)}\right)\gamma\\
&-\frac{\Phi_1|\gamma|^{\alpha+1}}{\alpha\Gamma(\alpha)(1+\alpha)}\\
&-1\le\gamma\le b
\end{aligned}\tag{21}$$

$$\begin{aligned}
\phi_2(\gamma)=\;&\Phi_1\left(\frac{M\left(b-|b|^{\beta+1}(\beta b+\beta+1)\right)}{2b\Gamma(\beta+2)}+\frac{\left(|b|^{\alpha+1}(1+\alpha+b\alpha)+b\right)}{2b\Gamma(\alpha+2)}\right)\\
&+\Phi_1\left(\frac{M\left(|b|^{\beta+1}(1+\beta b+\beta)+b\right)}{2b\Gamma(\beta+2)}-\frac{\left(|b|^{\alpha+1}(1+\alpha+b\alpha)+b\right)}{2b\Gamma(\alpha+2)}\right)\gamma\\
&-\Phi_1|\gamma|^{\beta+1}\frac{M}{\beta\Gamma(\beta)(1+\beta)}\\
&1\le b\le\gamma
\end{aligned}\tag{22}$$

the turbulence index of each flow is estimated from equation (14), calculating the Reynolds number corresponding to each flow as:

$$R_{e,1}=\frac{2HV}{\mu_1}\tag{24}$$

$$R_{e,2}=\frac{2HV}{\mu_2}$$

where V is the average mass velocity corresponding to the two phases of flow.

Predicted behavior of pressure drop to flow composition of turbulent two phases flow
if it is defined:

$$f_1(\gamma)=\frac{\phi_1(\gamma)}{\Phi_1} \quad (25)$$

$$f_2(\gamma)=\frac{\varphi_2}{\Phi_1}(\gamma)$$

then the average velocity isgiven by:

$$\langle\phi\rangle=\frac{\Phi_1\int_0^y\left(\int_{-1}^b f_1(\gamma)\,d\gamma+\int_b^1 f_2(\gamma)\,d\gamma\right)dy}{\int_0^y\int_{-1}^1 d\gamma\,dy} \quad (26)$$

where:

$$\langle\phi\rangle=1 \quad (27)$$

$$\Phi_1=\frac{H^2\rho_1\Delta P}{4\mu_1 VL}$$

therefore:

$$\Delta P=\frac{8\mu_1 VL}{\left(\int_{-1}^b f_1(\gamma)\,d\gamma+\int_b^1 f_2(\gamma)\,d\gamma\right)\rho_1 H^2} \quad (28)$$

the interface position b depends on the flow two phases composition, which is calculated as:

$$y=\frac{\int_{-1}^b\phi_1(\gamma)\,d\gamma}{\int_{-1}^b\phi_1(\gamma)\,d\gamma+\int_b^1\phi_2(\gamma)\,d\gamma} \quad (29)$$

where y represents the mass fraction of the fluid 1. Notice that the determination of b from equation (29) may require numeric methods of solution.

3.RESULTS AND DISCUSSIONS

Liquid-liquid system

For the predicted results, it was considered a hypothetical system with the characteristics shown in table 1:

Table 1. Fluid properties and system characteristics considered to analyze the predicted results

Case	μ_1Pas	μ_2Pas	$\rho_1$kgm^{-3}	$\rho_2$kgm^{-3}	α	β	ρ_{mezcla}
A	10^{-7}	10^{-3}	1000	1000	6. 4045		1000
B	10^{-3}	10^{-3}	20	1000	1	1. 7560	$\frac{20.0}{0.98y+0.02}$
C	10^{-7}	10^{-3}	20	1000	4. 6423		$\frac{20.0}{0.98y+0.02}$
D	10^{-7}	10^{-3}	1000	20	6. 404	1	$\frac{1000}{50-49y}$
$H=0.1m$							
$L=1000m$							
$V_q=0.1\frac{m}{s}$							
$r=0$							

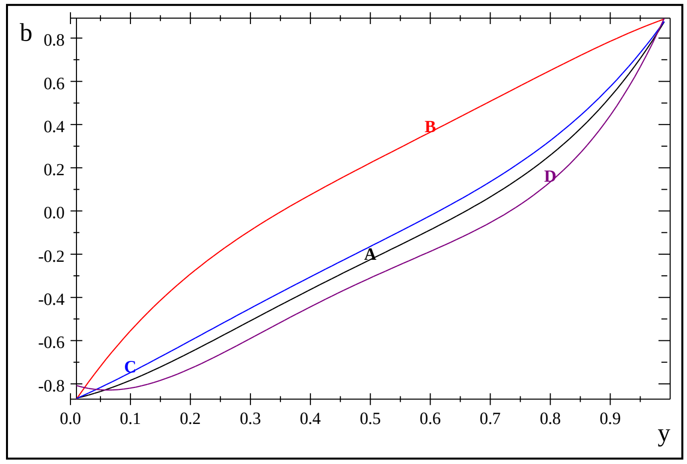

Figure 4. behavior of interface position b forthe mass fraction of fluid 1.

The behavior of interface position to flow composition for each considered case is shown in Figure 4. As expected, the interface position increases as the mass fraction of fluid 1 to all possibilities. Notice that this increase was most significant when both fluids have equal viscosity and different densities. The dimensionless mass velocity profiles corresponding to each case areshown in figure 5.

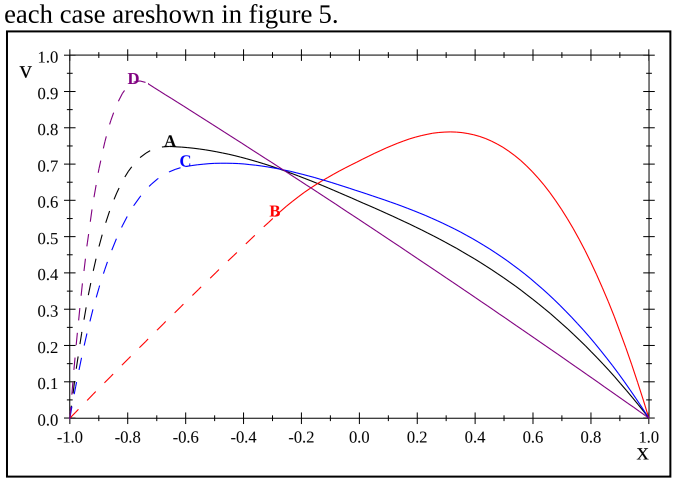

Figure 5 mass velocity profile for each considered case for y = 0.2. The dashed line represents fluid 1.

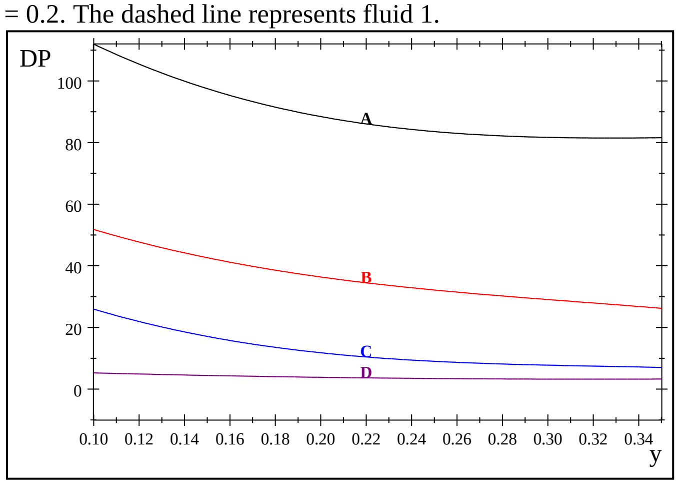

Figure 6. Behavior of the pressure drop forthe mass fraction of fluid 1 considering the same volumetric velocity for each considered case

The behavior of pressure drop concerning fluid composition is shown in figure 6. It can be observed the following predicted facts. The most significant pressure drop corresponds to case A, in which both fluids have immense density. As expected, the pressure drop decreases as composition flow because fluid 1 having the most negligible viscosity. Inthis case, the pressure drop reduction is getting due to viscosity effects.

The pressure drop decreases as flow composition in case of b when both fluids have the same viscosity and different densities, where fluid 1 has the most negligible density. The pressure drop reduction is getting due to density effects.

For case, C and D the fluid 1 have the most negligibleviscosity. Still, it has the smallest density in C. It has the most considerable density in D. For both cases; the pressure drop decreases with flow composition due to viscosity effects.

It is well known that using two phases of flow is a common practice used to transport fluid with high viscosity. The predicted results are indicating that the improver flow must have both small viscosity and small densityvalue.

Liquid-gas system

The proposed model can predict a gas-liquid system's behavior, in which fluid 1 corresponds to a gas. It is known that gas density is a function of both pressure and temperature. If it is considered an ideal gas, then:

$$\rho_1 = \frac{WP}{(8.31)T}$$

where W is the molecular weight. In this case:

$$\Phi_1 = \frac{H^2}{4V}\frac{\rho_1}{\mu_1}\frac{\partial P}{\partial L}$$

$$M = \frac{8.31T}{P}\frac{\mu_1}{\mu_2}\frac{\rho_2}{W}$$

and the pressure drop is obtained usinga solution of the differential equation:

$$\frac{\partial P}{\partial L} = \frac{8\mu_1 V}{\left(\int_{-1}^{b} f_1(\gamma,P)\,d\gamma + \int_{b}^{1} f_2(\gamma,P)\,d\gamma\right)\left(\frac{WP}{(8.31)T}\right)H^2}$$

the interface position b depends on the flow two phases composition and pressure, which is calculated as:

$$y = \frac{\int_{-1}^{b} f_1(\gamma,P)\,d\gamma}{\int_{-1}^{b} f_1(\gamma,P)\,d\gamma + \int_{b}^{1} f_2(\gamma,P)\,d\gamma}$$

.

4.CONCLUSION

This paper showed a model to describe a flow system's pressure drop behavior. There are two phases considering that the flow moves between two parallel plates in a steady-state for constant viscosity and density. It was shown every step for the model solution and the architecture of all applied equations, and a standardizing process that may help predictions of oil crude fluid flow.

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