ISSN 2347 - 3983
Volume 9. No. 4, April 2021
International Journal of Emerging Trends in Engineering Research
Available Online at http://www.warse.org/IJETER/static/pdf/file/ijeter17942021.pdf
https://doi.org/10.30534/ijeter/2021/17942021

Kahlerian manifolds related in H-projective recurrent curvature killing vector fields with vectorial fields

U.S. Negi[1] , Preeti Chauhan[2]
[1,2]Department of Mathematics,
H.N.B. Garhwal University(A Central University),
S.R.T. Campus Badshahithaul,Tehri Garhwal – 249199, Uttarakhand, India.
[1]usnegi7@gmail.com; [2]preetichauhan1011@gmail.com

ABSTRACT

Izumi and Kazanari [2], has calculated and defined on infinitesimal holomorphically projective transformations in compact Kaehlerian manifolds. Also, Malave Guzman [3], has been studied transformations holomorphic ammeters projective equivalentes. After that, Negi [5], have studied and considered some problems concerning Pseudo-analytic vectors on Pseudo-Kaehlerian Manifolds. Again, Negi, et. al. [6],has defined and obtained an analytic HP-transformation in almost Kaehlerian spaces. In this paper we have measured and calculated a Kahlerian manifolds related in H-projective recurrent curvature killing vector fields with vectorial fields and their holomorphic propertiesEinsteinian and the constant curvature manifoldsare established.Kaehlerian holomorphically projective recurrent curvature manifolds with almost complex structures by using the geometrical properties of the harmonic and scalar curvatures calculated overkilling vectorial fieldsare obtained.

Key words:H-projective, recurrent, curvature, Killing vector fields and Kaehlerian manifolds.

MSC 2010:53C15, 53B35,53A20.

1. INTRODUCTION

Kaehlerian manifolds in addition to complex hyper surfaces and other manifolds were measured implanted into special transformations with additive recurrent curvature properties and holomorphic projective correspondences and others. Considering (M, g, F) as a Kaehlerian manifold of $2n \geq 4$ dimension with $g = (g_{ij})$Riemannian metric and an almost-complex structure$F = F_{ij}$, where $F_{ij} = -F_{ji}$ and with Riemannian curvature tensor as well as Ricci tensor are:

$$R^h_{kji} = \partial_k \Gamma^h_{ji} - \partial_j \Gamma^h_{ki} + \Gamma^h_{ka}\Gamma^a_{ji} - \Gamma^h_{ja}\Gamma^a_{ki} ,$$
$$P_{ji} = R^a_{aji} (1.1)$$

Now, the Ricci tensor and the $r = g^{ba} P_{ba}$scalar curvature satisfy the following properties:

$$[P_{ji} = F_i^{\ b} F_i^a P_{ba} \quad , \quad H_{ji} = F_j^{\ a} P_{ai} \quad ,$$
$$H_{ji} + H_{ij} = 0$$

$$H_{ji} = F_i^b F_i^a H_{ba} , \quad F_i^a F_i^h = -\delta_i^h ,$$
$$F_j^a = -i\, \delta_j^a ,$$

$$g_{ij} = F_i^a F_j^b g_{ab} , \quad \nabla_j \tilde{F}_i = -\nabla_i \tilde{F}_j ,$$
$$\tilde{F}_i = F_i\, a\, F_a \,] \quad (1.2)$$

Where, $F^{ji} = g^{ja} F^i_a, H_{ji} = F^i_a P_{ai}$.

The Lie operator derivatives in the vectorial field direction $\boldsymbol{x}$ for R^h_{kji} and$h_{ji}$is represented respectively by,

$L_x R^h_{kji} = \nabla_k L_x \Gamma^h_{ji} - \nabla_j L_x \Gamma^h_{ki} y L_x \Gamma^h_{ji} = \nabla_j \nabla_i X^h + R^h_{aji} X^a$ (1.3)

And if X is a holomorphically projective transformation when:

$L_X \Gamma^h_{ji} = \delta^h_i V_i + \delta^h_j V_i - F^h_j F^a_i V_a - F^h_i F^a_j V_a$,(1.5)

Where $V = (V^i)$ is aexacting vector related to$X$.

Two metric $g = (g_{ij})$ and $\bar{g} = (\bar{g}_{ij})$ defined on Kaehlerian manifoldsK_n, they are holomorphic projective curvature correspondences if:

$\bar{\Gamma}^k_{ki} = \Gamma^k_{ji} + V_i \delta^k_i + V_j \delta^k_i - F^k_j \tilde{V}_i - F^k_i \tilde{V}_j$, (1.6)

Where $\widetilde{V}_I = F^a_i V_a$.

Now, Tensors for harmonic and scalar curvature are defined on the Kahlerian manifold $K_n$through the following relations:

$\nabla_a R^a_{kji} = \nabla_k P_{ji} - \nabla_j P_{ki}, R = g^{ba} P_{ba}$, (1.7)

Respectively where $P_{ji} = R^a_{aji} is$the Ricci tensor. The Laplacian of $\boldsymbol{f}$ is defined by:

$\Delta f = \nabla^a \nabla_a f = \Delta f$, (1.8)

Where $f = \frac{1}{n+2} \nabla_a X^a$ with $f \in C^\infty(K_n)$and$V_j = \nabla_j f$.

The characteristic commutative relationship of L_X and ∇ for a curvature tensor $Y$of(1,2) type is given by:

$L_X \nabla_k Y^h_{ji} - \nabla_k L_X Y^h_{ji} = (L_X \Gamma^h_{ka}) Y^h_{ji} - (L_X \Gamma^h_{kj}) Y^h_{ai} - (L_X \Gamma^a_{kj}) Y^h_{ai} - (L_X \Gamma^a_{ki}) Y^h_{ji}$(1.9)

Organism$X$a holomorphically projective curvature transformation with aVrelatedvector then the following identities are satisfied Izumi, H. [1]:

$2P^a_i V_a = -\nabla_i(\nabla f)$ (1.10)

$\nabla_j V_i = F^a_i F^b_j \nabla_b V_a$ (1.11)

$\nabla_k \nabla_j V_i = -F^b_k F^a_j R^c_{iab} V_c$. (1.12)

If X is a vectorial field then X is a Killing vector field, If satisfies:

$L_X g_{ji} = 0$, $i,j = \overline{\iota, n}$.(1.4)

From Malave Guzman, [3] gives.

$P_{ij} = \bar{\bar{P}}_{ij} + \tau(V_{ij} - \bar{V}_{ji})$,(1.13)

Where$V_{ij} = \nabla_i \nabla_j f$, τ – parameter.

An $A_n = (K_n, \nabla)$ space is a Peterson Codazzi one if $\nabla_k P_{ji} = \nabla_j P_{ki}$. If$\nabla_l R^h_{ijk} = V_l R^h_{ijk}$ it is a recurrent space where $V_l \neq 0$ or it is an Einstein space if$P = \lambda g$ taking Sas the Ricci tensor and g as the metric tensors and λ as a parameter.

Theorem (1.1): If compact Kaehlerian manifoldK_n of dimension n with a scalar curvature R and it admits a holomorphically projective curvature transformation then the following equation is satisfied:

$\nabla f = -\frac{2R}{n} f$and $P^a_i V_a = \frac{R}{n} V_i$

Proof. Since A_n is a recurrent curvature space and compact Kaehlerian manifoldK_n admits an holomorphically projective curvature transformation then we get:

$g^{hi} \nabla_b \nabla_j X_i + g^{hi} R_{abji} X^a - V_b \delta^h_j - V_j \delta^h_b + V_a F^a_b F^h_j + V_a F^a_j F^h_b = 0$ (1.14)

Multiplying (1.14) by g_{hk} and applying ∇^b it results that

$\nabla^b(\nabla_b \nabla_j X_i + R_{abji} X^a - V_b g_{ji} - V_j g_{bi} + V_a F^a_b F_{ji} + V_a F^a_j F_{bi}) = 0$ (1.15)

Now using Ricci's and Bianchi's identities we obtain

$(R_{abji} - 2R_{bjia}) \nabla^b X^a - R_{ai} \nabla_j X^a + R^a_j \nabla_a X_i - (\nabla_a R_{ji}) X^a = 0$, (1.16)

Finally, by applying ∇^j the result is

$-2\nabla_i R_{ba}\nabla^b X^a = 0 \Rightarrow \nabla_i R_{ba} L_X g^{ba} = 0$ (1.17)

$\Rightarrow -2RF_i = n\nabla_i(\Delta f)$

$\Rightarrow \nabla_i(\Delta f) = -\frac{2R}{n} f = \nabla_i\left(-\frac{2R}{n} f\right)$

$\Rightarrow \Delta f = -\frac{2R}{n} f,$ (1.18)

Due to $(n\Delta f + 2Rf)$ be constant for being:

$\int_M \Delta f d\sigma = \int_M f d\sigma = 0$ (1.19)

A compact K_n and dσ is a volumetric element of Kaehlerian manifold K_n.
Finally, we conclude that:

$$\Delta f = -\frac{2R}{n} f.$$

Again, the expression is attained by above theorem (1.1), then we have the following:

Theorem(1.2): Let Xisa holomorphically projective curvature transformation with aV related vector then:
$L_X P_{ji} = -(N+2)\nabla_j V_i$ (1.20)

Proof. We have by the definition of $L_X R^h_{kji}$we have

$$L_X P_{ji} = L_X R^h_{hji} = \nabla_h L_X \Gamma^h_{ji} - \nabla_j L_X \Gamma^h_{hi}$$

Because X is a holomorphically projective curvature transformation then:

$$\mathrm{L}_X P_{ji} = \nabla_j V_i + \nabla_i V_j - F_j^h F_i^a V_a - F_i^h F_j^a V_a - n\nabla_j V_i - \nabla_j V_i + i\, nF_i^a \nabla_j V_a - \nabla_j V_i$$

By considering the real part we obtain the desired result**:**

$$\mathrm{L}_X P_{ji} = -\mathrm{n}\,\nabla_j V_i - 2\,\nabla_j V_i = -(n+2)\nabla_j V_i$$

2. KAHLERIAN MANIFOLDS RELATED IN H-PROJECTIVE RECURRENT CURVATURE KILLING VECTOR FIELDS WITH VECTORIAL FIELDS:

Kaehlerian manifold to develop into a Peterson-Codazzi manifold under the assumption that the earlier is holomorphically projective then following theorem allocates**:**

Theorem (2.1): Let K_n be a Kaehlerian manifold and X be a holomorphically projective recurrent curvature killing vector field with arelatedto vectorial field V then:

$$L_X(\nabla_j P_{ki} - \nabla_k P_{ji}) = \{(n+2)R^a_{jki} - P_{ki}\delta^a_j + P_{ji}\delta^a_k - F^a_i H_{ki} + F^a_k H_{ji} + 2F^a_i H_{jk}\}V_a. \quad (2.1)$$

Proof. We have by the standard relation of commutation for a $(0,2)$ type tensor we obtain that:

$$(L_X\nabla_j P_{ki} - L_X\nabla_k P_{ji}) - (\nabla_j L_X P_{ki} - \nabla_k L_X P_{ji}) = (L_X\Gamma^a_{ki})P_{ja} - (L_X\Gamma^a_{ji})P_{ka} \quad (2.2)$$

But via hypothesis we consider X as a holomorphically projective curvature transformation by using (1.5) then we gets:

$$L_X\Gamma^a_{ji} = \delta^a_j V_i + \delta^a_i V_j - F^a_j F^h_i V_h - F^a_i F^h_j V_h \quad (2.3)$$

Additionally, treating to theorem (1.2),

$L_X P_{ji} = -(n+2)\nabla_j V_i$ (2.4)
And a rationally, we obtain $L_X\Gamma^a_{ki}$ and $L_X P_{ki}$.

Through Substituting (2.3) and (2.4) in (2.2), we get:

$$(L_X\nabla_j P_{ki} - L_X\nabla_k P_{ji}) - (\nabla_j[-(n+2)\nabla_k V_i] - \nabla_k[-(n+2)\nabla_j V_i])$$

$$= (\delta^a_k V_i + \delta^a_i V_k - F^a_k F^h_i V_h - F^a_i F^h_k V_h) P_{ja} - (\delta^a_j V_i + \delta^a_i V_j - F^a_j F^h_i V_h - F^a_i F^h_j V_h) P_{ka}$$

Through some operation and using generalization, we bring to a close that:

$$\{(n+2)R^a_{jki} - P_{ki}\delta^a_j + P_{ji}\delta^a_k - F^a_i H_{ki} + F^a_k H_{ji} + 2F^a_i H_{jk}\}V_a = L_X(\nabla_j P_{ki} - \nabla_{ki} P_{ji})$$

As of there on various submissions of the earlier outcomes will be given that:

A.If $\nabla_j P_{ki} = \nabla_k P_{ji}$ then K_n is Kaehler-Peterson-Codazzi manifolds and

$\{(n+2)R^a_{jki} - P_{ki}\delta^a_j + P_{ji}\delta^a_k - F^a_i H_{ki} + F^a_k H_{ji} + 2F^a_i H_{jk}\}V_a = 0$.(2.5)

Therefore, first significance is a Kaehler-Peterson-Codazzi manifold has a harmonic curvature in view of the fact that:

$$\nabla_j P_{ki} = \nabla_k P_{ji} \Leftrightarrow \nabla_a R^a_{jki} = 0.$$

Also, second significance is a Kaehler-Peterson-Codazzi manifold is an Einsteinian manifold if the previous has a constant scalar curvature. Exactly through multiplying g^{ki} into (2.5) then we get:

$$\{(n+2)g^{ki}R^a_{jki} - R\,\delta^a_j + g^{ai}P_{ji} - F^a_i g^{ki}H_{ki} + F g^{ki}H_{ji} + 2F^a_i g^{ki}H_{jk}\}V_a = 0.$$

While $V_a \neq 0$ and budding the three previous terms we comprise,

$$(n+2)g^{ki}R^a_{jki} - R\delta^a_j + g^{ai}P_{ji} - F^a_i\, F^b_k\, g^{ki}\, P_{bi} + 3F^a_k\, F^b_j g^{ki}\, P_{bi} = 0,$$

Next to creation the reduction $a = F$ and estimate starting $1\ to\ n$ then we get hold of:

$$g_{ki}(nR + 2R - nR + R)3\,P_{ki},$$

Hence like this, we complete that $P_{ki} = \frac{R}{n}\, g_{ki}$. then further expressions the Kaehler- Peterson-Codazzi manifold is an Einsteinian manifold.

B.If Kaehlerian manifold K_n is a recurrent curvature manifold, then:

$(n+2)\,R^a_{jki}\,V_a - L_X(R^a_{jki}V_a) = P_{ki}\delta^a_j - P_{ji}\delta^a_k + F^a_j H_{ki} - FH_{ji} - 2F^a_i H_{jk}V_a$.(2.6)

Hence, significance will be, if Kaehlerian manifold K_n is a harmonic curvature and $W = \{V = (V^i): V \neq 0\}$ with $V_j\, V^k = ||V||^2\ if\ (j = k)$ and $V_j V^k = 0\ \ if\ \ (k \neq j)$, Then Kaehlerian manifold K_n has a null scalar curvature.

In point of fact, if Kaehlerian manifold K_n confesses a harmonic curvature then building the reduction $l = a$ and summing up from $1\ to\ n$ in the relation:

$$\nabla_l R^a_{jkl} = R^a_{jkl}V_l$$

We get,

$$\nabla_l R^a_{jki} = R^a_{jki}V_l \Rightarrow R^a_{jki} = 0.$$

From (2.1) we get,

$$P_{ki}V_j - P_{ji}V_k + H_{ki}\tilde{V}_j - H_{ji}\tilde{V}_k - 2H_{jk}\tilde{V}_i = 0$$

And multiplying the earlier relation by g^{ki} it affects that:

$$g^{ki}P_{ji}V_k - g^{ki}P_{ki}V_j + g^{ki}H_{ji}\tilde{V}_k - g^{ki}H_{ki}\tilde{V}_j - 2g^{ki}H_{kj}\tilde{V}_i = 0$$

Hence

$$P^k_j V_k - rV_j = H^k_j \tilde{V}_k,$$

Somewhere in through relating V^j itproducts that $r = 0$. Thus we finish off that the Kaehlerian manifold K_n is natural.

Theorem (2.2): If exist Einstein dense Kaehlerian manifold $K_n = (M, \nabla)\ and\ \bar{K}_n = (M, \bar{\nabla})$, with metric $g = (g_{ij})\ and\ \bar{g} = (\bar{g}_{ij})$ holomorphic projective curvature identical, therefore we contract an term that recounts the scalar curvature R and $\bar{R}$.

Proof. We have using Martinez and Ramirez [4].

$P_{ij} = \bar{P}_{ij} + \tau(V_{ij} - \bar{V}_{ji}),$ (2.7)

Then in addition to like Einstein manifolds:

$P_{ij} = c_1 g_{ij}$, $\bar{P}_{ij} = c_2 \bar{g}_{ij}$or

$$P_{ij} = \frac{R}{n} g_{ij}, \quad \bar{P}_{ij} = \frac{\bar{R}}{n} \bar{g}_{ij},$$

It should be

$$\frac{R}{n} g_{ij} = \frac{\bar{R}}{n} \bar{g}_{ij} + \tau(V_{ij} - \bar{V}_{ji}), \quad \tau \in C,$$

After that, concerningg^{ij}gets the outcome:

$$R = \frac{\bar{R}}{n} g^{ij} \bar{g}_{ij} + \tau(g^{ij} V_{ij} - g^{ij} \bar{V}_{ji}),$$

$or R = \frac{\bar{R}}{n} g^{ij} \bar{g}_{ij} + \tau(\|V\| - g^{ij} \bar{V}_{ji}) \quad or \quad R\, g_{ij} = \frac{\bar{R}}{n} \bar{g}_{ij} + \tau(\|V\| g_{ij} - \bar{V}_{ij})$

Again concerning now$\bar{g}^{ij}$ gets the results:

$$R\, g_{ij} = \bar{R} \bar{g}_{ij} + \tau \|V\| g_{ij} (-\|\bar{V}\| \bar{g}_{ij}),$$

As ofat this time: $(R - \tau\|V\|) g_{ij} = (\bar{R} - \tau\|\bar{V}\|) \bar{g}_{ij}$,in that case

$$\frac{(R - \tau\|V\|) g_{ij}}{(\bar{R} - \tau\|\bar{V}\|) \bar{g}_{ij}} = \frac{\det(\bar{g}_{ij})}{\det(\bar{g}_{ij})}$$

Using by Malave Guzman [3] has studied Transformationes holomorficamemte proyectivas equivalentes article evidence then we have:

$$\text{In} \sqrt{\frac{\det(\bar{g}_{ij})}{\det(\bar{g}_{ij})}} = (n+2)h, \quad h \in C^{\infty}(K_n)$$

Thuswe conclude and find the relation:

$$(R - \tau\|V\|) = (\bar{R} - \tau\|\bar{V}\|) \exp[2h(n+2)].$$

Theorem (2.3): We obtain an expression that calculate the Ricci tensor in a dense Kahlerian manifold $K_n$confessing holomorphic projective curvature transformations with related vector V, if $A_n$this is recurrent curvature.

Proof: We have in this case:

$$\nabla_k R^h_{lji} = R^h_{lji} V_k,$$

By making the contraction $a = j$ and adding up from $1\ to\ n$ we get:

$\nabla_k P_{ji} = P_{ji} V_k,$ (2.8)

Although, we have

$$\nabla_k P_{ji} = \partial_k(P_{ji}) - \Gamma^a_{kj} P_{ai} - \Gamma^a_{ki} P_{ja}.$$

Concerningg^{ij} then outcome is:

$\nabla_k P_{ji} = \partial_k(P_{ji})$(2.9)

After then using (2.8) and (2.9) we find the results:

$$\partial_k(P_{ji}) - P_{ji}\, \partial_k f = 0.$$

Therefore, the solution of this partial differential equation is the Ricci tensor, hence is achieved the scalar curvature tensor.

REFERENCES

1.H. Izumi, A remark on infinitesimal holomorphically projective transformations, Mat. Japonica, 39, pp. 47-60, 1994.

2.H. Izumi,and Y. Kazanari. On infinitesimal holomorphically projective transformations in compact Kaehlerian manifolds, Hokkaido Math. J., 8, pp. 65-79, 1970.

3.R. Malave Guzman. Transformationes holomorficamemteproyectivas equivalentes, Department Mathematic de la Universidad de oriente, (tesis de maestria) 2007.

4.R. Martinez,andR. Ramirez. Lyra spaces,their application to mechanics, Jadronic, J, 12, pp. 123-236, 1992.

5.U.S. Negi.Some problems concerning Pseudo-analytic vectors on Pseudo-Kaehlerian Manifolds. AARJMD, vol. 1, issue 17, pp. 106-113, 2014.

6. U. S. Negi et., al. An analytic HP-transformation in almost Kaehlerian spaces., Aryabhatta Journal of Mathematics & informatics, Vol. 11, No. 1, pp. 103-108, 2019.