



Advanced Optimization of Road Network: Pedestrian Crossings with Calling Devices

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ABSTRACT

Pedestrian crosswalks are the part of road network which exerts significant influence on the time of motion of both transport and pedestrians. Using traffic lights with strict control mode in the places where the intensity of pedestrian flows is highly uneven during the day leads to excessive time consumptions. In such cases road traffic can be optimized by pedestrian crosswalks equipped with pedestrian push buttons. It is highly important to develop such evaluation model which could use mesoscopic data detailing and would obtain instant results together with their predictions for the near future. This article presents the model of pedestrian flow near road crosswalk with pedestrian push button and fixed time of Walk signal. Oriented motion of pedestrians to crosswalk with push button is presented in the form of random cluster stream. Hierarchical agglomerative procedure has been applied to subdivide pedestrians into clusters. The main property for combination of objects into clusters is the distance between neighboring pedestrians and the distance to cluster center. The algorithm of cluster formation of pedestrian flow at certain time has been developed. Evaluation of average waiting time for possibility to resume movement over crosswalk has been determined. Cumulative delay of vehicles near crosswalk with push button has been evaluated as well as average delay of a single vehicle for each traffic lane.

Key words: traffic flow, pedestrian flow, mathematical model, average delay, pedestrian crosswalk, pedestrian push button.

1. INTRODUCTION

Pedestrian crosswalks are the part of road network which exerts significant influence on the time of motion of both transport and pedestrians. Due to high traffic intensity, uncontrolled crosswalks become unreasonable and unsafe. Using traffic lights with strict control mode in the places where the intensity of pedestrian flows is highly uneven during the day leads to excessive time consumptions [1-3]. In such cases road traffic can be optimized by pedestrian crosswalks equipped with push buttons, such as Puffin, Pelican, Toucan.

Selection of optimum operation mode of pedestrian push buttons as well as prediction of vehicle and pedestrian travel time should be based on analytical evaluation of delay of vehicle and pedestrian flows on crosswalks with push

buttons. Moreover, taking into account implementation of intelligent transportation systems, **it is highly important** to develop such evaluation model which could use mesoscopic data detailing and would obtain instant results together with their predictions for the near future. Therefore, this work is aimed at development of evaluation methods of delays of vehicles and pedestrians at crosswalks with push buttons.

2. METHODS

2.1 General description

Recently the TIMeR_Mod mesoscopic model of traffic flow distribution over road network had been developed [4]. It was assumed that time intervals between traffic flows along each lane were governed by the generalized Erlang distribution. The authors' model of vehicle and pedestrian flows will be developed in the frames of assumptions of TIMeR_Mod model.

2.2 Algorithm

At present simulation of pedestrian flows attracts great attention due to demands in numerous fields. For instance, upon designing public buildings and structures, mass transit stations, airports and railway stations, during public events, it is required to simulate human behavior in various situations, including panic situation. Herewith, each case requires for various degree of accuracy, special importance of various aspects in behavior of pedestrian flow [5]. Presently microscopic, mesoscopic, and macroscopic models of pedestrian flow are available [6-8]. Microscopic models consider for position, speed, and forces acting on each individual. It is possible to mention Benefit Cost Cellular Model, Cellular Automata Model, Magnetic Force Model, and Social Force Model [9, 10]. In the Benefit Cost Cellular Model all space is subdivided into cells where a person can exist or not exist. Whether a cell is occupied, free, or neighboring with occupied one, appropriate cost is assigned to this cell. Pedestrian motion is described in the model as variation of cell cost.

The Magnetic Force Model describes motion of single pedestrian who moves to his/her target and avoids collisions by means of forces of attraction or repulsion with regard to other objects and pedestrians.

In the Cellular Automata Model travelling is simulated as motion along grid cells. In order to describe motion, a matrix is constructed the elements of which are probabilities of pedestrians to occupy neighboring cells.

The Social Force Model is considered to be the best model of microscopic models and is developed in sufficient details. According to this model, a pedestrian is the object motivated by social forces. A pedestrian attempts to reach the target by the most comfortable manner. In addition, each pedestrian attempts to retain comfort distance to other pedestrians and objects. On the basis of these provisions, equations are derived describing pedestrian behavior in flow.

The models based on queue theory (mass service theory) are considered either as microscopic or as mesoscopic models. They are applied in the case when a flow moves in predefined direction, for instance, upon simulation of evacuation from a building. This type of models can also be applied upon simulation of pedestrian flow over crosswalks.

Initial data of the model of pedestrian crosswalk with push button can be obtained using video records for determination of variables of pedestrian flow and vehicle traffic along each lane arriving at the crosswalk. This will be aided by statistical and cluster analysis.

3. RESULTS

3.1 Description of device.

Let us consider a crosswalk with Pelican push button. The push button is located between crosswalks at considerable distance. Not far away sources of pedestrian flow are located, for instance, a public transport stop, a school, an entertainment center, etc. When the button is pushed, the pedestrians during the time t_w wait for Walk signal. Then, during fixed time interval t_{gp1} the pedestrians cross the road. During the time t_{over} Stop signal is active both for the pedestrians and for vehicles. During this time the pedestrians complete their motion.

3.2 Description of pedestrian flow

As mentioned above, the models based on description of pedestrian flow as random using random functions are used when the flow moves in preset direction. Hence, they can be applied to simulate the pedestrian flow over crosswalk. Pedestrian flow approaching crosswalk can be considered as a flow of clusters moving in one direction [11]. The clusters are formed as a consequence of passengers leaving the public transport after arrival at the stop, after termination of cinema performance, after the end of classes, etc. A cluster is a dense congregation of humans which requires for single pushing of button to cross the road. Hence, it would be reasonable to consider this case not as a flow of single pedestrians but as a flow of clusters (Fig. 1).

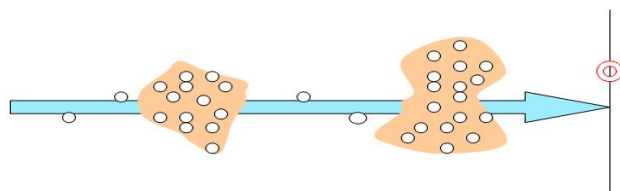


Figure 1: Pedestrian flow to crosswalk with push button

Let us consider a cluster as a separate object. Let us consider that the distribution of intervals in time between the objects (clusters and separate single pedestrians) is governed by the exponential rule with the variable λ_{0p} . Each cluster is characterized by the fact that single pushing of button is sufficient for everybody to cross road.

Assuming that the flow is distributed according to exponential rule, its intensity is defined as $\lambda_{0p} = \frac{1}{T_{0p}}$,

where T_{0p} is the average time interval (in seconds) between consecutive events (in this case: clusters following each other).

3.3 Data cluster analysis for simulation of pedestrian flow

The main task of cluster analysis is to subdivide a set of objects into classes the homogeneity of which is determined by certain preset properties. This is a multidimensional statistic method which allows to work with huge data and to consider simultaneously for several n properties characterizing objects [11]. Objects are presented as points in n -dimensional space, and homogeneity is determined by the distance between these points. Generally, the distance can be defined by various methods, it is important to select correct scale along each axis. This is aided by data normalization. For instance, it is possible to apply linear distance, Euclidian distance, squared Euclidian distance, generalized Minkowski distance, Chebyshev distance, Manhattan distance. Subdivisions into clusters are classified as hierarchical and non-hierarchical.

The main stages of cluster analysis are as follows: formulation of the problem; selection of distance measurement method; selection of clusterization place; selection of cluster amount; interpretation and profiling of clusters; evaluation of clusterization confidence.

In order to subdivide pedestrians into clusters, let us apply hierarchical agglomerative procedure sequentially combining the pedestrians into separate groups. The main property to combine objects is the distance between neighboring pedestrians and the distance to the cluster center. In addition, let us introduce restriction D_{max} for the cluster diameter and D_{free} for the distance between neighboring objects. The constant D_{max} is used for model calibration with respect to certain crosswalk. It is defined by the condition of guaranteed road crossing by all objects in cluster per one cycle at known variables: average speed of separate pedestrian and, lane width. The constant D_{free} defines pedestrian motion outside the cluster.

Two approaches to automated data acquisition are possible. In the first case the current data from video detector about object locations are acquired, their Cartesian coordinates in the plane Oxy : (x_i, y_i) are fixed. Then, the distance

between two objects d_{ij} is determined as common Euclidian distance:

$$d_{ij} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} \quad (1)$$

Objects are combined into one cluster if the distance between object to cluster center does not exceed $D_{max}/2$, and at least one object of formed cluster is in the circle with the radius D_{free} . When a new object is added to the cluster, the cluster diameter and center are recalculated. The diameter dc is the

distance between two outermost objects, and the center is the coordinates of diameter center.

In the second case only one coordinate x_i is recorded for each time reflecting object projection on the axis of motion direction. Then, the distance d_{ij} between two objects is:

$$d_{ij} = \sqrt{(x_i - x_j)^2} = |x_i - x_j| \quad (2)$$

The diameter in this case is the maximum distance between object projections on the axis of motion direction (Fig. 2).

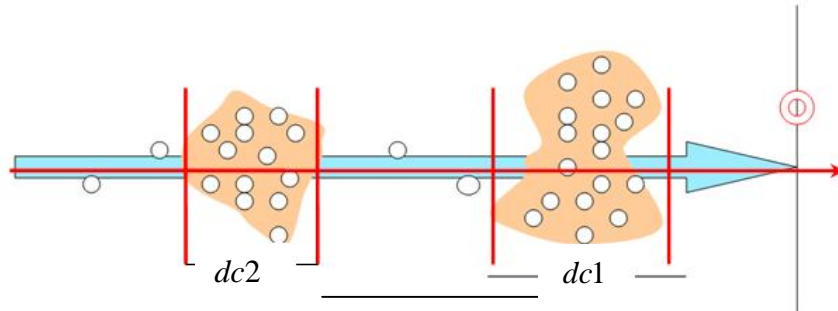


Figure 2: Cluster diameter upon simulation of pedestrian flow

While approaching crosswalk, data acquisition motion is simulated by moderate segment which can be approximated by straight line. Otherwise, if the motion direction is presented by curve and its approximation by straight line is unreasonable, then the coordinates are presented by projections onto the curve. The distance d_{ij} is calculated as the arc length of curve to which the points are projected.

For this model, the second case of cluster formation is preferable. On the basis of behavior of pedestrians, their distance from crosswalk along motion direction is important. The pedestrians arranged into "narrow" but "wide" cluster start their motion over crosswalk simultaneously.

3.4 Data processing algorithm for pedestrian flow

The distribution density of exponential law for a random value is as follows:

$$f(t) = \lambda_{0p} \cdot e^{-\lambda_{0p}t} \quad (3)$$

It is one-parametric and can be applied for flows of moderate intensity [4]. We can apply it upon subdivision of pedestrian flow into clusters. In order to obtain the variable λ_{0p} of exponential law, it is required to use experimental data for determination of average time interval between consecutive events T_{0p} . Then, $\lambda_{0p} = \frac{1}{T_{0p}}$.

While planning experiment, it is required to obtain the following measurements: time intervals Δt_i between consecutive arrivals of centers of pedestrian flow clusters to

the given point at approaches to crosswalk. Then, the sample mean is:

$$\bar{t} = \frac{\sum_{i=1}^n \Delta t_i}{n}, \quad (4)$$

where n is the number of measurements and $T_{0p} \approx \bar{t}$.

The data processing algorithm for pedestrian flow is as follows:

1) at the time $s = t_1$ the coordinates x_i of point projections (the pedestrians) onto the axis of motion direction are detected;

2) subdivision into clusters is carried out (Fig. 3):

2.1) the points are positioned in ascending order of the coordinates x_i ;

2.2) the distance from the point $x^* = x_N$ with the maximum coordinate to neighboring one is calculated;

2.3) if the distance $d_{ij} < D_{free}$, the points are combined into one cluster;

2.4) the cluster center m_c is determined as the middle point of segment between the two points;

2.5) the distance d_{ij} from the last point included into the cluster x_i to the next descending point x_{i-1} and the distance dm_{i-1} from the point x_{i-1} to the cluster center m_c are determined;

2.6) if both conditions $d_{ij} < D_{free}$ and $dm_{i-1} < \frac{D_{max}}{2}$ are valid, then the point x_{i-1} is included into the current cluster;

2.7) the cluster center as the middle point between x_{i-1} and x^* is determined:

$$m_c = \frac{x^* + x_{i-1}}{2};$$

2.8) if any of the conditions of item 2.6 of the current algorithm is invalid, then the point x_{i-1} is included into a new cluster and $x^* = x_{i-1}$ is assumed for the new cluster;

2.9) items 2.5–2.8 are repeated until all data for the time $s = 1$ are subdivided into clusters;

3) the coordinate of the first cluster center on the motion direction axis $X_{control}$ is detected;

4) the time $s = t_2$ is detected when the second cluster center reaches $X_{control}$;

5) $\Delta t_1 = t_2 - t_1$ is calculated;

6) items 2–5 for the time $s = t_2$ are repeated for the required number of times n ;

$$\bar{t} = \frac{\sum_{i=1}^n \Delta t_i}{n}$$

7) \bar{t} is calculated.

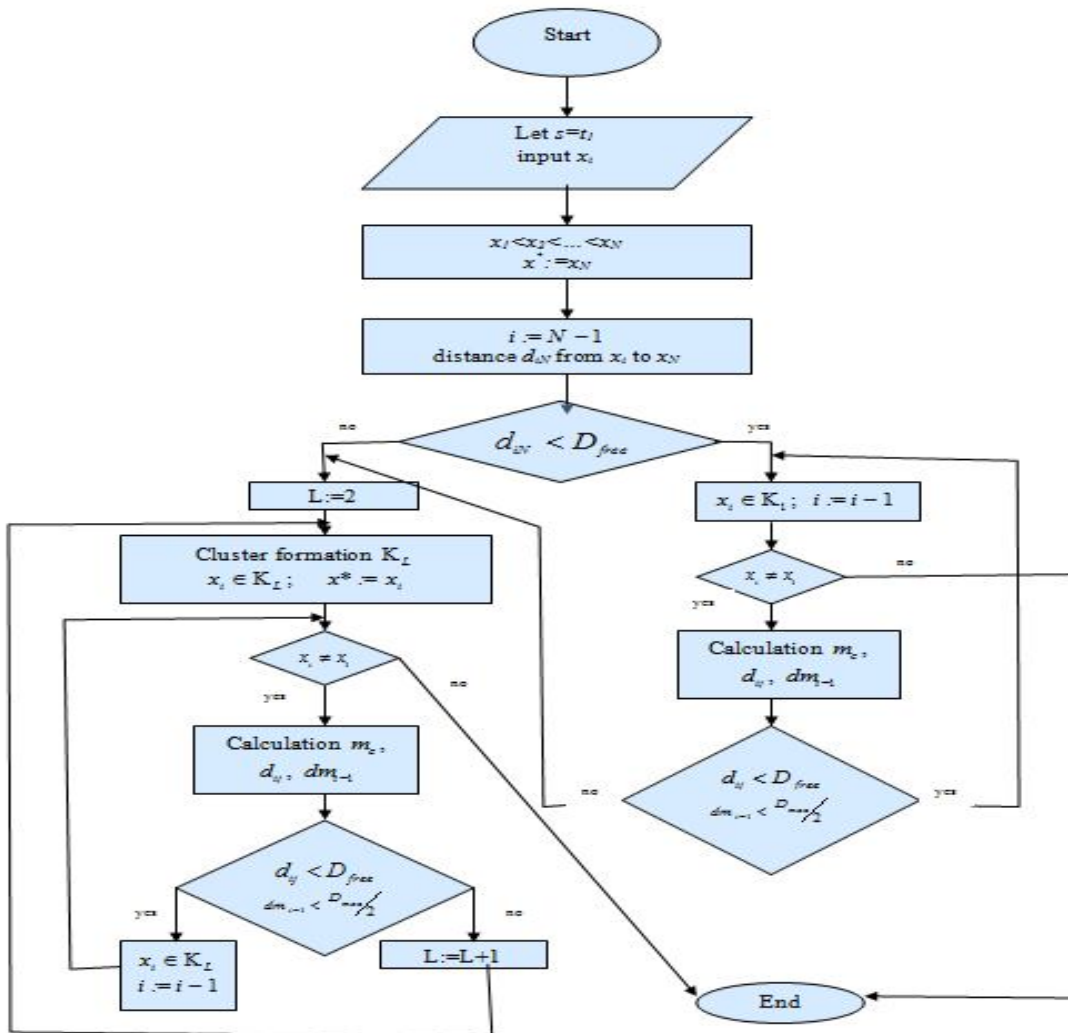


Figure 3: Algorithm of cluster formation of pedestrian flow at certain time

3.5 Calculation of delays for vehicle traffic

According to the TIMeR_Mod model [12], the flow of incoming vehicles is the Palm flow and the time intervals between vehicles are governed by the generalized Erlang distribution.

Let us detect the vehicle average delay arriving randomly to crosswalk. A delay is defined as the wait time to activation of Walk signal of traffic lights. Delays caused with decrease in speed while arriving crosswalk are not taken into account. If a vehicle arrives at crosswalk when pedestrians are absent, then it crosses without stop. Otherwise, it should wait to activation of Walk signal (Fig. 4).

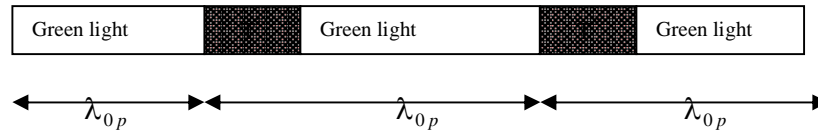


Figure 4: Activation flowchart of Stop signal for vehicles

A. Calculation of average delay of a vehicle arriving at crosswalk.

Stop signal for vehicles is active at random time intervals according to the exponential law with variable λ_{0p} (Fig. 4). Duration of Stop signal is fixed and equals to $T^* = t_{gpl} + t_{over}$. Let X is the random variable, the wait time to continue motion. Since the variable λ_{0p} characterizes average frequency of event occurrence per unit time, then:

$$M(X) = (\lambda_{0p} \cdot T^*) \cdot \frac{T^*}{2} = \lambda_{0p} \cdot \frac{(t_{gpl} + t_{over})^2}{2} \quad (5)$$

Equation (1) presets average delay of one vehicle arriving at crosswalk in the case of low intensity of traffic along each lane.

B. Calculation of average cumulative delay of vehicles arriving at crosswalk with push button

Let us calculate assumed cumulative delay of the considered flow in T^* seconds: the time when vehicle motion along the given lane in the considered direction is prohibited. Let us assume that there is no queue at the time $t = 0$.

In the case of the generalized Erlang distribution, the time interval between consecutive requirements passes k stages of T_0, T_1, \dots, T_{k-1} , the duration of these stages is characterized by exponential distribution with the variables $\lambda_0, \lambda_1, \dots, \lambda_{k-1}$, respectively. For the sake of convenience let us denote $\lambda = \{\lambda_0, \lambda_1, \dots, \lambda_{k-1}\}$. If all variables λ_i are different, then the generalized Erlang distribution is as follows:

$$f_k(t) = (-1)^{k-1} \cdot \prod_{i=0}^{k-1} \lambda_i \cdot \sum_{\substack{n=0 \\ n \neq j}}^{k-1} \frac{e^{-\lambda_i t}}{\prod_{j=0}^{k-1} (\lambda_j - \lambda_n)} \quad (6)$$

The expectance $M(T)$ and dispersion $D(T)$ for the generalized Erlang distribution are, respectively:

$$\mu = M(T) = M\left(\sum_{i=0}^{k-1} T_i\right) = \sum_{i=0}^{k-1} \frac{1}{\lambda_i};$$

$$\sigma^2 = D(T) = D\left(\sum_{i=0}^{k-1} T_i\right) = \sum_{i=0}^{k-1} \frac{1}{(\lambda_i)^2} \quad (7)$$

where $H(t)$ is the restore function of vehicle flow.

Let us subdivide the interval $(0; T^*)$ into n segments by the points t_1, t_2, \dots, t_n ; $\Delta t_i = t_i - t_{i-1}$. Then, the cumulative delay of all requirements of this flow per one cycle T^* is approximately:

$$W(T^*, \lambda) \approx \sum_{i=1}^n H(t_i) \cdot \Delta t_i \quad (8)$$

Transferring to the limit at $\max\{\Delta t_i\} \rightarrow 0$, we obtain:

$$W(T^*, \lambda) = \int_0^{T^*} H(t) dt;$$

$$W(T^*, \lambda) = \int_0^{T^*} \left(\frac{t}{\mu} + \frac{\sigma^2 - \mu^2}{2\mu^2} + R(t) \right) dt = \left(\frac{1}{\mu} \frac{t^2}{2} + \frac{\sigma^2 - \mu^2}{2\mu^2} t \right) \Big|_0^{T^*} + \int_0^{T^*} R(t) dt \quad (9)$$

It has been proved in [10] that the function $R(t)$ can be comprised only of the following terms:

- 1) $Ae^{s_p t}$;
- 2) $A_1 e^{s_p t} + A_2 t e^{s_p t}$;
- 3) $A \cdot e^{\alpha t} \cos \beta t + B e^{\alpha t} \sin \beta t$.

The respective terms in the function $W(T^*, \lambda)$ for each considered case are determined in [4]:

$$1) \quad \int_0^{T^*} A e^{s_p t} dt = \frac{A}{s_p} (e^{s_p T^*} - 1) \tag{10}$$

$$2) \quad \int_0^{T^*} (A_1 e^{s_p t} + A_2 t e^{s_p t}) dt = \frac{A_1}{s_p} (e^{s_p T^*} - 1) + \frac{A_2}{s_p} (e^{s_p T^*} - 1) - \frac{A_2}{s_p^2} (e^{s_p T^*} - 1) \tag{11}$$

$$3) \quad \begin{aligned} & \int_0^{T^*} (A \cdot e^{\alpha t} \cos \beta t + B e^{\alpha t} \sin \beta t) dt = \\ & = A \frac{\alpha^2}{\alpha^2 + \beta^2} \left(e^{\alpha T^*} \cdot \left(\frac{1}{\alpha} \cos(\beta T^*) + \frac{\beta}{\alpha^2} \sin(\beta T^*) \right) - \frac{1}{\alpha} \right) + \\ & + B \frac{\alpha^2}{\alpha^2 + \beta^2} \left(e^{\alpha T^*} \cdot \left(\frac{1}{\alpha} \sin(\beta T^*) - \frac{\beta}{\alpha^2} \cos(\beta T^*) \right) + \frac{\beta}{\alpha^2} \right) \end{aligned} \tag{12}$$

Average cumulative delay (in hours) of requirements of this flow per unit time, one hour, is expressed as follows:

$$W(T^*, \lambda) \cdot \frac{3600}{\left(\frac{1}{\lambda_0}\right)} \cdot \frac{1}{3600} = \lambda_0 \cdot W(T^*, \lambda) \tag{13}$$

$$\text{If } H(T^*) - \frac{\left(\frac{1}{\lambda_{0p}}\right) - T^*}{h} < 0, \tag{14}$$

then the queue in this direction will be eliminated per one cycle. In addition, upon subdivision of pedestrian flow into clusters, the following condition is taken into account:

$$\frac{1}{\lambda_{0p1} + \lambda_{0p2}} > t_w + t_{gp1} + t_{over}. \tag{15}$$

In this case it possible to adjust Eq. (1) for higher intensities of vehicle flows:

$$M(X) = W(T^*, \lambda) / H\left(\frac{1}{\lambda_{0p}}, \lambda\right) \tag{16}$$

In the case of high intensity of pedestrian or vehicle flows, a queue can be generated near crosswalk, this is characterized by the following inequality:

$$H(T^*) - \frac{\left(\frac{1}{\lambda_{0p}}\right) - T^*}{h} > 0. \tag{17}$$

Then, the vehicle average delay can be determined as follows:

$$M(X) = n \cdot \frac{1}{\lambda_{0p}} + W(T^*, \lambda) / H\left(\frac{1}{\lambda_{0p}}, \lambda\right), \tag{18}$$

where n is the number of total cycles of vehicle while waiting for possibility to continue motion.

C. Calculation of pedestrian delays at crosswalk.

According to the assumption, the distribution of time intervals between the objects (clusters and separate single pedestrians) is governed by exponential law with the variable λ_{0p} . Let us calculate average wait time of pedestrians to start crossing road. Let us apply item 5.2.

$H(t) = \lambda_{0p} \cdot t$ is the restore function for exponential law (number of objects arrived in time t).

Let us subdivide the interval $(0; t_w)$ into n parts by the points t_1, t_2, \dots, t_n ; $\Delta t_i = t_i - t_{i-1}$ and calculate the cumulative delay of all pedestrians arriving at crosswalk for the time t_w . Herewith, each cluster is identified with one object. This will be considered upon detection of average delay of one pedestrian.

$$\begin{aligned} W(t_w, \lambda_{0p}) & \approx \sum_{i=1}^n H(t_i) \cdot \Delta t_i = \sum_{i=1}^n \lambda_{0p} t_i \Delta t_i \\ W(t_w, \lambda_{0p}) & = \int_0^{t_w} \lambda_{0p} t dt = \lambda_{0p} \frac{t_w^2}{2} \end{aligned} \tag{19}$$

Average number of pedestrians arriving at crosswalk is determined by the restore function. The pedestrians arriving

at crosswalk when Walk signal is active, with rare exception, would cross the road quickening the pace and would finish their motion during the time t_{over} . Thus, they would not waste their time for waiting.

Therefore, the average wait time $M(T_{pc})$ of possibility to continue motion is defined as follows:

$$M(T_{pc}) = \frac{W(t_w, \lambda_{0p})}{H(t_w + t_{gpl} + t_{over})} = \frac{t_w^2}{2 \cdot (t_w + t_{gpl} + t_{over})} \quad (20)$$

D. Calculation of pedestrian and vehicle delays at crosswalk with push buttons at both sides of traffic lane

Let us consider a Pelican crosswalk with push buttons at both sides of traffic lane [1]. Pedestrian flows from both sides will be considered as the simplest cluster flows with intensities λ_{0p1} and λ_{0p2} , respectively. When two independent simplest flows with the intensities λ_{0p1} and λ_{0p2} are added, the result is again the simplest flow with the intensity $\lambda_{0p} = \lambda_{0p1} + \lambda_{0p2}$.

Similarly to item 6 in this article, it will be considered that the restore function in this case is as follows:

$$H(t) = \lambda_{0p} t = (\lambda_{0p1} + \lambda_{0p2}) \cdot t \quad (21)$$

And the average cumulative delay of all pedestrians from both sides awaiting to continue their motion if they arrive before activation of Walk signal is approximately:

$$W(t_w, \lambda_{0p}) \approx \sum_{i=1}^n H(t_i) \cdot \Delta t_i = \sum_{i=1}^n (\lambda_{0p1} + \lambda_{0p2}) t_i \Delta t_i \quad (22)$$

Let us go to the limit at $\Delta t \rightarrow 0$, then:

$$W(t_w, \lambda_{0p}) = \int_0^{t_w} (\lambda_{0p1} + \lambda_{0p2}) t dt = (\lambda_{0p1} + \lambda_{0p2}) \frac{t_w^2}{2} \quad (23)$$

Average number of the pedestrians arriving to both sides of crosswalk per one cycle is determined by the restore function:

$$H(t_w + t_{gpl} + t_{over}) = (\lambda_{0p1} + \lambda_{0p2}) \cdot (t_w + t_{gpl} + t_{over}) \quad (24)$$

Therefore, the average delay $M(T_{pc})$ of a pedestrian per one cycle can be expressed as follows:

$$M(T_{pc}) = \frac{W(t_w, \lambda_{0p})}{H(t_w + t_{gpl} + t_{over})} = \frac{t_w^2}{2 \cdot (t_w + t_{gpl} + t_{over})} \quad (25)$$

Average cumulative delay of a vehicle on each lane is calculated similarly to item 5.2. And average delay of vehicle on each lane for motion is as follows:

$$M(X) = W(T^*, \lambda) / H\left(\frac{1}{\lambda_{0p1} + \lambda_{0p2}}, \lambda\right) \quad (26)$$

4. CONCLUSION

The developed model of vehicle and pedestrian flows over controlled crosswalks requires for initial data which could be acquired and updated automatically using video detectors. IT advances make it possible to update these data in online mode. This model of vehicle and pedestrian flows over crosswalks with push button is in agreement with previous model [10, 13] of traffic distribution over road network. Hence, it is possible to expand the range of traffic problems to be solved by this model. The developed evaluation of delays of traffic participants at crosswalk is based on analytical methods at limited amount of initial data. Herewith, these data can be adjusted by results acquired from video detectors. Therefore, the presented model can be applied in intelligent transportation systems.

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