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Mathematical model of the eroding effect on the surface morphology of Compressed Earth Bricks

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ABSTRACT

A solid-surface morphology has a rough character that prevents it from being described by Euclidean geometry; fractal geometry is plausible. From considering that the deposition of particles and their detachment significantly influences roughness, an expression based on stochastic modeling techniques was obtained to predict the fractal dimension of a surface based on the dynamics of the processes that occur in it. The model obtained was used to characterize whether fiber in building materials made of poured earth influences the surface's morphology and the effects of erosion.

Key words : Bricks eroding, stochastic degradation model, Surface analysis.

1.INTRODUCTION

The use of earth and vernacular housing construction techniques is currently an ecological alternative by using sustainable materials (Catalán-Diez, R. 2018).Compressed earth bricks (CEB) with other techniques as adobe or bahareque are standard in some places in México, reducing emissions to the environment. Some properties like thermal properties and energy required for production are enhanced (Morales-Cristobal, R. et al. 2020; Roux-Gutierrez, RS et al. 2016), allowing comfortable places, increasing in many cases the resistance capacity and internal forces growth, with the possibility of a versatile design (Galarza, M. et al. 2018)

Although Cimva-ram and manual production are typical, it is necessary to consider greater control in the manufacture, allowing a better interaction among compounds and dosing some additives or stabilizers according to the soil used (Vázquez, M. et al. 2015; Sitton, J. D. 2016). Mucilage from nopal or other cactus is preferred with some fibers for seismic cities, increasing the brick's impermeability (Roux-Gutierrez, RS et al. 2015; Zhang, F. 2020), preventing capillarity by reducing the porosity according to Suárez-Domínguez, E.J. et al. 2020.

According to the stabilizer are the Surface properties are the possibilities of use and apparent finishing (Paredes-Avilés, FL et al. 2017; Amaya, I. et al. 2020); it also reduces the entering of humidity or prevents degradation or pores (Suarez-Dominguez, E. J. et al. 2020b).

In the present work, a model based on mesoscopic formalism and fractal geometry is proposed to predict the behavior of the fractal dimension of a surface as a function of the maximum height of the interface and a parameter related to the speed constant of the process of surface erosion. The proposed model was used to know if the presence of fiber placed to stabilize and enhance the earth's compressed bricks in construction materials obtained from poured soil influences the surface's morphology before and after it is subjected to erosion processes.

2.MESOSCOPIC MODEL

We establish the following considerations and assumptions to obtain a mesoscopic model that describes the particle deposition effect on the surface:

1.On the microscopic scale, the variable describing the system under study is the total number N of virtual particles of average volume v that make up a solid interface of surface area A and thickness H; the latter is the maximum height difference between two interface points.

2.The non-dimensional intensive variable z that describes the behavior of the system is identified with the height difference between two adjacent points and the maximum height difference:

$$z = \frac{h}{H} \tag{1}$$

where the relationship between the intensive variable and the microscopic variable is given by

$$N = z\frac{AH}{v} \tag{2}$$

3.The values of N and z change over time due to two fundamental processes: i) the deposition and adsorption of particles on the surface and ii) the detachment of particles due to erosion. These processes occur randomly, where the transition probability per unit time WN+1/N associated with the deposition of particles is assumed a priori as:

$$W_{N+1/N} = Q \tag{3}$$

where Q is a constant (s-1) while the probability transition by unity in time WN-1/N associated to the unplugs of particles is:

$$W_{N-1/N} = kN \tag{4}$$

k (s-1) is a velocity constant associated with the particles detachment, depending on the interface height:

$$k = k_0 h \tag{5}$$

From the assumed transition probabilities, the master equation that describes the behavior of the probability P is obtained due to observing N particles at time t:

$$\frac{\partial P(N;t)}{\partial t} = \left(\mathbf{E}^{-1} - 1\right) Q P(N;t) + \left(\mathbf{E}^{+1} - 1\right) k N P(N;t) \tag{6}$$

E is the up-down operator that acts on the functions of continuous variables. Suppose we assumed that the change that takes place when an individual microscopic process occurs is practically negligible for N. In that case, N can be considered a continuous variable, in such a way that from the master equation, the Fokker - Planck equation is

$$\frac{\partial P(N;t)}{\partial t} = -\frac{\partial}{\partial N}(Q - kN) P(N;t) + \frac{1}{2}\frac{\partial^2}{\partial N^2}(Q + kN) P(N;t) \tag{7}$$

Equation (7) consider:

$$N = z\frac{AH}{v} \tag{8}$$

$$P(N;t) = P(z;t)\frac{\partial z}{\partial N} = P(z;t)\frac{AH}{v} \tag{9}$$

$$\frac{\partial}{\partial N} = \frac{\partial z}{\partial N}\frac{\partial}{\partial z} = \frac{v}{AH}\frac{\partial}{\partial z} \tag{10}$$

obtaining:

$$\frac{\partial P(z;t)}{\partial t} = -\frac{\partial}{\partial z}\left(Q_0 - k_0 H z^2\right) P(z;t) + \frac{1}{2}\frac{1}{\Omega}\frac{\partial^2}{\partial z^2}\left(Q_0 + k_0 H z^2\right) P(z;t) \tag{11}$$

where:

$$Q_0 = \frac{Qv}{AH} \; : \; \frac{1}{\Omega} = \frac{v}{AH} \tag{12}$$

In steady-state *P*(*z*;*t*) has a normal distribution, with expected value:

$$Z = \sqrt{\frac{1}{H}\frac{Q_0}{k_0}} \tag{13}$$

And variance:

$$\sigma = \frac{1}{\Omega}\frac{Q_0 + k_0 H Z^2}{4HZk_0} = \frac{1}{2\Omega}\sqrt{\frac{1}{H}\frac{Q_0}{k_0}} \tag{14}$$

3.FRACTAL DIMENSION ESTIMATION

The stochastic nature of the processes that occur at the surface level manifested in that the height of the interface fluctuates randomly concerning the spatial position, a phenomenon known as surface roughness

Surface roughness can be quantified by different methods, all of which require experimental observation of the surface and statistical techniques. The application of fractal geometry makes it possible to quantify an irregular line's roughness from the surface's intersection with a plane perpendicular to it, as shown in Figure 1. The value of f is determined by the box-counting method, making use of an appropriate image treatment program, while we estimate the real length of the irregular line with.”

$$L = w.l^{f} \tag{15}$$

where w is a parameter depending on image magnification and the measurement precision, and l is the distance from the Euclidean straight line that joins the initial and ends of the irregular line (Suárez-Domínguez, E. et al. 2020c). The fractal dimension of a line is between 1 and 1.5, such that an irregular line has a greater length than the equivalent Euclidean line.

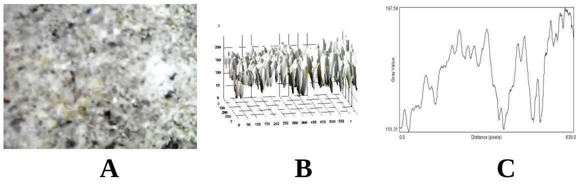


Figure 1. A: Image of a solid surface; B: 3D reconstruction of the surface where the height of the interface is identified with the color intensity of the pixels; C: irregular line that shows the average height of the pixels, which has a fractal dimension equal to 1.1533 and a maximum value of maximum height difference $\frac{(197-151)}{255} = 0.18039$

To estimate the theoretical fractal dimension behavior, considering in steady-state the surface divides into sites whose size is equivalent to that of the particles that make up the surface, and the probability that a specific site has a height of the interface h comes from the probability obtained by the mesoscopic model in the limit $\Omega \rightarrow 1$. From this consideration, the length of the irregular line is:

$$L = \int \rho dz \tag{16}$$

ρ is identified with the expected value of the probability function:

$$\rho = \int [P(z)] P(z) dz = C \frac{1}{\sigma^{\frac{1}{2}}} \tag{17}$$

and

$$\frac{1}{\sigma^{\frac{1}{2}}} \equiv Z^{\mu} \tag{18}$$

where:

$$\mu = \lim_{\Omega \rightarrow 1} \lim_{Z \rightarrow 1} \frac{\partial \ln \left(\sigma^{-\frac{1}{2}}\right)}{\partial Z} \left(\frac{\partial \ln Z}{\partial Z} \right)^{-1}$$

$$\mu = \frac{1}{2} \frac{(1 - Hb)}{1 + Hb} \tag{19}$$

$$b = \frac{k_0}{Q_0} \tag{20}$$

Substituting (eq 17-19) on eq (16) and considering eq (15) we obtain:

$$f = \frac{1}{2} \frac{Hb + 3}{Hb + 1} \simeq \frac{3}{2} - Hb \tag{21}$$

H is the maximum height difference between two sites on the surface; b is a parameter representing the relationship between velocity constant associated with the process of particle detachment or surface erosion and the associated with the deposition of these.

Figure 2 shows the fractal dimension has predicted theoretical behaviors for H and b, predicting that the fractal dimension decreases concerning both parameters.

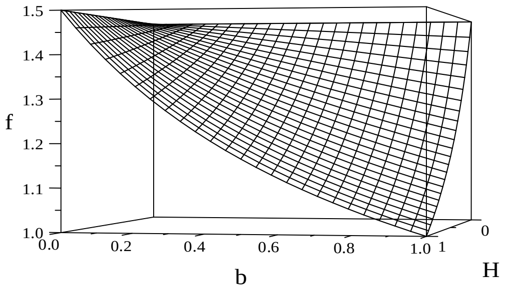
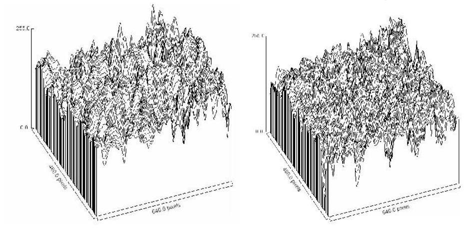


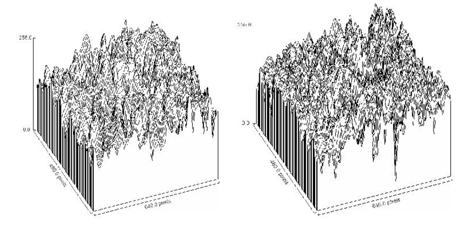
Figure 2. Predicted behavior of the fractal dimension for the maximum height difference H in the interface and relation b between the detachment velocity coefficient and particles deposition

4.RESULTS AND DISCUSSION

Samples were taken from the surfaces of two solids A and B, formed from poured earth, where the difference between the two is that it presents fiber as an additional component in the case of B. Ten samples were taken from each of the surfaces, determining for each sample the maximum height difference found on the surface and the fractal dimension. Both parameters were determined experimentally from the image of the surfaces taken with an x1000 magnification microscope. The image treatment software determined the fractal dimension, and the maximum height of the interface was ImageJ. Each image converts into an 8-bit image, obtaining the average intensity profile of the pixels. The fractal dimension value and the maximum height difference in a non-dimensional way were determined (see Figure 1 b). This procedure was also applied to study the surfaces after the abrasion process. For each sample, we took ten surface images, before and after every surface were subjected to an erosion process through mechanical abrasion, for which a total of 40 images were taken and treated with ImageJ. Then we determined the fractal dimension f and the maximum height difference of the H interface for each of the images. An increase in the magnitude of spatial fluctuations was observed after the erosion process, as illustrated in Figure 3.



Solid A original eroded



Solid B original eroded

Figure 3. 3D Surface morphology for two solids with and without eroding

A statistical comparison of the samples surfaces fractal dimension obtained for each solid before and after eroding are in Table 1 and the box-and-whisker plot in Figure 4.We obtained differences between the interface morphology before and after eroding, but there are no differences between both surfaces (A and B) for similar cases. We found that fiber presence in the composition does not affect the surface's morphology, where the most critical factor influencing the morphology was the erosion of the surface of both solids.

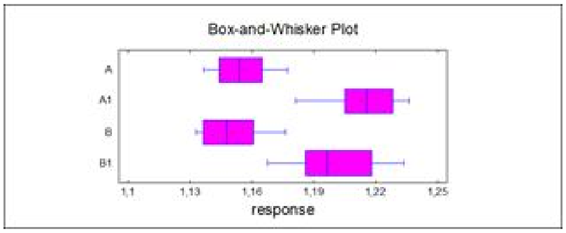


Figure 4. Samples Comparision for fractal dimensión in solids A and B original and after eroding (A1 y B1)

Figure 5 shows the behavior of the fractal dimension f concerning the maximum height of the H interface for each of the solids before erosion (NE) and after erosion (E) and the lines of a linear trend whose slopes allow estimating the value of parameter b. In this case, it can be seen that for solid B without erosion, there is a better correlation between both parameters about solid A without erosion, with an estimated coefficient of b equal to 0.32, practically twice the value obtained for case A, where the estimated value of b is equal to 0.18.

On the other hand, we did not find a significant decrease in the maximum height, but significant differences between the fractal dimension before and after erosión are observed. For A, the increase in the fractal dimension also implies an increase in the coefficient b, showing that the probability of particle detachment was higher in the case of solid B. However, the fractal dimension is increased, the value of parameter b decreases, and the correlation coefficient between both variables

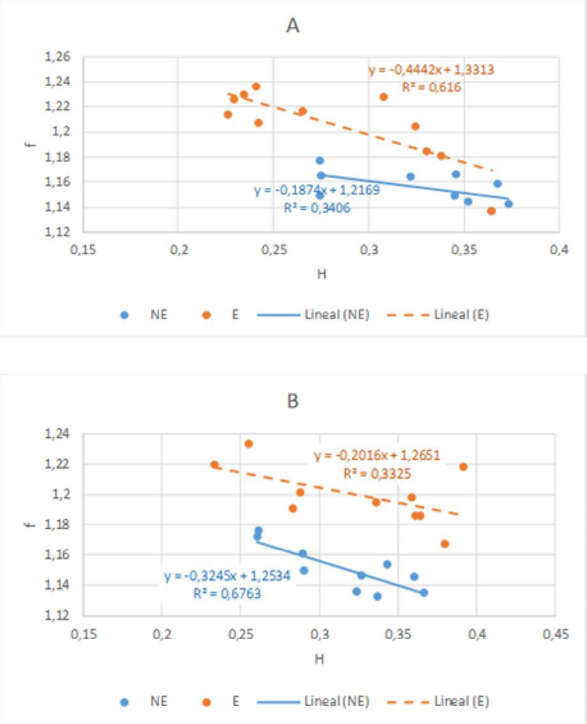


Figure 5. It shows the interface maximum height fractal dimension behavior for each solid A and B before erosion (NE) and after erosion (E)

5.CONCLUSION

From stochastic modeling techniques and the fractal dimension, we obtained a model allowing describe how the fractal dimension of a surface behaves as a function of the dynamics of the particle deposition and detachment processes. This model was used to study the morphology of the brick surface of two solids, one formed by poured earth and the other by a mixture of poured earth and fiber, for all images of the treated surface using the ImageJ software. The experimental results observed the fractal dimension's behavior to the maximum height of the interface correspond to those predicted theoretically. A statistical comparison of the fractal dimension samples of both solids before and after erosion was carried out, finding that fiber does not influence the surface; however, it is significantly modified when the surfaces were subjected to an erosion process by mechanical abrasion.

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