

## PI Controller Design for Networked Control Systems with Random Time Delay

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### ABSTRACT

This paper presents an approach to PI controller design for networked control systems (NCSs) with random time delay. In the proposed approach, the time delay induced by the closed loop over the network is supposed to be bounded, and its upper bound is known. A PI controller is designed for the control of the system with the maximum time delay using a practical approach based on stability region locus. The obtained PI controller is then used to control the actual networked control system. To show the validity and effectiveness of the proposed approach, a simulation example in Matlab/Simulink and TrueTime is given.

**Key words:** Matlab/Simulink, networked control systems, PI controller, random time delay, Truetime.

### 1. INTRODUCTION

A Networked Control System (NCS) is a kind of control system in which the feedback loop is closed over a real network. In such a system, the main components (i.e., controllers, actuators, and sensors) exchange control signals and information (e.g., reference input, control input, output signals) via a communication network [1].

In the past few decades, NCSs have attracted considerable research interest as the digital communication technology develops rapidly (e.g., [2-10] and references therein). In addition, due to their many advantages (e.g., reducing wiring and cost, easy maintenance, the ability to extend systems easily, etc.) NCSs have been finding applications in a wide range of fields, such as automated highway systems, unmanned aerial vehicles, remote surgery, haptics collaboration over the Internet, mobile sensor networks, and more [11].

Random time-delay induced by the communication network represents one of the most important challenges in the control of networked control systems. These time delays may degrade the performance of the system or even lead to instability. Hence, there have been considerable number of studies that have tried to address the control problem of networked control systems with random delays (e.g., [12-15]). Despite the fact that the control problem of networked control

systems with time-delay has been, as mentioned, widely studied in the literature, to the best of the author's knowledge, this problem has not been fully solved to date. Moreover, almost all of the methods proposed in the literature are, in general, difficult to apply in real cases and/or need to make some assumptions about the nature of the time-delay. In this paper, we propose a practical and easy method for designing a PI controller for a networked control system with random time delay. In this approach, we only assume that the time-delay is bounded and that its upper bound is known.

This paper is organized as follows. In Section 2, an overview of NCSs is given. The method of designing a PI controller for a networked control system with random time delay is presented in Section 3. In Section 4, a simulation example in Matlab and TrueTime/Simulink showing the effectiveness and validity of the proposed method is presented. Finally, the conclusions are provided in Section 5.

### 2. REVIEW OF NCSs

#### 2.1. Single-loop NCS structure

As mentioned above, a networked control system (NCS) is a spatially distributed system that consists of the plant to be controlled, the controller, the actuator, the sensor, and a shared, band-limited communication network, which is used by the components of the system to exchange information and control signals between them. Figure 1 shows a general, single-loop NCS layout.

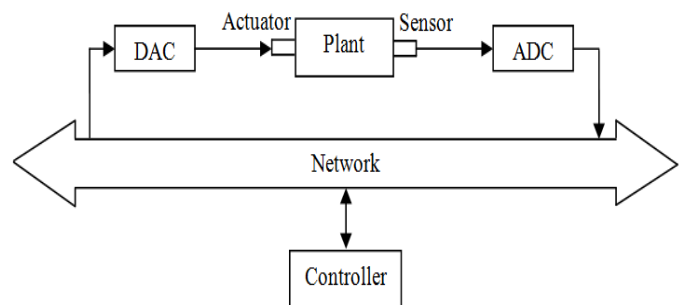


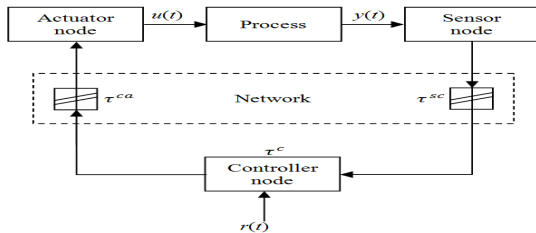
Figure 1: Single-loop NCS layout

The transmission of data packets over the shared network introduces network-induced delays to the networked control system. These delays may degrade the performance or even lead to instability of the control system. In the next subsection

we give a brief presentation on the types of network-induced delays that can occur in a NCS.

### 2.2. Network-induced delays

Figure 2 shows the structure of a networked control system with induced-network time delays.



**Figure 2:** NCS with induced-network time delays

As can be seen in Fig. 2, there are three kinds of delays in NCSs [16]:

- communication delay between the sensor and the controller, ( $\tau^{sc}$ )
- computational delay in the controller, ( $\tau^c$ )
- communication delay between the controller and the actuator, ( $\tau^{ca}$ )

Although computational delay in the controller,  $\tau^c$ , always exists, this delay is usually small compared to the network delays and can be neglected [17]. On the other hand, the two network-induced delays,  $\tau^{sc}$  and  $\tau^{ca}$ , may have different characteristics [18, 19].

According to the types of the communication networks used in NCSs, the characteristics of the network-induced delay vary as follows [18, 23, 24, 25].

- (i) Cyclic service networks (e.g., Token-Ring and Token-Bus). These delays are often bounded and can be regarded as constant for most occasions.
- (ii) Random access networks (e.g., Ethernet and CAN). These delays are often random and unbounded.
- (iii) Priority order networks (e.g., DeviceNet). These delays are bounded for the data packets with higher priority and unbounded for those with lower priority.

In most cases, these delays can be lumped together as one network delay [18, 20-22].

In this study, computational delay in the controller ( $\tau^c$ ) is neglected and all other delays are lumped together as induced-network delay  $\tau$  as follows:

$$\tau = \tau^{sc} + \tau^{ca}$$

In the next section, a method for PI controller design for compensation for induced-network delay in NCSs is presented.

## 3. DESIGN OF PI CONTROLLER FOR NCSS WITH RANDOM TIME-DELAY

### 3.1. Assumptions

- The model of the system to be controlled and its parameters are known.
- The induced-network delay is bounded and its upper bound is known.
- The free-delay system is a time-invariant system and is stable.

### 3.2. PI Controller design

Here, a single-loop networked control system with induced-network time delays (like the one shown in Fig. 2) is considered. Assume that the overall closed-loop networked control system is represented by the following transfer function:

$$T(s) = \frac{C(s)G(s)e^{-\tau s}}{1 + C(s)G(s)e^{-\tau s}} \tag{1}$$

where  $\tau$ ,  $G(s)$  and  $C(s)$  are the induced-network time delay, the transfer function of the plant to be controlled, and the transfer function of a PI controller, respectively. Assume also that the time delay  $\tau$  is bounded, and its upper bound is  $\tau_u$ .

The objective is to determine the parameters of a PI controller that stabilizes the overall closed-loop system and ensures good reference tracking by the system output.

The method we propose to meet this objective is as follows.

First, one determines a point inside the stability region of the closed-loop system for  $\tau = \tau_u$  as follows.

Let the closed-loop transfer function of the system when the delay is maximum (i.e.,  $\tau = \tau_u$ ) be

$$T_u(j) = \frac{C(s)G(s)e^{-\tau_u s}}{1 + C(s)G(s)e^{-\tau_u s}} \tag{2}$$

where  $G(s) = \frac{N(s)}{D(s)}$  and  $C(s) = K_p + \frac{K_i}{s}$ .

The substitution  $s = j\omega$  in the equation (2) and the decomposition of the numerator and denominator into their even and odd parts lead to:

$$T_u(j\omega) = \frac{(wK_p - jK_i)(N_e + jwN_o)e^{-jw\tau_u}}{K_1wN_o e^{-jw\tau_u} + jw^2K_pN_o e^{-jw\tau_u} + jD_o w^2 - jK_1N_e e^{-jw\tau_u} + K_pN_e w e^{-jw\tau_u} + wD_e} \tag{3}$$

Inserting Euler's identity for  $e^{-jw\tau_u} = \cos(\tau_u w) - j\sin(\tau_u w)$  into equation (3), the characteristic polynomial can be expressed as:

$$P_c(j\omega) = R_Q(\omega) + jI_Q(\omega) \tag{4}$$

where

$$R_Q(w) = (wD_e(N_o w^2 \sin(w\tau_u) + wN_e \cos(w\tau_u)) - w^2 D_o(N_o w^2 \cos(w\tau_u) - wN_e \sin(w\tau_u)))K_p + (wD_e(N_o w \cos(w\tau_u) - N_e \sin(w\tau_u)) - w^2 D_o(-N_e \cos(w\tau_u) - wN_o \sin(w\tau_u)))K_i - D_o^2 w^4 + D_e^2 w^2$$

$$I_Q(w) = (wD_e(w^2 N_o \cos(w\tau_u) - wN_e \sin(w\tau_u)) + w^2 D_o(wN_e \cos(w\tau_u) + w^2 N_o \sin(w\tau_u)))K_p + (wD_e(-wN_e \cos(w\tau_u) - wN_o \sin(w\tau_u)) + w^2 D_o(wN_o \cos(w\tau_u) - N_e \sin(w\tau_u)))K_i + 2w^3 D_o D_e$$

Thus, the characteristic equation of the system is  $P_c(jw) = R_Q(w) + jI_Q(w) = 0$  (5)

Then, dropping  $w$  for simplicity and equating the real and imaginary parts of  $P_c(jw)$  to 0 results in the following equations system:

$$\begin{cases} P_{11}K_p + P_{12}K_i = M \\ P_{21}K_p + P_{22}K_i = Z \end{cases} \quad (6)$$

where

$$P_{11} = (wD_e(N_o w^2 \sin(w\tau_u) + wN_e \cos(w\tau_u)) - w^2 D_o(N_o w^2 \cos(w\tau_u) - wN_e \sin(w\tau_u)))$$

$$P_{12} = (wD_e(N_o w \cos(w\tau_u) - N_e \sin(w\tau_u)) - w^2 D_o(-N_e \cos(w\tau_u) - wN_o \sin(w\tau_u)))K_i$$

$$P_{21} = (wD_e(w^2 N_o \cos(w\tau_u) - wN_e \sin(w\tau_u)) + w^2 D_o(wN_e \cos(w\tau_u) + w^2 N_o \sin(w\tau_u)))$$

$$P_{22} = (wD_e(-wN_e \cos(w\tau_u) - wN_o \sin(w\tau_u)) + w^2 D_o(wN_o \cos(w\tau_u) - N_e \sin(w\tau_u)))$$

$$M = -D_o^2 w^4 + D_e^2 w^2$$

$$Z = 2w^3 D_o D_e$$

Solving the equations system (6) for  $K_p$  and  $K_i$  gives

$$K_p = -\frac{w^2 D_o N_o \cos(w\tau_u) + w D_e N_o \sin(w\tau_u) + D_e N_e \cos(w\tau_u) - w D_o N_e \sin(w\tau_u)}{N_o^2 w^2 + N_e^2} \quad (7)$$

$$K_i = \frac{w(-w D_e N_o \cos(w\tau_u) + w^2 D_o N_o \sin(w\tau_u) + w D_o N_e \cos(w\tau_u) + D_e N_e \sin(w\tau_u))}{N_o^2 w^2 + N_e^2} \quad (8)$$

The choice for  $w \geq 0$  gives a set of pairs  $K_p$  and  $K_i$ . Plotting the dependency of  $K_i$  on  $K_p$  and the axis  $K_i = 0$  splits the  $(K_p, K_i)$ -plan into several regions, as shown in Fig 3. To obtain a pair  $(K_p, K_i)$  stabilizing the system to be controlled, one chooses a point inside the region delimited by the curves  $(K_p, K_i)$  and the axis  $K_i = 0$  and checks the stability of the closed-loop system.

In the approach proposed in this paper, the pair  $(K_p, K_i)$  to be chosen is

$$K_{pc} = \frac{1}{N} \sum_w K_p(w) \quad (9)$$

$$K_{ic} = \frac{1}{N} \sum_w K_i(w) \quad (10)$$

$K_{pc}$  and  $K_{ic}$  are the coordinates of the weighted geometrical center of the stability region on the  $(K_p, K_i)$ -plan.  $N$  is the number of points considered over an appropriate frequency range.

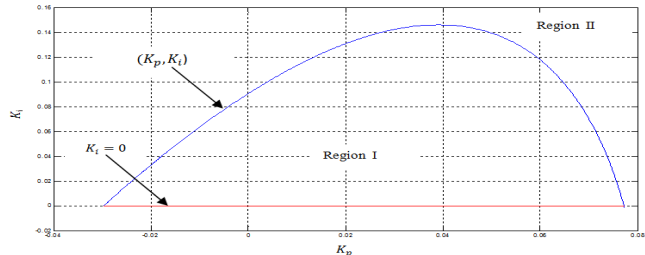


Figure 3: Stability regions locus

Second, since the free-delay system is stable, the weighted geometrical center is inside all the stability regions of the closed-loop system that could be obtained for any  $0 \leq \tau \leq \tau_u$ , and the pair  $(K_{pc}, K_{ic})$  gives the parameters of a PI controller that stabilizes the system (1) and ensures good reference tracking by the output of the system. This result can be proved by doing the same above calculations for  $\tau$  and taking into consideration that the free-delay system is stable.

#### 4. SIMULATION EXAMPLE

In order to show the validity and effectiveness of the proposed approach, a simulation example is given.

The implementation of the considered networked control system as TrueTime's blocks is shown in Fig.4. The system includes four nodes (interference node – Node 1; actuator node – Node 2; sensor node – Node 3; and controller node – Node 4).

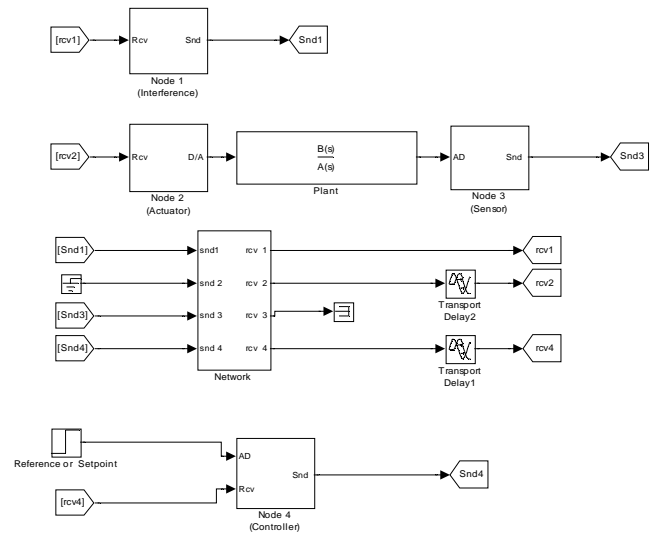


Figure 4: Networked Control System scheme[15]

The plant subject to control is defined by the transfer function [16],

$$G(s) = \frac{2029.826}{(s+26.29)(s+2.296)} \tag{11}$$

Thus, the even and odd parts of the numerator and denominators are  $N_e(j\omega) = 2029.826$ ,  $N_o(j\omega) = 0$ ,  $D_e(j\omega) = -\omega^2 + 60.36184$ , and  $D_o(j\omega) = 28.586$ , respectively. The induced-network random time-delay is bounded and its upper bound is equal to 0.33 seconds.

The objective is to determine the values of the parameters of a PI controller to control the above NCS. The stability boundary locus for induced-network time delay equal to 0.33 is calculated using equations (7) and (8) and plotted as shown in Fig.5.

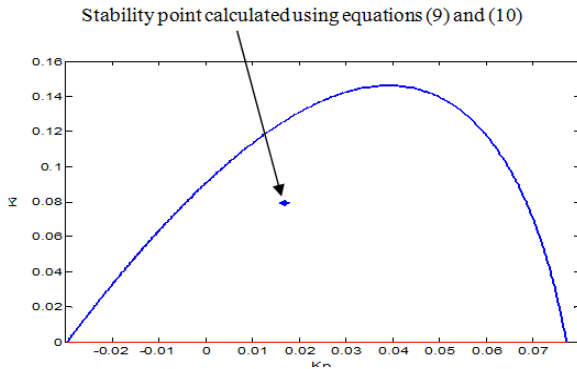


Figure 5: Stability regions locus for the system (11)

The coordinates of the weighted geometrical center of the stability region are:  $K_{pc} = 0.0169$ ,  $K_{ic} = 0.0794$ .

Figure6 shows the boundary stability for different random values of the induced-network time delay.

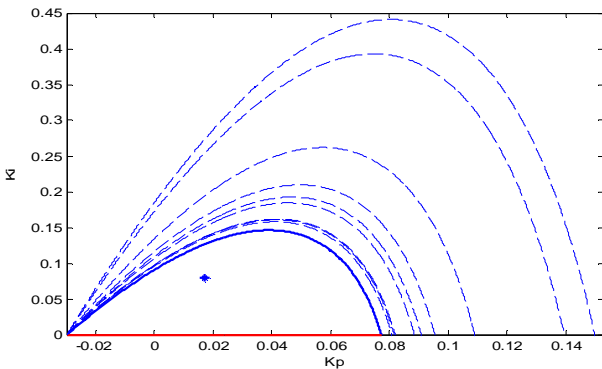


Figure 6: Stability boundaries( solid line: stability boundary for  $\tau = \tau_u = 0.33$ ; dashed lines: stability boundaries for different random values of induced-networked time delay  $\tau$ ; \*: stability point for  $\tau = \tau_u = 0.33$ )

As can be seen in Fig. 6, the point  $(K_p, K_i) = (0.0169, 0.0794)$  is inside of all the stability boundaries obtained for various random values of time delay.

The system (11) is stable. Hence, according to the

discussion presented in Section 3, the PI controller whose parameters are  $K_p = 0.0169$ ,  $K_i = 0.0794$  stabilizes the system and ensures good tracking reference for any random value of the induced-network time delay.

To show the results of the control of the considered NCS with the PI controller whose parameters are  $K_p = 0.0169$ ,  $K_i = 0.0794$  in the presence of a random time delay, a numerical simulation using the TrueTime toolbox/Simulink was carried out. During the simulation study, the sampling period was 0.01s, the network communication mode was CSMA/CD (Ethernet), the transmission rate was 80Kbit/s, and the network delays were  $\tau^{sc} = \tau^{ca} = \frac{\tau}{2} = 0.33$  s.

The performance of the system is shown in Fig.7.

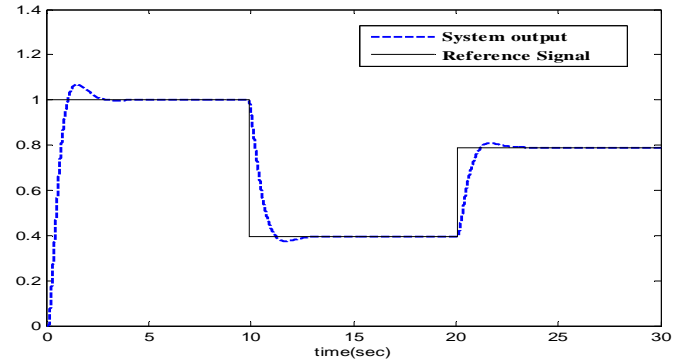


Figure 7: .Simulation results (solid black line: reference signal; dashed blue line: system output)

From Fig.7, it can be seen that the system is stable and the system output tracks the reference signal very well.

### 5. CONCLUSION

In this paper, a method based on the stability boundary locus has been presented to design a PI controller for an NCS with random time delay where the random time delay is assumed to be bounded and its upper bound is known. The center point of the PI controller stability region obtained when the time delay is equal to the upper bound has been determined, and it has been shown that this point gives the parameters of a PI controller stabilizing the closed loop system with random time delay. The validity and effectiveness of the proposed method have been shown through a simulation example.

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