ISSN 2347 - 3983
Volume 8. No. 10, October 2020
International Journal of Emerging Trends in Engineering Research
Available Online at http://www.warse.org/IJETER/static/pdf/file/ijeter1578102020.pdf
https://doi.org/10.30534/ijeter/2020/1578102020

On the applicability of HAM to seek periodic solution for truly nonlinear oscillator

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ABSTRACT

Homotopy perturbation method (HPM) is claimed to be a simple analytic approximation method suitable for solving nonlinear differential equations. It assures solution series convergence by transferring the nonlinear problem into a number of linear sub-problems. A differential equation which is really nonlinear oscillator is considered for obtaining periodic solution using the HPM. To examine the adequacy of HPM, the phase diagram made from the approximate solution is studied with respect to the actual phase diagram.

Key words: Amplitude; Equation of motion; Frequency parameter; Homotopy perturbation method; Phase diagram.

1. INTRODUCTION

Perturbation and asymptotic approximations are generally applicable for weakly nonlinear ODEs (ordinary differential equations) and PDEs (partial differential equations) having small/large physical parameters. Khatami et al. [1] have successfully applied the DTM (differential transform method) and obtained solutions for nonlinear Duffing oscillators. The modified DTM provides inconsistency in the periodic solutions of the nonlinear Duffing oscillators having asymmetric oscillations [2-4]. Many nonlinear differential equations are solved applying the Homotopy Perturbation method (HPM) and the traditional Adomian decomposition method [5-27]. HPM is claimed to be a simple analytic approximation method suitable for nonlinear problems. It transfers into a number of linear sub-problems and assures the solution series convergence. The objective of this research article is to study the competence of the HPM for a truly nonlinear oscillator possessing actual solution.

2. MOTION EQUATION

A DE of order two with nonlinearity of motion for undamped free vibrations is of the form

$$\frac{d^2y}{dt^2}+f(y)=0 \tag{1}$$

The cubic polynomial restoring force function, $f(y)$ is

$$f(y)=\alpha\, y+\beta\, y^2+\gamma\, y^3 \tag{2}$$

The Duffing equation is a well-known eg. of system which is nonlinear [28-33]. The non-linear vibration characteristics are studied on laminated beams and plates [34-38].
The truly nonlinear oscillator having the power-law type restoring force function considered here is [39-41]

$$\frac{d^2y}{dt^2}+y^3=0 \tag{3}$$

$$y = 1,\ \frac{dy}{dt} = 0 \text{ at } t = 0 \qquad (4)$$

For the restoring force function, $f(y) = y^3$ in equation (3), the frequency is expected to increase with increasing amplitude [42]. This hardening nonlinearity is depicted by rubber pads in the mounting of machinery under compression. Displacement's cube in the restoring force function can also be depicted by the motion of a ball-bearing oscillating in a U-shaped vertically standing glass tube. Whineray [43] has demonstrated experimentally by constructing a cubic-law air track oscillator. Actual solution is carried out for equation (3) and the improved MDTM is incapable to provide accurate results [4]. A trail has beencarried out here to verify the competence of homotopy perturbation method (HPM) by solving equations (3) and (4). Applying the HPM [44, 45] to equations (3) and (4), the first-order approximation obtained is

$$y(t) = \cos(0.866t) + 0.0322\cos(2.598t) \qquad (5)$$

By using the constraints in (4) and taking anti derivatives of (3) it is evident that $(y')^2 = \frac{1}{2}(1-y^2)(1+y^2)$ (6)

Figure 1. describes the comparative study of phase diagrams generated from equations (5) and (6).

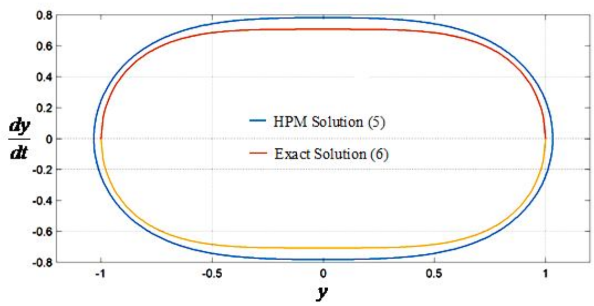

Figure 1. Phase diagrams generation from equations (5) and (6)

The actual solution for (3) and (4) is obtained in Jacobi elliptic cosine function terms of (cn) as [4]

$$y(t) = cn\left(t, \frac{1}{\sqrt{2}}\right) \qquad (7)$$

The angular frequency, $\omega = 0.8472$, and it is 0.866 by the 1st approximation of HPM (see equation (5)). Though, slight difference is noticed in the values of angular frequency and amplitude, the trend in the phase diagram from the HPM solution is seen like the actual phase diagram.

$y(t)$ in equation (3) satisfies the initial conditions (4) is assumed in the form

$$y(t) = A_1\cos(\omega t) + (1 - A_1)\cos(3\omega t) \qquad (8)$$

By applying the HARMONIC BALANCE METHOD applications and basic trigonometry principles in order to retain only constant terms and terms of $\cos(\omega t)$ and $\cos(3\omega t)$ two equations are obtained. These equations give $A_1 = 0.9571$ and $\omega = 0.8488$. The solution of equations (3) and (4) obtained from equation (8) is

$$y(t) = 0.9571\cos(0.8488t) + 0.0429\cos(2.5464t) \qquad (9)$$

Figure-2 shows the excellent matching of the PHASE DIAGRAMS created from equations (6) and (9). From Figures 1 and 2, the use of the HBM with higher order harmonics provides results close to the exact, whereas little discrepancy is noticed in the results of the problem using HPM.

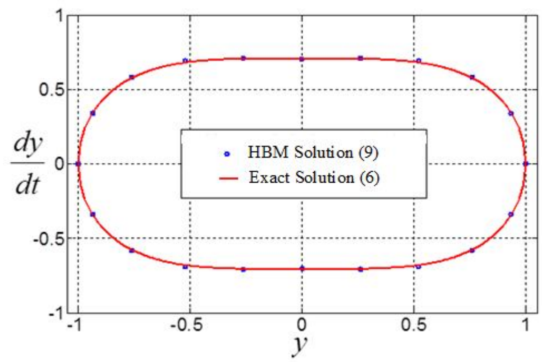

Figure 2. Comparative study of the PHASE DIAGRAMS generated from equations (6) and (9).

3. CONCLUSION

The homotopy perturbation method (HPM) provides approximate solution for the problem of truly nonlinear oscillations comparable with exact solution. In the above conversation we have established that the restoring force function in the truly nonlinear oscillations displacement cube. Usage of the method of harmonic balance with higher order harmonics presents realistic results. This innovative research study demonstrates the drawbacks in the usage of MDTM to get the periodic solution of a simple truly nonlinear oscillator differential equation having exact solution.

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