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Modified Minimum depth-span ratio of beams and slabs

Mereen H. Fahmi[1], Ayad Z. Saber[2]
[1]Professor, Erbil Polytechnic University, Erbil Technical Engineering College, Dept. of Civil engineering. Erbil-Iraq, mereen.akrawi@epu.edu.iq
[2]Assistant Professor, Erbil Polytechnic University, Erbil Technical Engineering College, Dept. of Civil engineering. Erbil-Iraq, ayad.saber@epu.edu.iq

ABSTRACT

The deflection control and determining of minimum thickness of slabs and beams are included in different buckling codes and standards, also the ACI Code includes provisions of determining immediate and time dependent deflection and depth-span limitations which is tabulated for normal weight concrete and specified yield strength of steel reinforcement and correction factors are provided for other values of steel reinforcement and light weight concrete. Correction factors are suggested in this study to include the effect of concrete strength and steel reinforcement yield strength on the ACI Code limitation factors of the depth-span ratio of beams and slabs with different boundary conditions;(simply supported, fixed ended, propped and cantilevers).

Key words: Deflection, Deflection control, Depth-span ratio.

1. INTRODUCTION

Deflection of slabs and beams can be controlled by addition of steel reinforcement bars in tension and compression zones or using pre-stressing concrete, in addition to that deflection are influenced by different factors such as loading type and value, material properties (E), section properties (I), type of the member, i.e. type of the boundary condition (simply supported, fixed or free), and time dependent or long term deflection due to creep and shrinkage of concrete during the life of the structure.

Building design codes and ACI code [1] calculate the deflection under loads up to the full service load to ensure that stresses in the stream fiber in both steel and concrete remain within elastic range, and the uncracked section properties are used to determine the immediate deflection, then long term or time dependent deflection is calculated due to creep and shrinkage along the life of the structure. Lee et al. [2] compared provisions of different codes and standards about minimum thickness, they concluded that the CSA and ACI provisions have limited application and the proposed equation is recommended for calculation the minimum thickness. Beal and Thomason [3], presented an approximated depth-span ratio for the preliminary design specification in term of (M/bd^2) rather than (A_s/bd) to include the effect of steel design stress. Shehata et al [4], presented a theoretical study for the minimum steel ratio is required for bending, shear and torsion for beams with different concrete strengths. ACI code [1], provides the minimum depth of one-way slab and beams shown in Table (1) for non-prestressed conditions normal weight concrete $(\gamma_c = 145\ \text{lb/ft}^3\ or\ 2320\ \text{kg/m}^3)$ and steel reinforced yielding strength $(f_y = 60{,}000\ psi\ or\ 414\ MPa)$, correction factors are used for light weight concrete and other values of (f_y) as shown below:

$$\lambda_1 = 1.65 - 0.005\gamma_c \geq 1.09 \tag{1}$$

$$\lambda_2 = 0.4 + \frac{f_y}{100{,}000} \tag{2}$$

Table 1: Minimum thickness ACI code limitation [1], [5]-[8]

Support type	One way slab	Beam
Simply supported	$L/20$	$L/16$
One end continuous (propped)	$L/24$	$L/18.5$
Two end continuous (fixed ended)	$L/28$	$L/21$
Cantilever	$L/10$	$L/8$

The minimum thickness calculated by the code provisions to ensure that the beam or slab will be stiff enough and the deflection within the permissible range. Generally, the deflection is a function of the load, span length, beam cross section represented by moment of inertia (I), material property represented by (Modulus of elasticity E), and the support condition (simple, fixed or free) at both ends. Elastic deflection can be expressed in the following general form: [5-8]

$$\Delta = \frac{f\ (load\ , span\ , \sup portcondition)}{EI} \tag{3}$$

where;

E = modulus of elasticity (MPa)

I = moment of inetia of the cross section (mm^4)

$f\ (load\ , span, \sup portcondition)$: is a function of load, span length and support condition which is determined by elastic analysis. Table (2) shows the maximum deflection for different type beams and loadings.

Table 2: Maximum deflection of different types of beams and loadings [9]-[11]

Support type	Loading type	α
Simply supported	Uniform distributed load	$5/384$
One end continuous (propped)	Uniform distributed load	$1/185$
Two end continuous (fixed ended)	Uniform distributed load	$1/384$
Cantilever	Uniform distributed load	$1/8$
Simply supported	Concentrated load at midspan	1/48
One end continuous (propped)	Concentrated load at midspan	$1/(48\sqrt{5})$
Two end continuous (fixed ended)	Concentrated load at midspan	1/192
Cantilever	Concentrated load at free end	1/3

where:

$$\delta_{\max} = \frac{\alpha\, w\, L^4}{E\, I} \quad \text{for distributed load.}$$

$$\delta_{\max} = \frac{\alpha\, P\, L^3}{E\, I} \quad \text{for concentrated load.}$$

Orvin and Anik [12] determined the minimum thickness of reinforced concrete slabs to resist undesirable vibration, and compare the results with other study. They conclude that American Concrete Institute (ACI) minimum thickness limit is not satisfactory for vibration. Three dimensional finite element modelling is carried out to study the natural floor vibration, and the results are verified by ANSYS model and ETABS modeling. Several parameters as slab thickness, span length and floor panel aspect ratio are taken into consideration. Akmaluddin [13] presented an improvement model of the effective moment of inertia to predict to predict the short term deflection of reinforced light weight concrete beam. The proposed model is verified and compared with experimental results of nine beams, good agreement is obtained with the experimental results and in some cases have similar trend to the ACI and SNI previsions. Ho et al [14], developed a simplified method for providing minimum flexural ductility and evaluation of maximum values of tension steel ratio and neutral axis depth corresponding to the proposed minimum curvature ductility factor for various concrete grades and steel yielding strengths. Islam Khan et al [15] investigated reinforced concrete building analysis by using three dimensional finite element modelling to determine the minimum slab thickness to prevent undesirable vibration. The developed finite element model is applied on post experiments which are validated the applicability of the model for further parametric study. Different slab thickness, span length and floor aspect ratio are studied. Empirical equation is suggested which provide minimum slab thickness of a short span reinforced concrete building.

2. THEORETICAL ANALYSIS

Uncracked section property (I_g) is used in the calculation of deflection up to cracking moment when tensile stress at the stream fiber reached to the tensile strength of the concrete (f_r), but beyond this limit, effective moment of inertia (I_e) is used which is lied between cracking and uncracked section, moment of inertia as given in the following equation:

$$I_e = \left(\frac{M_{cr}}{M_a}\right)^3 I_g + \left[1 - \left(\frac{M_{cr}}{M_a}\right)^3\right] I_{cr}$$

(1)

where: I_{cr} = Moment of inertia of cracked transformed section (mm^4).

I_g = Moment of inertia of uncracked transformed section (mm^4).

M_a = Maximum bending moment due to the service load $(kN.mm)$.

M_{cr} = Cracking bending moment due to service load and equal to

$$M_{cr} = \frac{f_r . I_g}{y_t} \tag{5}$$

$f_r =$ Modulus of rupture of the concrete (MPa).

$y_t =$ Distance from the neutral axis of the section to the extreme fiber at the

tension face (mm).

For rectangular section without reinforcement: $I_g = \dfrac{bh^3}{12}$

where: b = width of the cross section (mm).

h = total depth of the cross section (mm).

For beam with tension reinforcement:

$$\bar{y} = \frac{bh\left(\frac{h}{2}\right) + (n-1)A_s . d}{bh + (n-1)A_s}$$

where: d = effective depth (mm).

A_s = Area of tension reinforcement (mm^2)

$$n = \frac{E_s}{E_c}$$

E_s = Modulus of elasticity of steel reinforcement (MPa) = 200,000 MPa

E_c = Modulus of elasticity of the concrete (MPa) = $4730\sqrt{f_c^{'}}$

$f_c^{'}$ = cylinder compressive strength of the concrete (MPa)

$$I_g = \frac{bh^3}{12} + bh(\bar{y} - \frac{h}{2})^2 + (n-1)A_s(d-\bar{y})^2 \tag{6}$$

$$I_{cr} = \frac{bc^3}{3} + (nA_s)(d-c)^2 \tag{7}$$

Where (c) is the depth of the compressive zone at the cracked condition, and determined from the following equation:

$$c = \frac{bc(\frac{c}{2}) + nA_s d}{bc + nA_s} \tag{8}$$

$$\frac{bc^2}{2} + nA_s c - nA_s d = 0 \tag{9}$$

equation (9) is solved to determine value of (c).

By equating the moment of inertia of the beam with tension reinforcement with the equivalent section without reinforcement which giving the same deflection, the new equivalent depth (h_1) is determined from the following equations:

$$\frac{bh_1^3}{12} + bh_1[\frac{bh_1(\frac{h_1}{2}) + (n-1)(\rho bd_1)d_1}{bh_1 + (n-1)(\rho bd_1)} - \frac{h_1}{2}]^2 + (n-1)(\rho bd_1)[d_1 - \frac{bh_1(\frac{h_1}{2}) + (n-1)(\rho bd_1)d_1}{bh_1 + (n-1)(\rho bd_1)}]^2 = \frac{bh^3}{12} \tag{10}$$

Where $d_1 = h_1 -$ central cover

ρ = reinforcement index = $\frac{A_s}{bd}$

Assuming $(d_1 = 0.25h_1)$

The depth of neutral axis $\bar{y} = \psi h$

$$\text{where} \quad \psi = \frac{0.5 + 0.7225(n-1)\rho}{1 + 0.85(n-1)\rho} \tag{11}$$

The neutral moment of inertia including the effect of reinforcement and concrete compression strength is:

$$I_g = \lambda \frac{bh^3}{12} \tag{12}$$

Where

$$\lambda = [1 + 12(\psi - 0.5)^2 + 10.2(n-1)\rho(0.85 - \psi)^2 \tag{13}$$

The new depth (h_1) is calculated from (I_g) which is obtained from Eq.(12).

For beam with dimension (width=250mm), taking different values of compressive strength of concrete ($f_c^{'}$= 21, 28, 35, 42, 63, and 84 MPa) and steel yielding strength (f_y = 420 and 525 MPa). The equivalent depth (h_1) is determined for beams with different reinforcement index (ρ). The reinforcement index is taken as a ratio to the balancing reinforcement index (ρ / ρ_b), also (h_1) is determined at maximum reinforcement index (ρ_{max}) and (ρ_t), where:

$$\rho_b = (0.85\beta_1 \frac{f_c^{'}}{f_y})(\frac{\varepsilon_u}{\varepsilon_u + \varepsilon_y})$$

$$\rho_{max} = (0.85\beta_1 \frac{f_c^{'}}{f_y})(\frac{0.003}{0.003 + 0.004})$$

$$\rho_t = (0.85\beta_1 \frac{f_c^{'}}{f_y})(\frac{0.003}{0.003 + 0.004})$$

$$\varepsilon_y = \frac{f_y}{E_s}$$

The results of the ratio of (h_1 / h) for different values of concrete compressive strength ($f_c^{'}$)and steel yielding strength (f_y) are shown in Table (3).

Table 3: Results of (h_1 / h) or (h / h_o)

$f_c^{'}$ (MPa)	21	28	35	42	63	63	84
f_y (MPa)	420	420	420	420	525	420	525
$f_c^{'} / f_y$	0.050	0.067	0.083	0.10	0.12	0.15	0.16
$\rho/\rho_b = 0$	1.0	1.0	1.0	1.0	1.0	1.0	1.0
$\rho/\rho_b = 0.5$	0.968	0.964	0.963	0.962	0.971	0.962	0.968
$\rho/\rho_b = 0.634$	0.960	0.956	0.955	0.954	0.981	0.954	0.958
$\rho/\rho_b = 0.724$	0.956	0.951	0.950	0.949	0.957	0.948	0.953
$\rho/\rho_b = 1.0$	0.943	0.937	0.936	0.935	0.949	0.9343	0.944

The results shown in Table (3) show that the depth ratio (h_1 / h) decreased with increasing the reinforcement index ratio (ρ / ρ_b), this mean that smaller depth is required as the tension reinforcement area is increased.

Two variables are suggested (ρ / ρ_b) to represent the effect of reinforcement amount as a ratio to balancing reinforcement index and other variable (f_c' / f_y) to represent the effect of the material strengths. General equation is suggested to determine the depth ratio (h_1 / h) in the following form:

$$\frac{h_1}{h} = k_0 + k_1\left(\frac{\rho}{\rho_b}\right) + k_2\left(\frac{f_c'}{f_y}\right) \tag{14}$$

Using the data of Table (3), applying the principle of least square and regression analysis method; the following equation is determined:

$$\frac{h_1}{h} = 0.997 - 0.0604\left(\frac{\rho}{\rho_b}\right) + 0.013\left(\frac{f_c'}{f_y}\right) \tag{15}$$

The calculated results from this equation are shown in Table (4) and Fig (1).

The above equation can be written in another way:

$$h_1 = \alpha \; h_{ACI} \tag{16}$$

Where (α) is the correction factor including the effect of reinforcement amount and material strengths.

$$\alpha = \left[0.997 - 0.0604\left(\frac{\rho}{\rho_b}\right) + 0.013\left(\frac{f_c'}{f_y}\right)\right] \tag{17}$$

or can be written as,

$$h_1 = \left[0.997 - 0.0604\left(\frac{\rho}{\rho_b}\right) + 0.013\left(\frac{f_c'}{f_y}\right)\right] h_{ACI} \tag{18}$$

The corrected total depth can be expressed in another form:

$$h_1 = \frac{L}{\beta N} \tag{19}$$

Where (N) is constant depend on the support condition as shown in Table (1), and (L) is the span of the member. General equation in the form of Eq. (17) is suggested to determine the correction factor (β) in term of the same variables (ρ / ρ_b *and* f_c' / f_y) as shown below:

$$\beta = 1.00384 - 0.0595\left(\frac{\rho}{\rho_b}\right) + 0.0048\left(\frac{f_c'}{f_y}\right) \tag{20}$$

$$h_1 = \frac{L}{\left(1.00384 - 0.0595\dfrac{\rho}{\rho_b} + 0.0048\dfrac{f_c'}{f_y}\right)N} \tag{21}$$

Table 4: Modification Factor (α)

f_c' / f_y	ρ/ρ_b	f_c' / f_y	α	$\alpha_{cal.}$	$R = \alpha/\alpha_{cal.}$
3/60	0	0.05	1	0.9974	0.9974
	0.5	0.05	0.968	0.9672	0.9991
	0.634	0.05	0.9606	0.9591	0.9984
	0.724	0.05	0.956	0.9536	0.9975
	1	0.05	0.943	0.9370	0.9936
4/60	0	0.0667	1	0.9976	0.9976
	0.5	0.0667	0.9642	0.9674	1.0033
	0.634	0.0667	0.9563	0.9593	1.0031
	0.724	0.0667	0.9513	0.9539	1.0027
	1	0.0667	0.9373	0.9372	0.9999
5/60	0	0.0833	1	0.9978	0.9978
	0.5	0.0833	0.9631	0.9676	1.0047
	0.634	0.0833	0.955	0.9595	1.0047
	0.724	0.0833	0.95	0.9541	1.0043
	1	0.0833	0.936	0.9374	1.0015
6/60	0	0.1	1	0.9980	0.9980
	0.5	0.1	0.9628	0.9678	1.0052
	0.634	0.1	0.9547	0.9597	1.0053
	0.724	0.1	0.9495	0.9543	1.0050
	1	0.1	0.9352	0.9376	1.0026
9/75	0	0.12	1	0.9983	0.9983
	0.5	0.12	0.9714	0.9681	0.9966
	0.7	0.12	0.9618	0.9560	0.9940
	0.8	0.12	0.9572	0.9500	0.9924
	1	0.12	0.9493	0.9379	0.9880
9/60	0	0.15	1	0.9987	0.9987
	0.5	0.15	0.9622	0.9685	1.0065
	0.634	0.15	0.954	0.9604	1.0067
	0.724	0.15	0.9487	0.9549	1.0066
	1	0.15	0.9343	0.9383	1.0042
12/75	0	0.16	1	0.9988	0.9988
	0.5	0.16	0.9685	0.9686	1.0001
	0.7	0.16	0.9581	0.9565	0.9983
	0.8	0.16	0.9533	0.9505	0.9970
	1	0.16	0.944	0.9384	0.9941
			St.Dv. =	0.0209	
			Correl.=	0.9781	
			Var.=	0.0004	
			Ravg.=	1.00005	

The results obtained from Eq. (20) shown very good correlation as shown in Fig. (2) and Table (5). According to the above results and equations, the modified depth-span ratio is determined for beams and slabs for different types of boundary conditions (simply supported, fixed ended, propped and cantilevers) as shown in Tables 6 and 7.

The following examples are solved to check and verify the modified equations, in all examples the results are very close, where the moment of inertia for the section with reinforcement and new modified depth is exactly equal to the original section without reinforcement with depth according to ACI-Code limitations.

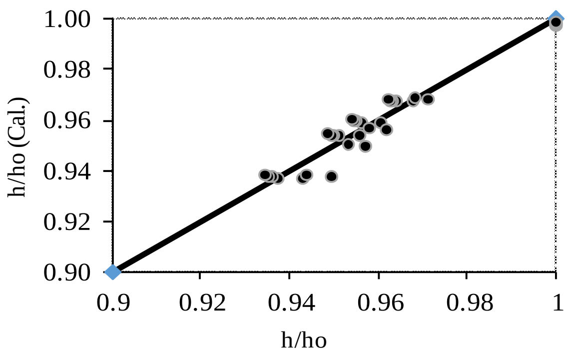

Figure 1: Calculated ($\alpha = h/h_o$) from equation (17)

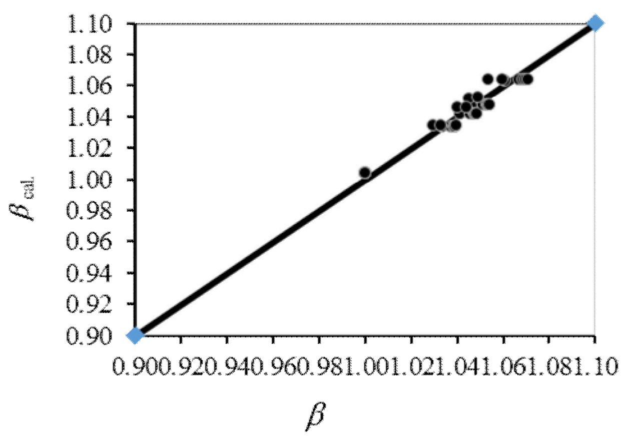

Figure 2: Calculated (β) from equation (20)

Table 5: Modification Factor (β)

f_c'/f_y	ρ/ρ_b	f_c'/f_y	β	$\beta_{cal.}$	$R = \beta/\beta_{cal.}$
3/60	0	0.05	1.0000	1.0041	1.0041
	0.5	0.05	1.0331	1.0338	1.0008
	0.634	0.05	1.0410	1.0418	1.0008
	0.724	0.05	1.0460	1.0472	1.0011
	1	0.05	1.0604	1.0636	1.0030
4/60	0	0.0667	1.0000	1.0042	1.0042
	0.5	0.0667	1.0371	1.0339	0.9969
	0.634	0.0667	1.0457	1.0419	0.9964
	0.724	0.0667	1.0512	1.0472	0.9962
	1	0.0667	1.0669	1.0637	0.9970
5/60	0	0.0833	1.0000	1.0042	1.0042
	0.5	0.0833	1.0383	1.0340	0.9958
	0.634	0.0833	1.0471	1.0420	0.9951
	0.724	0.0833	1.0526	1.0473	0.9950
	1	0.0833	1.0684	1.0637	0.9957
6/60	0	0.1	1.0000	1.0043	1.0043
	0.5	0.1	1.0386	1.0341	0.9956
	0.634	0.1	1.0474	1.0420	0.9948
	0.724	0.1	1.0532	1.0474	0.9945
	1	0.1	1.0693	1.0638	0.9949
9/75	0	0.12	1.0000	1.0044	1.0044
	0.5	0.12	1.0294	1.0342	1.0046
	0.7	0.12	1.0397	1.0461	1.0061
	0.8	0.12	1.0447	1.0520	1.0070
	1	0.12	1.0534	1.0639	1.0100
9/60	0	0.15	1.0000	1.0046	1.0046
	0.5	0.15	1.0393	1.0343	0.9952
	0.634	0.15	1.0482	1.0423	0.9943
	0.724	0.15	1.0541	1.0476	0.9939
	1	0.15	1.0703	1.0641	0.9942
12/75	0	0.16	1.0000	1.0046	1.0046
	0.5	0.16	1.0325	1.0344	1.0018
	0.7	0.16	1.0437	1.0463	1.0024
	0.8	0.16	1.0490	1.0522	1.0031
	1	0.16	1.0593	1.0641	1.0045
				0.0221	
				0.9778	
				0.0005	
				1.00003	

Table 6: Modified minimum Span-Depth ratio for Beams $(N = L/h)$

f_c' (MPa)	21	28	35	42	63	63	84
f_y (MPa)	420	420	420	420	525	420	525
ρ/ρ_b	Simply supported beam						
0	16	16	16	16	16	16	16
0.5	16.52	16.59	16.61	16.61	16.47	16.62	16.52
0.634	16.65	16.73	16.75	16.75	16.63	16.77	16.7
0.724	16.73	16.81	16.84	16.85	16.71	16.86	16.78
1	16.96	17.07	17.09	17.10	16.85	17.12	16.94
ρ/ρ_b	Fixed ended beam						
0	21	21	21	21	21	21	21
0.5	21.69	21.78	21.80	21.81	21.61	21.82	21.68
0.634	21.86	21.96	21.99	21.99	21.83	22.01	21.92
0.724	21.96	22.07	22.10	22.11	21.93	22.13	21.02
1	22.26	22.40	22.43	22.45	22.12	22.47	22.24
ρ/ρ_b	Propped beam						
0	18.5	18.5	18.5	18.5	18.5	18.5	18.5
0.5	19.11	19.18	19.20	19.21	19.04	19.22	19.10
0.634	19.25	19.34	19.37	19.37	19.23	19.39	19.30
0.724	19.35	19.44	19.47	19.48	19.32	19.5	19.40
1	19.61	19.73	19.76	19.78	19.48	19.80	19.59
ρ/ρ_b	Cantilever beam						
0	8	8	8	8	8	8	8
0.5	8.26	8.29	8.30	8.30	8.23	8.31	8.26
0.634	8.32	8.36	8.37	8.38	8.31	8.38	8.35
0.724	8.36	8.41	8.42	8.42	8.35	8.43	8.39
1	8.48	8.53	8.54	8.55	8.42	8.56	8.47

Table 7: Modified minimum Span-Depth ratio for Slabs $(N = L/h)$

f_c' (MPa)	21	28	35	42	63	63	84
f_y (MPa)	420	420	420	420	525	420	525
ρ/ρ_b	Simply supported beam						
0	20	20	20	20	20	20	20
0.5	20.66	20.74	20.76	20.77	20.58	20.78	20.65
0.634	20.82	20.91	20.94	20.94	20.79	20.96	20.87
0.724	20.92	21.02	21.05	21.06	20.89	21.08	20.98
1	21.20	21.33	21.36	21.38	21.06	21.40	21.18
ρ/ρ_b	Fixed ended beam						
0	28	28	28	28	28	28	28
0.5	28.92	29.04	29.07	29.08	28.82	29.1	28.91
0.634	29.14	29.28	29.31	29.32	29.11	29.35	29.22
0.724	29.28	29.43	29.47	29.48	29.25	29.51	29.37
1	29.69	29.87	29.91	29.94	29.49	29.96	29.66
ρ/ρ_b	Propped beam						
0	24	24	24	24	24	24	24
0.5	24.79	24.89	24.92	24.92	24.70	24.94	24.78
0.634	24.98	25.09	25.13	25.13	24.95	25.15	25.05
0.724	25.10	25.28	25.26	25.27	25.07	25.29	25.17
1	25.45	25.60	25.64	25.66	25.28	25.68	25.42
ρ/ρ_b	Cantilever beam						
0	10	10	10	10	10	10	10
0.5	10.33	10.37	10.38	10.38	10.29	10.39	10.32
0.634	10.41	10.45	10.47	10.47	10.39	10.48	10.43
0.724	10.46	10.51	10.52	10.53	10.44	10.54	10.49
1	10.60	10.66	10.68	10.69	10.53	10.70	10.59

3. NUMERICAL EXAMPLES

3.1 A Simply supported beam, with length (L = 6 m). Concrete compressive strength (f_c' = 28 MPa), steel yielding strength (f_y = 420 MPa) and cross section width (b = 200 mm).

Section without Reinforcements: minimum depth according to ACI-Code = $L/16$ = 375mm

$$I_g = \frac{bh^3}{12} = 878,906,250 mm^4$$

For applied load w =15 kN/m

Maximum central deflection

$$\delta_{\max} = \frac{5wL^4}{384EI} = 11.506\ mm\ ;\ \frac{\delta_{\max}}{L} = 0.00192$$

Which is less than $\delta_{\max} = \dfrac{L}{360} = 16.667mm$ o.k.

Section with Reinforcements: $\rho = \rho_b$, b = 200 mm

$$\rho_b = 0.25\beta_1 \frac{f_c'}{f_y}\frac{0.003}{0.003+\varepsilon_y} = 0.0285$$

From Table (6); $\dfrac{\rho}{\rho_b} = 1.0$ and f_c' = 28 MPa ;

$$h_{\min} = \frac{L}{17.07} = 351.5\ mm$$

$$n = \frac{E_s}{\varepsilon_c} = \frac{200000}{4730\sqrt{28}} = 8;\ n-1 = 7$$

Take $d = 0.85h = 298.775\ mm$

$$A_s = \rho bd = 1703.0175\ mm^2$$

Neutral axis depth

$$\bar{y} = \frac{bh(\frac{h}{2}) + (n-1)A_s d}{bh + (n-1)A_s} = 193.587\ mm\ ,$$

$$\psi = \frac{\bar{y}}{h} = 0.5507$$

Check ψ from Eq. (11) $= \dfrac{0.5 + 0.7225(n-1)\rho}{1 + 0.25(n-1)\rho} = 0.5507$

Total moment of inertia

$$= \frac{bh^3}{12} + bh(\bar{y} - \frac{h}{2})^2 + (n-1)A_s(d - \bar{y})^2$$

$= 878105564.3\ mm^4$ this value is very close to the section without reinforcement

$$\frac{I(\rho = \rho_b)}{I(\rho = 0)} = \frac{878105564.6}{878906250} = 0.999 \approx 1.0$$

$$I_0 = \frac{bh^3}{12} = 723810264.6\ mm^4$$

$$\lambda = \frac{I_{total}}{I_0} = 1.21317$$

Check λ from Eq. (13)

$$= 1 + 12(\psi - 0.5)^2 + 10.2(n-1)\rho(0.85 - \psi)^2 = 1.21317$$

The original depth equals 375mm for section without reinforcement. The modified depth equal to 351.5mm for section with reinforcement ($\rho = \rho_b$)

The ratio $\dfrac{h}{h_1} = 0.9373$, by using Eq.(17);

$$\frac{h}{h_1} = \alpha = [0.997 - 0.0604(\frac{\rho}{\rho_b}) + 0.013(\frac{f_c'}{f_y})]$$

For $\dfrac{\rho}{\rho_b} = 1.0\ and\ \dfrac{f_c'}{f_y} = \dfrac{28}{420};\alpha = 0.9372$ which is exactly equal to that obtained before.

3.2 Fixed-ended beam; with length (L = 7m). Concrete compressive strength (f_c' = 21 MPa), steel yielding strength (f_y = 420 MPa), applied load w =30 KN/m, $\rho/\rho_b = 0.5$, cross section width (b = 300 mm).

Section without Reinforcements:

minimum depth according to ACI-Code =

$$\frac{L}{21} = 333.333\ mm$$

$$I_g = \frac{bh^3}{12} = 925923148.2\ mm^4$$

$$E_c = 4730\sqrt{21} = 21675.583\,MPa;\ n = \frac{E_s}{E_c} = 9.227$$

$$\delta_{\max} = \frac{wL^4}{384EI} = 9.346\ mm\ ;$$

$$\frac{\delta_{\max}}{L} = 0.001335\ <\ \frac{1}{360}$$

Section with Reinforcements:

$\rho = \frac{1}{2}\rho_b$, b= 300 mm

From Table (6); and $f_c^{'}$= 21 MPa;

$$h_{\min} = \frac{L}{21.694} = 322.67\ mm$$

Take $d = 0.85h = 274.27\,mm$

$$\rho_b = 0.85(0.85)(\frac{21}{420})\frac{0.003}{0.003+\frac{420}{200000}} = 0.021376$$

$$\rho = \frac{\rho_b}{2} = 0.010688$$

$$\bar{y} = 169.19mm = 0.52434h$$

Check $\psi = \dfrac{0.5+0.7225(n-1)\rho}{1+0.25(n-1)\rho} = 0.52434$

$$I_0 = \frac{bh^3}{12} = 839877169.5\ mm^4$$

$I_g = 925737360\ mm^4$ which is very close to the section without reinforcement.

$$I_g = 0.9998\ I_g\ (\rho = 0)$$

Check

$$\frac{h_1}{h} = 0.9967 - 0.0604(0.5) + 0.013(\frac{21}{420}) = 0.9672$$

$$\frac{h_1}{h} = \frac{322.67}{333.333} = 0.968$$

$$\lambda = \frac{I_g}{I_0} = 1.10223$$

Check

$$\lambda = 1 + 12(\psi - 0.5)^2 + 10.2(n-1)\rho(0.25-\psi)^2 = 1.104$$

3.3 A propped beam; with length (L = 8m). Concrete compressive strength ($f_c^{'}$= 42 MPa), steel yielding strength (f_y = 420 MPa), applied load w = 50 KN/m, $\rho = \rho_t$, cross section width (b = 300 mm).

Section without Reinforcements:

minimum depth according to ACI-Code

$$h_{\min} = \frac{L}{18.5} = 432.432\ mm$$

$$I_g = \frac{bh^3}{12} = 2021591866\ mm^4$$

$$E_c = 4730\sqrt{21} = 30653.9035\,MPa;\ n = 6.5245$$

$$\delta_{\max} = \frac{wL^4}{185EI} = 17.864\ mm\ ;$$

$$\frac{\delta_{\max}}{L} = 0.002233\ <\ \frac{1}{360}$$

Section with Reinforcements:

$$\rho = \rho_t\ ,\ \beta_1 = 0.75$$

$$\rho_t = 0.85\beta_1\frac{f_c'}{f_y}\frac{0.003}{0.003+0.004} = 0.0239;\ \rho = 0.0239$$

$$\rho_b = 0.85\beta_1\frac{f_c'}{f_y}\frac{0.003}{0.003+\varepsilon_y} = 0.0377;\ (n-1)\rho = 0.132$$

From table (6); $h_{\min} = \dfrac{L}{19.378} = 412.84\ mm$

$$\alpha = \frac{412.84}{432.432} = 0.9547$$

Check; $\psi = \dfrac{0.5+0.7225(n-1)\rho}{1+0.25(n-1)\rho} = 0.5353$

$$I_0 = \frac{bh^3}{12} = 1760702800\ mm^4$$

$$I_g = 2021949326\ mm^4$$

$$I_g = 1.000177\ I_g\ (\rho = 0)$$

$$\lambda = \frac{I_g}{I_0} = 1.1483$$

Check

$$\lambda = 1 + 12(\psi - 0.5)^2 + 10.2(n-1)\rho(0.25 - \psi)^2 = 1.1483$$

3.4 Cantilever beam; with length (L= 4m). Concrete compressive strength (f_c'= 35 MPa), steel yielding strength (f_y = 420 MPa), applied load w = 20 KN/m, $\rho = \rho_b$, cross section width (b = 250 mm).

<u>Section without Reinforcements:</u>

minimum depth according to ACI-Code

$$h_{\min} = \frac{L}{8} = 500\,mm$$

$$I_g = \frac{bh^3}{12} = 260416666.7\ mm^4$$

$$E_c = 4730\sqrt{35} = 27983.06\,MPa;\ n = 7.1472$$

$$\delta_{\max} = \frac{wL^4}{8EI} = 8.7825\,mm\ ;\ \frac{\delta_{\max}}{L} = 0.0022\ <\ \frac{1}{360}$$

<u>Section with Reinforcements:</u>

$$\rho = \rho_k\ ,\ \beta_1 = 0.8$$

$$\rho_b = 0.85\beta_1 \frac{f_c'}{f_y}\frac{0.003}{0.003 + \frac{f_y}{E_s}} = 0.03353;\ \rho = \rho_b = 0.03353$$

$$(n-1)\rho = 0.206116$$

From table (6); $h_{\min} = \dfrac{L}{8.547} = 468\ mm$

$$\alpha = \frac{468}{500} = 0.936$$

$$d = 0.25h = 397.8$$

Check;

$\alpha = 0.9967 - 0.0604(1) + 0.013(\dfrac{35}{420}) = 0.9374$ which is very close to that obtained above.

$$\bar{y} = 258.42 = 0.552h$$

Check; $\psi = \dfrac{0.5 + 0.7225(n-1)\rho}{1 + 0.25(n-1)\rho} = 0.552$

$$I_0 = \frac{bh^3}{12} = 2135484000\ mm^4$$

$I_g = 2603469471\ mm^4$ which is very close to the original section

$$I_g = 0.9997\,I_g\ \ (\rho = 0)$$

$$\lambda = \frac{I_g}{I_0} = 1.219147$$

Check

$$\lambda = 1 + 12(\psi - 0.5)^2 + 10.2(n-1)\rho(0.25 - \psi)^2 = 1.219148$$

4. CONCLUSION

1. A modification of the ACI code depth-span ratio is suggested in this study to include the effect of reinforcement area in addition to the correction for concrete type and yielding strength of steel bars.
2. The correction factors (α and β) are determined in term the reinforcement indices ratio (ρ / ρ_b) and the material strengths ratio (f_c'/f_y).
3. The correction factor (α) decreased with increasing the value of the reinforcement indices ratio (ρ / ρ_b) for all values of (f_c'/f_y).
4. The correction factor (β) increased with increasing the value of the reinforcement indices ratio (ρ / ρ_b) for all values of (f_c'/f_y).

5. Suitable equations are proposed to predict (α *and* β) in term of the variables (ρ / ρ_b) and $(f_c^{'} / f_y)$, and the calculated results showed excellent correlation.
6. Numerical examples are solved for simply supported, fixed ended, propped and cantilever beams to verify the proposed equation, the results shown very close results between the original section without the reinforcement and modified section with specified reinforcement.

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