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Application of Homotopy Analysis Method for Investigating Nonlinear Oscillations

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ABSTRACT

Homotopy Analysis Method (HAM) is a well organized method to get the periodic solution of Non-Linear Oscillatory Duffing Equation of Motion. By having a glance on Helmholtz Equation of Motion an attempt has been made to explore the proficiency of HAM. This research article explores on Helmholtz Equation of Motion possessing non-odd restoring force function. Moreover in this equation the behavior of oscillations is different for the same magnitude of +ve and –ve amplitudes. This phenomenon concerns with asymmetric oscillations which are nonlinear. In the case of large amplitude vibrations a greater amount of inconsistency has been noticed here. Furthermore the incapability of HAM in differentiating the non-periodic solution of Motion has been extensively discussed.

Key words : Homotopy Analysis Method (HAM); Duffing Equation; Helmholtz's equation of Motion (HEM) ; Periodic solution; Phase-plane diagram, Differential Equation (DE)
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1. INTRODUCTION

A larger number of problems in Engineering Sciences are modeled by Non-Linear Duffing Equation of Motion namely

$$\frac{d^2x}{dt^2} + g(x) = G(t) \tag{1}$$

Here restoring force function which is a polynomial is given by

$$g(x) = ax + bx^3 + cx^5 + ... + dx^m \tag{2}$$

G (t) =Forcing Function (Periodic)
Here stiff constant is a.
b, c,..d are parameters.
$m \in \{1,3,5,...\}$.

(2) is an odd function so that one can expect oscillations which are symmetric. To the equation of motion or a harmonically forced undamped single degree of freedom oscillator, Helmholtz added the nonlinearity. The behavior of the eardrum is like an asymmetric oscillation with restoring function

$$g(x) = a_0 x + a_1 x^2 \tag{3}$$

The Helmholtz equation of motion is

$$n\frac{d^2x}{dt^2} + a_0 x + a_1 x^2 \tag{4}$$

Khatami et al. [1] presented general solutions for Duffing oscillations which are not linear with 5th,6th and 7th degree polynomial odd restoring force functions. They incorporate DTM and received fruitful results. In the case of an odd degree polynomial restoring function with symmetric oscillations of the system this method can give good results. But in the case of non-odd restoring force functions with asymmetric oscillations uncertainties are observed. In this scenario a large number of researchers made an attempt by adopting HPM in order to crack many DEqs. The primary goal of this talk is to test the acceptability HPM for Duffing Oscillators with non-odd restoring force function

2. HOMOTOPY ANALYSIS METHOD

HEM is

$$\frac{d^2x}{dt^2} + x + \nu x^2 = 0 \qquad (5)$$

$$x = B, \frac{dx}{dt} = 0 \quad \text{at } t = 0 \qquad (6)$$

ν = nonlinearity of x.

B= Amplitude of x.

In the light of Liao [10], the HAM is applied to the NLDE (5) and a Homotopy namely d $\Omega \times [0,1] \to R$ is introduced and follows

$$q[N(x)+L(x_0)]-[L(x_0)-L(x)]=0 \qquad (7)$$

Here $L(x) = \frac{d^2x}{dt^2} + x$ and $N(x) = \nu x^2$

(7) is approximated with

$$x = x_0 + qx_1 + q^2x_2 + ... \qquad (8)$$

And $w = \lim_{q \to 1} x = x_0 + x_1 + x_2 + ...$ (9)

Put (8) in (7) and by making comparison we can have

$$L(x_1) + L(x_0) + \nu x_0^{\ 2} = 0 \qquad (10)$$

$$L(x_2) + 2\nu x_0 x_1 = 0 \qquad (11)$$

$$L(x_3) + 2\nu(x_1^{\ 2} + x_0x_2) = 0 \qquad (12)$$

Let the beginning approximation (5) be

$$x_0(t) = B(COS(\alpha t)) \qquad (13)$$

$$L(x_0) = B(COS\,\alpha t)(1-\alpha^2) \qquad (14)$$

$\alpha(\nu)$ is fixed and not equal to 0 and its value at 0is 1.

(10) is simplified as

$$\frac{d^2x_1}{dt^2} + x_1 + (1-\alpha^2)B(COS\alpha t) + 0.5B^2\nu + 0.5B^2\nu COS2\alpha t = 0 \qquad (15)$$

The solution of (15), gives

$$x_1(t) = (B + 0.5B^2\nu + 0.5B^2\nu(1-4\alpha^2)^{-1})COSt - BCOS\alpha t - 0.5B^2\nu - 0.5B^2\nu(1-4\alpha^2)^{-1}COS2\alpha t \quad (16)$$

The secular term in (16) is eliminated by setting

$$B + 0.5B^2\nu + 0.5B^2\nu(1-4\alpha^2)^{-1} = 0$$

$$\Rightarrow \alpha = 0.5(SQRT(1+B\nu)/(1+0.5B\nu)) \qquad (17)$$

α *is* 0 *gives* $1 + B\nu = 0 \ \Rightarrow B = -(\nu)^{-1}$

First Order Approximation to (9) is given by

$$x(t) = x_0(t) + qx_1(t)$$

Now

$$x(t) = BCOS\alpha t + q[-BCOS\alpha t - 0.5B^2\nu - 0.5B^2\nu(1-4\alpha^2)^{-1}COS2\alpha t] \quad 18)$$

At $q = 1$

$$x(t) = -0.5B^2\nu + (B + 0.5B^2\nu)COS\,2\alpha t \qquad (19)$$

For B=1 & $\nu = 0.1$ (19) is

$$x(t) = (1.05)COS\,(1.02353t) - 0.05 \qquad (20)$$

(19) gives

$$\frac{dx}{dt} = (((1+B\nu)/(1+0.5B\nu))(B-x)(B+B^2\nu+x))^{\frac{1}{2}}$$

numerically. (21)

This is used in comparing the results with actual solution.

3. PHASE DIAGRAM GENERATION

To create PHASE DIAGRAM, one can see the relation as $\left(\frac{dx}{dt}\right)^2 = \left[(B+x) + \frac{2\nu}{30}(B^2+Bx+x^2)\right](B-x)$

by(5) (22)

The PHASE DIAGRAM of DE (5) with boundary condition (6) is represented by the plot of $\frac{dx}{dt}$ Vs x. For B=1 and $\nu = 0.1$ the plot of $\frac{dx}{dt}$ versus x created from (21) depicts a boundary which is closed. From (22) the magnitude of +ve amplitude of Non-Linear Oscillations is 1. From (22) the magnitude of –ve amplitude of Non-Linear Oscillations is -1.0717. This gives asymmetry of the PHASE DIAGRAM w.r.t $\frac{dx}{dt}$ axis, whereas it is symmetric w.r.t x-axis. (1) describes the PHASE DIAGRAM created from (22) and it shows magnitudes of +ve and -ve amplitudes which are unequal.

At B=1 and $\nu = 0.1$ the singularities are origin and (-10,0). Origin is the singular point. (-10, 0) is the saddle point. In

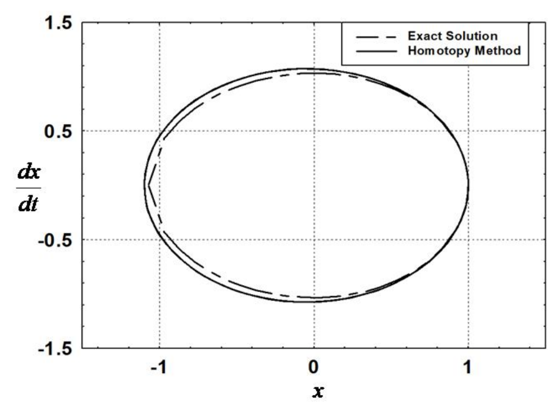


Figure 1. Helmholtz equation Vs exact solution

In order to get the periodic solution the range of amplitudes should lie in the interval (-10, 5).The periodic solution is impossible Out-side of this range.

In Fig.2, 3, 4, 5; the PHASE DIAGRMS are created for $B \in \{2,4,5,6\}$ and $\nu = \frac{1}{10}$. More over these are being compared with HOMOTOPY METHOD's solution. PHASE DIAGRAM concerning with x(0)=5 stands for the separatrix.

In the case of x(0)=1 It stands for the boundary which is closed and possessing periodicity. PHASE DIAGRAM created for x (0) =5 by using (21) is possessing close magnitudes of +ve & –ve amplitudes. Here the separatrix stands for the big amount of difference in the magnitudes of +ve & -ve amplitudes. The HOMOTOPY solution stands exactly close to the domain at the particular boundary constraints. Fig.6 & 7 depict distinct PHASE DIAGRAMS from (21) and (22). HOMOTOPY METHOD is incapable to differentiate non-periodic &periodic behavior of the solutions.

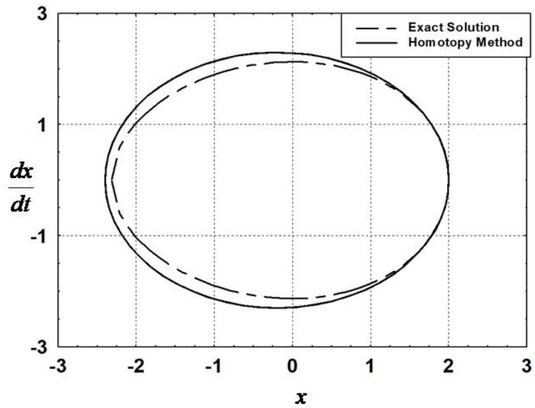
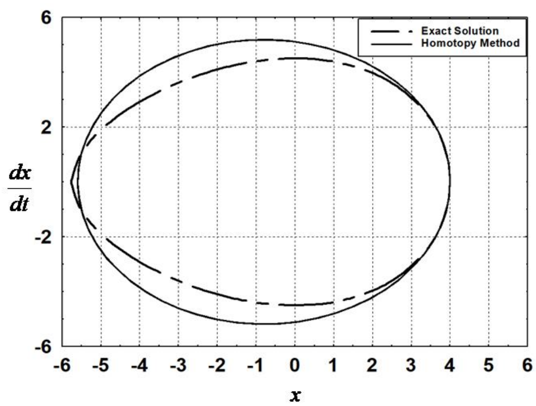


Figure 2. Comparing the PHASE DIAGRAMS for x(0) =2



Fiure 3. Comparing the PHASE DIAGRAMS for x (0) = 4.

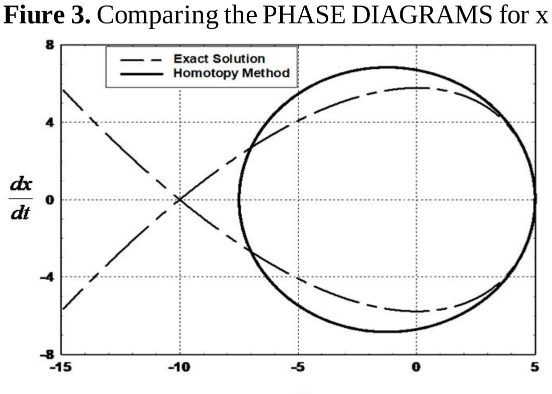


Figure 4. comparing the PHASE DIAGRAMS generated for x (0) =5

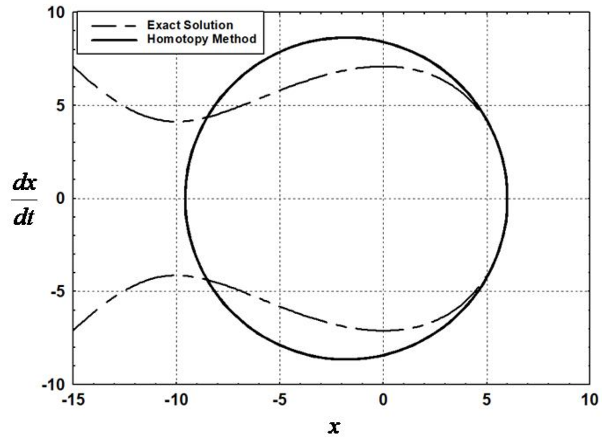


Figure 5. Comparing the PHASE DIAGRAMS for x (0) =6

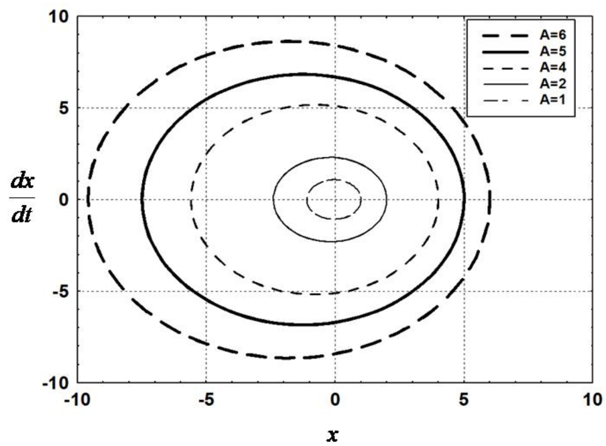


Fig. 6: PHASE DIAGRAMS generated for different amplitudes obtained by the Homotopy analysis method

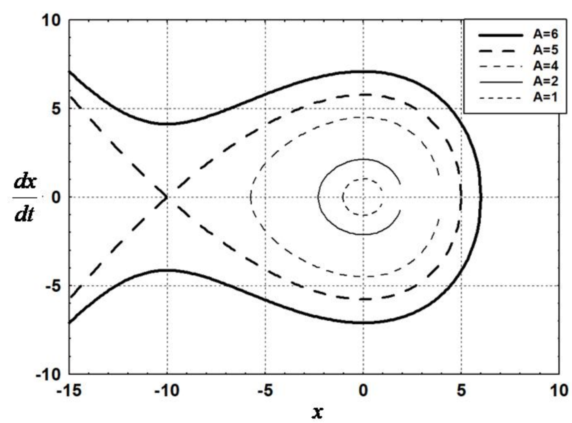


Fig.7: PHASE DIAGRAMS created out of exact solution for various amplitudes

4. CONCLUSION

The capability of the Homotopy Analysis Method is investigated by observing the nonlinear oscillations of Helmholtz Equation of Motion. Large discrepancy is observed in magnitudes of positive and negative amplitudes using the Homotopy Analysis Method in case of large amplitudes. Therefore Homotopy Analysis Method is incapable to differentiate non-periodic and periodic solutions. In the case of B exceeding 5 and $\nu = 0.1$, phase plane diagram obtaining by exact solution is not closed and depicts non-periodic nature, whereas the Homotopy Analysis Method gives the closed phase plane diagram (i.e., periodic) for all values of B. Hence, Homotopy Analysis Method is incapable to differentiate the non-periodic solutions.

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