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Response of an Undamped Forced Oscillator via Rohit Transform

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ABSTRACT

In this paper, the response of an undamped forced mechanical oscillator, as well as an undamped forced electrical oscillator, is obtained by the integral transform Rohit transform. This paper put forward a new technique for obtaining the response of an undamped forced mechanical oscillator as well as an undamped forced electrical oscillator and reveals that the Rohit transform is an effective technique for obtaining the response of an undamped forced mechanical oscillator as well as an undamped forced electrical oscillator.

Key words: Response, Rohit Transform, undamped forced mechanical oscillator, electrical oscillator.

1. INTRODUCTION

The Rohit transform is a new integral transform that has been proposed by the author Rohit Gupta in recent years [1]. It has been applied to solve many initial value problems in science and engineering such as solving the Schrodinger equation for a quantum mechanical particle [2], for the analysis of RLC circuits with exponential excitation sources [3], for the analysis of one-way streamline flow between parallel plates [4], for the analysis of basic series inverter [5], for the analysis of electric network circuits with sinusoidal potential [6], for the analysis of uniform infinite fin [7], for obtaining the response of a permanent magnet moving coil instrument [8], determining the heat conducted through the fins of varying cross-sections [9], for the analysis of damped mechanical and electrical oscillators [10]. This paper put forward the Rohit transform for obtaining the response of an undamped forced mechanical oscillator as well as an undamped forced electrical oscillator. There is no. of techniques like Convolution theorem approach [11]-[13], Laplace transform [14]-[16], Mohand transform [17], [18], Matrix method [19], [20], Residue theorem approach [21], Gupta transform [22]-[26], Abaoub-Shkheam transform [29], Shehu transform [30], SEE transform [31], [32], Aboodh transform [33], [34], Elzaki transform [35] etc. for solving the governing differential equations with boundary conditions. This paper proves the applicability of the Rohit transform for obtaining the response of an undamped forced mechanical oscillator as well as an undamped forced electrical oscillator. It shows that the Rohit transform is an effective tool for obtaining the response of an undamped forced mechanical oscillator as well as an undamped forced electrical oscillator. The Rohit transform of a function g(t) [1]-[10] is defined as

$R\{g(t)\} = q^3 \int_0^\infty e^{-qt} g(t)dt$, provided that the integral is convergent, where q may be a real or complex parameter.

The RT of some basic functions is given by

- $R\{t^n\} = \frac{n!}{q^{n-2}}$,
  $where\ n\ is\ zero\ or\ a\ positive\ integer.$
- $R\{e^{bt}\} = \frac{q^3}{q-b}$,
- $R\{sinbt\} = \frac{b\,q^3}{q^2+b^2}$,
- $R\{cosbt\} = \frac{q^4}{q^2+b^2}$,

The Rohit transform of some derivatives of a function g(t) is

$R\{g'(t)\} = qR\{g(t)\} - q^3 g(0)$,
$R\{g''(t)\} = q^2 R\{g(t)\} - q^4 g(0) - q^3 g'(0)$,
and so on.

2. MATERIAL AND METHOD

A. Response Of An Undamped Forced Mechanical Oscillator

The differential equation of the forced mechanical oscillator [12]-[16] is given by

$m\ddot{x}(t) + b\dot{x}(t) + kx(t) = F\cos\omega t \ldots\ldots..(1)$

For an undamped forced mechanical oscillator, damping constant i.e. b = 0, therefore, equation (1) becomes

$$m\ddot{x}(t) + kx(t) = F\cos\omega t$$

Or

$$\ddot{x}(t) + \omega_0{}^2x(t) = \frac{F}{m}\cos\omega t \;\; \ldots.(2)$$

$$where\ \omega_0 = \sqrt{\frac{k}{m}}$$

The initial boundary conditions are as follows [18], [19], [24]:

(i) x (0) = 0.

(ii) $x(0) = 0.$

The Rohit Transform of (2) provides

$$q^2\overline{x}(q) - q^4x(0) - q^3\dot{x}(0) + \omega_0{}^2\overline{x}(q) = \frac{F}{m}\frac{q^4}{q^2+\omega^2} \;\; \ldots.(3)$$

Here $\overline{x}(q)$ denotes the Rohit Transform of $x(t)$.

Put $x(0) = 0$ and $\dot{x}(0) = 0$ and simplifying (3), we get

$$q^2\overline{x}(q) + \omega_0{}^2\overline{x}(q) = \frac{F}{m}\frac{q^4}{q^2+\omega^2}$$

$$\overline{x}(q) = \frac{F}{m}\frac{q^4}{(q^2+\omega^2)(q^2+\omega_0{}^2)}$$

$$\overline{x}(q) = \frac{F}{m}\{\frac{q^4}{(-\omega_0{}^2+\omega^2)(q^2+\omega^2)} + \frac{q^4}{(-\omega^2+\omega_0{}^2)(q^2+\omega_0{}^2)}\}$$

Taking inverse Rohit transform, we have

$$x(t) = \frac{F}{m}\{\frac{1}{(-\omega_0{}^2+\omega^2)}\cos\omega t + \frac{1}{(-\omega^2+\omega_0{}^2)}\cos\omega_0 t\}$$

$$x(t) = \frac{F}{m}\{\frac{1}{(\omega^2-\omega_0{}^2)}\cos\omega t - \frac{1}{(\omega^2-\omega_0{}^2)}\cos\omega_0 t\}$$

$$x(t) = \frac{F}{m}\frac{1}{(\omega^2-\omega_0{}^2)}(\cos\omega t - \cos\omega_0 t)$$

$$x(t) = \frac{2F}{m}\frac{1}{(\omega_0{}^2-\omega^2)}\sin\frac{(\omega_0-\omega)t}{2}\sin\frac{(\omega_0+\omega)t}{2} \;\; \ldots.(4)$$

If ω is slightly greater than ω_0, then the above expression is thought to be the product of two terms: $\frac{2F}{m}\frac{1}{(\omega_0{}^2-\omega^2)}\sin\frac{(\omega_0-\omega)t}{2}$ and $\sin\frac{(\omega_0+\omega)t}{2}$.

Since ω is slightly greater than ω_0, therefore the magnitude of their difference is small, the term $\sin\frac{(\omega_0+\omega)t}{2}$ has a much higher frequency than the term $\frac{2F}{m}\frac{1}{(\omega_0{}^2-\omega^2)}\sin\frac{(\omega_0-\omega)t}{2}$. In such a case, the solution provided in equation (4) represents an oscillation of high frequency with an amplitude that is modulated by an oscillation of low frequency.

B. Response Of An Undamped Forced Electrical Oscillator

The differential equation of the forced electrical oscillator [12]-[16] is given by

$$L\ddot{Q}(t) + R\dot{Q}(t) + \frac{1}{C}Q(t) = V\cos\omega t$$

Or

$$\ddot{Q}(t) + \frac{R}{L}\dot{Q}(t) + \omega_0{}^2Q(t) = \frac{V}{L}\cos\omega t \;\; ..(5)$$

For an *undamped forced electrical oscillator, resistance R = 0,* *therefore, equation (5) becomes*

$$\ddot{Q}(t) + \omega_0{}^2Q(t) = \frac{V}{L}\cos\omega t \;\; \ldots.(6)$$

$where\ \omega_0 = \sqrt{\frac{1}{LC}}\ and$ $Q(t)$ is the instantaneous charge.

The initial boundary conditions are as follows [20]--[22]:

(i) Q (0) = 0.

(ii) $\dot{Q}(0) = 0.$

The Rohit Transform of equation (6) provides

$$q^2\overline{Q}(q) - q^3Q(0) - q^2\dot{Q}(0) + \omega_0{}^2\overline{Q}(q) = \frac{V}{L}\frac{q^4}{q^2+\omega^2} \ldots.(7)$$

Here $\overline{Q}(q)$ denotes the Rohit transform of $Q(t)$.

Put $Q(0) = 0$ and $\dot{Q}(0) = 0$, and simplifying (7), we have

$$q^2\overline{Q}(q) + \omega_0{}^2\overline{Q}(q) = \frac{V}{L}\frac{q^4}{q^2+\omega^2}$$

$$\overline{Q}(q) = \frac{V}{L}\frac{q^4}{(q^2+\omega^2)(q^2+\omega_0{}^2)}$$

$$\overline{Q}(q) = \frac{V}{L}\{\frac{q^4}{(-\omega_0{}^2+\omega^2)(q^2+\omega^2)} + \frac{q^4}{(-\omega^2+\omega_0{}^2)(q^2+\omega_0{}^2)}\}$$

Taking inverse Rohit transform, we have

$$Q(t) = \frac{V}{L}\{\frac{1}{(-\omega_0{}^2+\omega^2)}\cos\omega t + \frac{1}{(-\omega^2+\omega_0{}^2)}\cos\omega_0 t\}$$

$$Q(\mathrm{t}) = \frac{V}{L}\{\frac{1}{(\omega^2 - {\omega_0}^2)}\cos\omega t - \frac{1}{(\omega^2 - {\omega_0}^2)}\cos\omega_0 t\}$$

$$Q(\mathrm{t}) = \frac{V}{L}\frac{1}{(\omega^2 - {\omega_0}^2)}(\cos\omega t - \cos\omega_0 t)$$

$$Q(\mathrm{t}) = \frac{2V}{L}\frac{1}{({\omega_0}^2 - \omega^2)}\sin\frac{(\omega_0 - \omega)t}{2}\sin\frac{(\omega_0 + \omega)t}{2} \quad \ldots.(8)$$

If ω is slightly greater than ω_0, then the above expression is thought to be the product of two terms: $\frac{2V}{L}\frac{1}{({\omega_0}^2-\omega^2)}\sin\frac{(\omega_0-\omega)t}{2}$ and $\sin\frac{(\omega_0+\omega)t}{2}$.
Since ω is slightly greater than ω_0, therefore the magnitude of their difference is small, the term $\sin\frac{(\omega_0+\omega)t}{2}$ has a much higher frequency than the term $\frac{2V}{L}\frac{1}{({\omega_0}^2-\omega^2)}\sin\frac{(\omega_0-\omega)t}{2}$. In such a case, the solution provided in equation (8) represents an oscillation of high frequency with an amplitude that is modulated by an oscillation of low frequency.

3. CONCLUSION

In this paper, the response of an undamped forced mechanical oscillator, as well as an undamped forced electrical oscillator, has been successfully obtained by the integral transform Rohit transform. This paper exemplified the Rohit transform for obtaining the response of an undamped forced mechanical oscillator as well as an undamped forced electrical oscillator. A new method is exploited for obtaining the response of an undamped forced mechanical oscillator as well as an undamped forced electrical oscillator. In an undamped forced mechanical oscillator, if ω is slightly greater than ω_0, then the magnitude of their difference is small. The term $\sin\frac{(\omega_0+\omega)\mathrm{t}}{2}$ has a much higher frequency than the term $\frac{2\mathrm{F}}{\mathrm{m}}\frac{1}{({\omega_0}^2-\omega^2)}\sin\frac{(\omega_0-\omega)\mathrm{t}}{2}$. In such a case, the solution of the differential equation of undamped forced mechanical oscillator represents an oscillation of high frequency with an amplitude that is modulated by an oscillation of low frequency. In an undamped forced electrical oscillator, if ω is slightly greater than ω_0, then the magnitude of their difference is small. The term $\sin\frac{(\omega_0+\omega)\mathrm{t}}{2}$ has a much higher frequency than the term $\frac{2\mathrm{V}}{\mathrm{L}}\frac{1}{({\omega_0}^2-\omega^2)}\sin\frac{(\omega_0-\omega)\mathrm{t}}{2}$. In such a case, the solution of the differential equation of undamped forced electrical oscillator represents an oscillation of high frequency with an amplitude that is modulated by an oscillation of low frequency.

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