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The Bayes Rule of Decision Making in Joint Optimization of Search and Detection of Objects in Technical Systems

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ABSTRACT

The main results of solving the problems of search and detecting objects in technical systems for various purposes are analyzed. The differential characteristics of the Bayes criterion of minimum average risk are introduced. The term of the current survey area was introduced The Bayes rule of decision making is clarified when joint optimization of search and detection of objects in the current survey area.

Key words : Technical system, search and detection of objects, Bayes rule of decision making, criterion of average risk.

1. INTRODUCTION

Today, in the development of technical systems (for example, radar [1]–[5], optoelectronic [4]–[11], laser [12]–[13], and others), the main issues are the joint optimization of the stages of search and detection of objects. A number of significant scientific results were obtained in optimizing the search and detection of objects. However, existing optimization methods consider search as a single task to review space, process signals and make decisions only in the production plan.

Solutions are obtained only for individual components of the task. The solution to the problem as a whole has not been received. A unified approach to the selection of an efficiency criterion that adequately reflects the tasks of a technical system at the stage of searching and detecting objects of interest has not been formulated.

1.1 Problem analysis

To find out the reasons for such difficulties in solving the problem of joint optimization of the search and detection of objects, we analyze the classical approach to solving the detection problem from the standpoint of the theory of statistical solutions [5], [7], [9], [14]–[20].

In the theory of statistical decisions in the presence of a complete set of a priori data, the criterion of average risk is used – the average value of the decision-making fee when testing statistical hypotheses [5], [14]. At the same time, the main characteristics of the average risk and its constituent elements are integral characteristics. Using these characteristics, we can get some indicators of the quality of the search and detection of an object in a given predetermined survey area as a whole.

Obviously, an infinite number of search strategies will satisfy the same integral quality data. This makes it difficult to find optimal decision rules for the case of joint optimization of procedures such as search and detection of an object.

The task of jointly optimizing search and detection of objects is the ultimate task of detection. Therefore, it is advisable to choose optimization criteria that characterize the efficiency of object detection as the final criteria for joint optimization of the search and detection of objects [5], [14]–[20].

The decision to detect an object is based on the analysis of signals received from the object. In this regard, we will search for an algorithm for joint search and detection optimization in the class of optimal Bayes decision-making algorithms. Using this algorithm, the minimum value (lower limit) of the average risk is reached [5], [10], [14], [17]–[20].

In [5], [10], [14], [17]–[20], the case of adopting a simple hypothesis against a simple alternative was considered and an expression was obtained for the average risk value. It was found that the main characteristics of the average risk and its constituent elements are such characteristics as:

p_j – a priori probabilities of hypotheses about the absence of an object H_0 and its presence H_1;

$W(y/H_j)$ – the likelihood function of the sample Y, provided that the hypothesis is true H_j;

$P(\gamma_i / H_j)$ – conditional probability of decision making γ_i in the area of survey Ω, provided that the hypothesis is true $H_j, i, j = 0,1$.

The indicated characteristics relate either to the area of survey as a whole or to its individual components and is their integral characteristics [14], [17]–[18]. With the help of integral characteristics, it is possible to obtain data and rules for detecting an object in a certain rear survey area Ω as a whole.

Moreover, the optimization parameters are the detection parameters, which are determined by conditional probabilities $P(\gamma_i / H_j)$. This is sufficient to solve the detection problem. The search parameters of the object are absent or present in the form of integral data p_j.

Obviously, an infinite number of search strategies will satisfy the same integral data of quality. It is also difficult to find the optimal decision rules both for the case of search-only optimization and for the case of joint search and detection optimization.

Integrated object detection features not only do not take into account the features of the object search, but also poorly comply with the conditions of object search and detection. The basic conditions of detection in technical systems for various purposes are systems [1], [5], [7], [10], [16], [18]:

large space scope of the survey areas;

uneven distribution of a priori objects in the survey area and etc..

Given these search and detection conditions of objects, we can conclude that the average risk within the survey area cannot be assumed uniform. In this regard, it is advisable to switch from integrated medium risk characteristics to medium risk differential characteristics.

2. MAIN MATERIAL

We introduce the differential characteristics of the average risk criterion. These characteristics will allow to take into account the features of the Bayes decision for each point and a separate section of the search and detection of objects. Differential characteristics will be considered in the following expression:

$u(x)$ – a priori density distribution of the object' location in a given survey area Ω by space coordinates x;

$dp_1(x) = u(x)dx$ – a priori probability of the presence of an object in the unit cell dx of the area of view Ω;

$dp_0(x) = \tilde{u}(x)dx$ – a priori probability of the absence of an object in the unit cell dx of the area of view Ω;

$\tilde{u}(x)$ – a priori probability density of the absence of an object in a given survey area Ω in space coordinates x;

$dR(x) = \dot{R}(x)dx$ – average risk when deciding on the presence or absence of an object in a unit cell dx;

$\dot{R}(x)$ – average risk density in the area of survey;

$P(\gamma_i / H_j, x)$ – conditional probability of making a decision γ_i provided that the hypothesis H_j in the unit cell dx of the area of survey Ω, $i, j = 0,1$ is true. The elements of the loss matrix are left unchanged.

Given the notation introduced, the differential value of the average risk $dR(x)$ for the two-alternative case can be calculated as:

$$dR(x) = d(p_0(x)r_0(x)) + d(p_1(x)r_1(x)), \tag{1}$$

where

$$d(p_0(x)r_0(x)) = I_{00}P(\gamma_0 / H_0, x)dp_0(x) + \\ + I_{01}P(\gamma_1 / H_0, x)dp_0(x),$$

$$d(p_1(x)r_1(x)) = I_{10}P(\gamma_0 / H_1, x)dp_1(x) + \\ + I_{11}P(\gamma_1 / H_1, x)dp_1(x) -$$

conditional risks in the unit cell dx of the area of survey Ω, which correspond to the hypothesis H_0 of the absence of the object and the alternative H_1 of the presence of the object;

γ_0 – the decision to accept the hypothesis H_0 in a unit cell dx;

γ_1 – decision to accept the hypothesis H_1 in a unit cell dx;

$P(\gamma_i, H_j, x)$ – conditional probability of decision making γ_i in a unit cell dx, provided that the hypothesis H_j, $i, j = 0,1$ is true;

$P(\gamma_1, H_0, x)$ – conditional probability to reject the correct hypothesis H_0 in the unit cell dx of the area of view Ω, the probability of an error of the first kind (significance level);

$P(\gamma_0, H_1, x)$ – conditional probability to reject the correct hypothesis H_1 in the unit cell dx of the survey area Ω, the probability of error of the second kind.

After simple transformations, expression (1) is rewritten in the expression (2):

$$dR(x) = I_{00}dp_0(x) + I_{10}dp_1(x) - ((I_{10} - I_{11}) \\ P(\gamma_1 / H_1, x)dp_1(x) - (I_{01} - I_{00})P(\gamma_1 / H_0, x)dp_0(x)). \quad (2)$$

Denoting $dR_0(x) = I_{00}dp_0(x) + I_{10}dp_1(x)$ and considering that $dR_0(x)$ is a non-negative constant in the unit cell dx, the Bayes algorithm for testing the simple hypothesis H_0 against the simple alternative H_1 in the unit cell dx of the survey area Ω is written as follows (3):

$$(I_{10} - I_{11})P(\gamma_1 / H_1, x)dp_1(x) - (I_{01} - I_{00}) \\ P(\gamma_1 / H_0, x)dp_0(x) \underset{\gamma_0}{\overset{\gamma_1}{\gtrless}} 0 \quad (3)$$

or

$$dl(x) = \frac{P(\gamma_1 / H_1, x)dp_1(x)}{P(\gamma_1 / H_0, x)dp_0(x)} \underset{\gamma_0}{\overset{\gamma_1}{\gtrless}} \frac{I_{01} - I_{00}}{I_{10} - I_{11}}, \quad (4)$$

where $dl(x)$ – unconditional likelihood ratio in elementary cell
dx of the survey area Ω.

The unconditional likelihood ratio refers to the ratio of the unconditional probability of the correct detection of the object $P(\gamma_1 / H_1, x)dp_1(x)$ to the unconditional probability of false alarm $P(\gamma_1 / H_0, x)dp_0(x)$.

Thus, the Bayes algorithm (4) for testing a simple hypothesis versus a simple alternative in the unit cell dx of the survey area Ω consists in comparing the likelihood ratio $dl(x)$ with the threshold:

$$c_b = \frac{I_{01} - I_{00}}{I_{10} - I_{11}}, \quad (5)$$

moreover, if $dl(x) \geq c_b$, then the solution γ_1 is made (the hypothesis H_0 is rejected), if $dl(x) < c_b$, then the solution γ_0 is made (the hypothesis H_0 is accepted). Minimizing average risk now reduces to maximizing the unconditional likelihood ratio.

It can be seen that the results obtained (1)–(5) do not contradict the general classical theory, which was adopted when solving the problem of testing a simple hypothesis against a simple alternative. The expression for medium risk and the rule for testing hypotheses in the unit cell dx of the survey area Ω also agree well.

We take into account that such introduced differential characteristics cannot be directly applied in practice. They suggest the calculation of the average risk and the unconditional likelihood ratio for each unit cell dx, which is practically impossible. In addition, it remains unknown which algorithm to use when solving the problem of survey elementary $\Omega(t)$, provided that $\Omega(t) \to \Omega$ for $t \to T$, where T is the time of survey for a given area Ω.

We pose the problem of finding the optimal Bayes decision-making algorithm in the current area of survey $\Omega(t)$ taking into account the introduced differential characteristics. With such a statement of the problem, an additional optimization parameter appears: the current size and position of the area $\Omega(t)$ in the general of the survey area Ω. Therefore, conditions are created for finding the optimal search strategy for the minimum average risk by the Bayes criterion.

The average risk can now be found as (6):

$$R(t) = \int_{\Omega(t)} dR(x) = \int_{\Omega(t)} \overset{\square}{R}(x)dx. \quad (6)$$

Substituting (2) into expression (6) after a series of transformations, we have:

$$R(t) = I_{00}\int_{\Omega(t)} dp_0(x) + I_{10}\int_{\Omega(t)} dp_1(x) - ((I_{10} - I_{11}) \\ \int_{\Omega(t)} P(\gamma_1 / H_1, x)dp_1(x) - (I_{01} - I_{00})\int_{\Omega(t)} P(\gamma_1 / H_0, x)dp_0(x). \quad (7)$$

We assume that $R_0(t) = I_{00}\int_{\Omega(t)} dp_0(x) + I_{10}\int_{\Omega(t)} dp_1(x)$ is a non-negative constant for the current area of survey $\Omega(t)$ at time t. The Bayes algorithm for testing the simple hypothesis H_0 against the simple alternative H_1 in the current area $\Omega(t)$ of the survey Ω is written as follows:

$$\frac{\int_{\Omega(t)} P(\gamma_1 / H_1, x)dp_1(x)}{\int_{\Omega(t)} P(\gamma_1 / H_0, x)dp_0(x)} = \\ = \frac{\int_{\Omega(t)} P(\gamma_1 / H_1, x)u(x)dx}{\int_{\Omega(t)} P(\gamma_1 / H_0, x)\tilde{u}(x)dx} = \\ = \frac{P_1(\gamma_1, t)}{P_0(\gamma_1, t)} \underset{\gamma_0}{\overset{\gamma_1}{\gtrless}} \frac{I_{01} - I_{00}}{I_{10} - I_{11}}, \quad (8)$$

where $P_1(\gamma_1,t)$ is current value of the unconditional probability of the correct detection of an object in the area $\Omega(t)$;

$P_0(\gamma_1,t)$ – current value of the unconditional probability of false alarm in the area $\Omega(t)$.

Passing to the unconditional likelihood ratio $l(t)=\dfrac{P_1(\gamma_1,t)}{P_0(\gamma_1,t)}$, we write expression (8) in the expression (9):

$$l(t) \underset{\gamma_0}{\overset{\gamma_1}{\gtrless}} \frac{I_{01}-I_{00}}{I_{10}-I_{11}}. \qquad (9)$$

Thus, the optimal Bayes algorithm (9) obtained on the basis of expressions (6)-(8) for testing a simple hypothesis against a simple alternative consists in maximizing the likelihood ratio $l(t)$ in the current area $\Omega(t)$ and comparing it with the threshold:

$$c_b - \frac{I_{01}-I_{00}}{I_{10}-I_{11}}, \qquad (10)$$

moreover, if $l(t) \geq c_b$, then the solution γ_1 is made (hypothesis H_0 is rejected), if $l(t) < c_b$, then the solution γ_0 is made (hypothesis H_0 is accepted).

In accordance with (8), optimization should be performed according to the parameters of the conditional probability of the correct detection of $P(\gamma_1/H_1,x)$ and the parameters of the current survey area $\Omega(t)$.

Consider an important special case. We assume that, similarly to the Neyman-Pearson criterion, the value of the unconditional probability of false alarm $P_0(\gamma_1,t)$ is fixed at a constant level. Then, according to (8), finding the maximum of the unconditional likelihood ratio reduces to finding the maximum of the unconditional probability of the correct object detection $P_1(\gamma_1,t)\int\limits_{\Omega(t)} P_1(\gamma_1/H_1,x)u(x)dx$.

Thus, in order to find the optimal Bayes decision-making algorithm in the current area $\Omega(t)$ of the Ω survey area, the problem of finding the optimal search strategy for the minimum average risk by the Bayes criterion should also be solved. The search strategy $\lambda(x,t)$ is a rule that at any time t establishes in which zone of area the search $\Omega(t)$ should be performed and with what energy costs.

For further research, we introduce the main restrictions on the search strategy, usually used in search theory. We require that the search strategy be T-truncated, that is, $\lambda(x,t)=0$ for $t>T$ and $x\in\Omega$. In other words, the condition for mandatory viewing of the survey area Ω during the search for T must be satisfied.

Obviously,

$$\lambda(x,t) > 0, \text{ for } x \in \Omega/\Omega(t); \qquad (11a)$$

$$\lambda(x,t) = 0, \text{ for } x \in \Omega/\Omega(t). \qquad (11b)$$

We assume that the search strategy should be constant for all coordinates surveyed at a fixed moment of time t. Moreover, the measure of the current survey area $\Omega(t)$ should be a non-decreasing function of time t, since the search strategy extends throughout the entire search time. Therefore, for each point of the area of survey Ω there is a moment of time $t(x)$, which determines the moment of the beginning of its survey, those

$$\lambda(x,t) > 0, \text{ for } t \in [t(x),T]; \qquad (12a)$$

$$\lambda(x,t) = 0, \text{ for } t \in [0,t(x)]. \qquad (12b)$$

We also additionally require that it satisfy the optimality condition, which is as follows: if each T - truncated strategy $\lambda(x,t)$ corresponds to the functional $P_1(\gamma_1,t)=P(\lambda(x,t))$ is the unconditional probability of the correct detection of the object for time t with the strategy $\lambda(x,t)$, then the strategy $\lambda_{opt}(x,t)$ will be optimal if

$$P(\lambda_{opt}(x,t)) = \sup P(\lambda(x,t)). \qquad (13)$$

We also require that the search strategy be optimal for any time moment T of the end of the search. That is, at what point in time the search would not be interrupted, up to this point in time it should be optimal according to the criterion of the maximum unconditional probability of correct detection.

From the analysis of the results on the choice of search strategies that were investigated in the theory of search, of all the strategies, the class of uniformly optimal search strategies satisfies most fully the expressions (11)–(13). A strategy $\lambda(x,t)$ is uniformly optimal if its every T-truncated strategy is optimal, those

$$P(\lambda(x,t)) = P(\lambda_{opt}(x,t)), \forall t \leq T. \qquad (14)$$

Thus, when solving the problem of finding, according to the Bayes criterion, the minimum average risk of the object search strategy, the optimal is a uniformly optimal search strategy, in accordance with which the current sizes and position of the area $\Omega(t)$ in the general survey area Ω .

To find the measure of the area $\Omega(t)$ of the distribution of the search strategy, it is necessary to find the area of the primary search Ω_c from the condition $u(x) > C$,

where C is a constant,

and then solve the Arkin differential expression with zero initial condition:

$$\frac{d\Omega(t)}{dt} = \frac{C\left(\Omega(t)\right)L_0}{\Omega(t)C'(\Omega(t))}, \tag{15}$$

where $L_0 = \varepsilon P_0$ is the power of the radar station P_0 ;

$\varepsilon = \dfrac{G^2 \lambda_R^2 \sigma_T}{(4\pi)^3 D^4 N_0}$ – a proportionality factor that is constant for a particular radar station;

G – gain of the antenna of the radar station;

λ_R – wavelength of the radar station;

σ_T – effective scattering surface of the object;

D – distance to the object;

N_0 – is the spectral power density of radiation noise.

3. CONCLUSION

Thus, the following refined rule for finding the optimal Bayes decision-making algorithm can be formulated. When solving the problem of testing a simple hypothesis against a simple alternative, joint optimization of the search and detection of objects is reduced to:

finding a uniformly optimal search strategy;

calculating the maximum unconditional likelihood ratio in the current survey area;

and comparing it with a threshold.

The formulated rule is quite easily extended to the case of a multi-alternative problem of testing hypotheses and remains valid for the case of discrete search.

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