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Comparison of MGBEKF and UKF Algorithms for Bearings-Only Tracking

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ABSTRACT

Inactive target tracking using bearings-only measurements is a crucial issue of underwater tracking. Target Motion Analysis (TMA) process is highly non-linear so the non linear algorithms like Modified Gain Bearings-only Extended Kalman Filter (MGBEKF) and Unscented Kalman Filter (UKF) are implemented and their performance is evaluated based on their solution convergence times. It is presumed that the target is moving in straight line path with constant speed. The algorithms are simulated for several scenarios which are close to reality using MATLAB. Monte-Carlo runs are performed to evaluate the capability of the algorithms.

Key words: Bearings-only tracking, Modified Gain Bearings-only Extended Kalman filter, Statistical signal processing, Target tracking, Unscented Kalman filter.

1. INTRODUCTION

Two dimensional tracking of targets with bearings-only measurements is often carried out in underwater applications [1]. A single observer platform is utilized to obtain the bearing measurements. The estimates for the target parameters (range, course and speed) are acquired from these bearing measurements only. The mathematical method for obtaining these parameters is provided in Section 2.1.

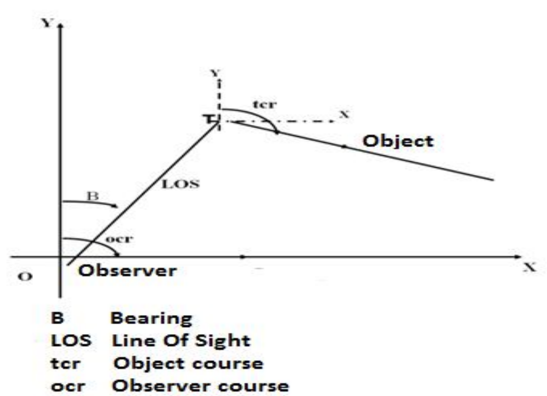

Figure 1: Initial target- observer scenario

The practicality of Speyer's modified gain extended Kalman filter (MGEKF) [3] along with the simpler version of algorithm introduced by Galkowski [4] are considered and the algorithm Modified Gain Bearings-only EKF (MGBEKF) is proposed in this paper. The algorithm for MGBEKF is given in section 2 of mathematical modelling.

Another algorithm that is employed for comparison is Unscented Kalman Filter (UKF) [5-7]. The algorithm for UKF is explained in section 3 of mathematical modelling.
The target observer scenario is as shown in figure 1. The observer is presumed to be initially at position 'O' and the target at position 'T'. The observer follows 'S' maneuver [8] for tracking the target.

Performance of the two algorithms is assessed based on the best convergence time of the solution for the three scenarios given in Table 1. Section 3 presents the process of simulation and the different scenarios on which the simulation is done. The results are plotted as graphs and analyzed in the tables. Section 4 gives the overall summary of the work done in this paper.

2. MATHEMATICAL MODELING

2.1 Target Motion Analysis

Consider the observer is at position 'O' initially and the target is moving with constant speed and course. The state vector at time instant 'τ' of the observer [8] is represented as

$$S_o(\tau) = [v_{xo}(\tau) \quad v_{yo}(\tau) \quad r_{xo}(\tau) \quad r_{yo}(\tau)]^T \qquad (1)$$

where $v_{xo}(\tau)$, $v_{yo}(\tau)$, $r_{xo}(\tau)$, $r_{yo}(\tau)$ are the speed and range components of the observer in x and y coordinates respectively. The change in the observer position is obtained from its course and speed as

$$dr_{xo}(\tau) = v_{xo}(\tau) * \sin ocr * t \qquad (2)$$

$$dr_{yo}(\tau) = v_{yo}(\tau) * \cos ocr * t \qquad (3)$$

where $dr_{xo}(\tau), dr_{yo}(\tau)$ are the change in x-coordinate and y-coordinates of observer and ocr is the observer course angle and t is the time period of one second [1]. The relative state vector [1, 3] of the target is represented as

$$S_s(\tau) = [v_x(\tau) \quad v_y(\tau) \quad r_x(\tau) \quad r_y(\tau)]^T \qquad (4)$$

where $v_x(\tau)$, $v_y(\tau)$, $r_x(\tau)$, $r_y(\tau)$ are relative components of speed and range in x and y coordinates respectively. The relative state vector for the next time period based on the present time state vector is calculated as

$S_s(\tau+1) = A(\tau)S_s(\tau) + b(\tau+1) + \omega C(\tau)$ (5)

where $A(\tau)$ is the system dynamics matrix calculated as

$$A(\tau) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ t & 0 & 1 & 0 \\ 0 & t & 0 & 1 \end{bmatrix} \quad (6)$$

$C(\tau)$ is the process noise and ω is calculated as

$$\omega = \begin{bmatrix} t & 0 \\ 0 & t \\ t^2/2 & 0 \\ 0 & t^2/2 \end{bmatrix} \quad (7)$$

$b(\tau)$ is a deterministic matrix and is calculated as

$$b(\tau+1) = \begin{bmatrix} 0 \\ 0 \\ -\left(r_{xo}(\tau+1) - r_{xo}(\tau)\right) \\ -\left(r_{yo}(\tau+1) - r_{yo}(\tau)\right) \end{bmatrix}^T \quad (8)$$

The covariance of the process noise is calculated as

$Q(\tau) = E[(\omega C(\tau))(\omega C(\tau))^T]$

$$Q(\tau) = \sigma^2 \begin{bmatrix} t^2 & 0 & t^3/2 & 0 \\ 0 & t^2 & 0 & t^3/2 \\ t^3/2 & 0 & t^4/4 & 0 \\ 0 & t^3/2 & 0 & t^4/4 \end{bmatrix} \quad (9)$$

where σ^2 represents variance in the process noise.
The measurement equation for this application has only bearing angles and the bearing angle $\beta(n)$ is represented as

$\beta_m(\tau) = \tan^{-1}(r_x(\tau)/r_y(\tau)) + \Upsilon_b$ (10)

where Υ_b is the noise in measurement which is assumed to be following Gaussian distribution with variance σ_B^2.

2.2 MGBEKF Algorithm

The plant noise and measurement noise are presumed to be independent to each other. The nonlinear equation (6) is linearized by using the Taylor series expansion. The measurement model matrix is calculated as

$$H(\tau+1) = \begin{bmatrix} 0 \\ 0 \\ r_y(\tau+1)/R^2(\tau+1) \\ r_x(\tau+1)/R^2(\tau+1) \end{bmatrix}^T \quad (11)$$

Since the actual values of range will not be known, the estimated range values will be used in the above equation. The predicted covariance matrix is calculated as

$P(\tau+1) = (A(\tau+1)P(\tau)A^T(\tau+1)) + \omega C(\tau+1)\omega^T$ (12)

The Kalman gain is

$G(\tau+1) = P(\tau+1)H^T(\tau+1)[\sigma_B^2 + H(\tau+1)P(\tau+$ $1HT(\tau+1)-1$ (13)

The updated state matrix is calculated as

$S_s(\tau+1) = S_s(\tau+1) + G(\tau+1)[\beta_m(\tau+1) -$ $M\tau+1,S s\tau+1$ (14)

where $M(\tau+1, S_s(\tau+1))$ is the bearing measurement obtained from predicted estimate at time index $(\tau+1)$. The updated covariance matrix is given in equation (11).

$P(\tau+1) = [I - G(\tau+1)g(\beta_m(\tau+1), S_s(\tau+$ $1P\tau+1I-G\tau+1g\beta m\tau+1,S s\tau+1T+\sigma B2G\tau+1GT\tau+1$ (15)

where g represents the modified gain function and is calculated as follows [12]

$$g = \begin{bmatrix} 0 \\ 0 \\ \left(\frac{\cos\beta_m}{r_x \sin\beta_m + r_y \cos\beta_m}\right) \\ \left(\frac{-\sin\beta_m}{r_x \sin\beta_m + r_y \cos\beta_m}\right) \end{bmatrix}^T \quad (16)$$

2.3 UKF Algorithm

A random variable x is considered to be propagating through a nonlinear function $y = U(x)$. Consider $\bar{x}$ as the mean of x and P_x as the covariance of x. The statistics of y are calculated by considering a matrix χ of sigma vectors χ_i with i having a maximum value of $2L_1 + 1$ (where L_1 is the dimension of x). The sigma vectors χ_i are assigned with corresponding weights W_i. The matrix χ is formed by using the following equations [13]:

$\chi_0 = \bar{x}$

$\chi_i = \bar{x} + \left(\sqrt{(L_1+\lambda) + P_x}\right)_i \qquad i = 1,2,\ldots\ldots, L_1$

$\chi_i = \bar{x} - \left(\sqrt{(L_1+\lambda) + P_x}\right)_{i-L_1} \qquad i = L_1+1,\ldots\ldots,2L_1$

$W_0^{(m)} = \lambda/(L_1+\lambda)$ (17)

$W_0^{(c)} = \lambda\,/\,((L_1+\lambda) + (1 - \vartheta^2 + \xi))$

$W_i^{(m)} = W_i^{(c)} = 1/(2(L_1+\lambda)) \qquad i = 1,2,\ldots,2L_1$

where $\lambda = \vartheta^2(L_1+\alpha) - L_1$ is a scaling parameter. ϑ is set to a small positive value (e.g., 1e-3) that determines how the sigma points are spread around the mean. α, which is set to zero, is a secondary scaling parameter and ξ incorporates prior knowledge of the distribution of x (for Gaussian distribution, $\xi = 2$ is optimal). $\left(\sqrt{(L_1+\lambda) + P_x}\right)_i$ represents the i^{th} row of the matrix square root. $W_0^{(m)}$, $W_0^{(c)}$, $W^{(m)}$ and $W^{(c)}$ represents the weights of initialized target state vector, state covariance matrix, state sigma point vector and state sigma point covariance matrix respectively. The nonlinear function used for propagating these sigma vectors is represented as

$y_i = U(\chi_i) \qquad i = 1,2,\ldots.,2L_1$ (18)

The weighted mean and covariance of posterior sigma points are utilized to estimate the mean and covariance of x [13].
The UKF implementation steps are as follows:

(a) Let L_1 be the dimension of target state vector. $(2L_1+1)$ state vectors are calculated from the initial points using sigma points

$$S(\tau) = \begin{bmatrix} S_s(\tau) \\ S_s(\tau) + \sqrt{(L_1+\lambda) + P(\tau)} \\ S_s(\tau) - \sqrt{(L_1+\lambda) + P(\tau)} \end{bmatrix}^T \quad (19)$$

(b) Based on the process model equation (2), transform the sigma points.

(c) The predicted state estimate at time $(\tau+1)$ with τ measurements is calculated as

$$S_s(\tau+1) = \sum_{i=0}^{2L_1} W_i^{(m)} S_s(i,(\tau+1)) \quad (20)$$

(d) The predicted covariance matrix, assuming additive and independent process noise, is calculated as

$$P(\tau+1) = \sum_{i=0}^{2L_1} W_i^{(c)} [S_s(i,(\tau+1)) - S_s(\tau+1)] \times Ssi,\tau+1-Ss\tau+1T+Q\tau \quad (21)$$

(e) The sigma points are updated using the predicted mean and predicted covariance as follows

$$S(\tau+1) = \begin{bmatrix} S_s(\tau+1) \\ S_s(\tau+1) + \sqrt{(L_1+\lambda)+P(\tau+1)} \\ S_s(\tau+1) - \sqrt{(L_1+\lambda)+P(\tau+1)} \end{bmatrix}^T \quad (22)$$

(f) Based on the measurement model given in equation (20), transform the predicted sigma points.

(g) Predicted measurement matrix is calculated as

$$\widehat{M}(\tau+1) = \sum_{i=0}^{2L_1} W_i^{(m)} Y(\tau+1) \quad (23)$$

where $$Y(\tau+1) = h(S_s(\tau+1)) \quad (24)$$

(h) The innovation covariance matrix is calculated as

$$P_{yy} = \sum_{i=0}^{2L_1} W_i^{(c)} [Y(i,(\tau+1)) - \widehat{M}(\tau+1)] [Y(i,(\tau+1-M\tau+1T+\sigma B2(\tau) \quad (25)$$

(i) The cross covariance matrix is calculated as

$$P_{xy} = \sum_{i=0}^{2L_1} W_i^{(c)} [S_s(i,(\tau+1)) - S_s(\tau+1)] [S_s(i,(\tau+1-Ss\tau+1T \quad (26)$$

Kalman gain is calculated as

$$G(\tau+1) = P_{xy} P_{yy}^{-1} \quad (27)$$

(j) The estimated state is calculated as

$$S(\tau+1) = S(\tau+1) + G(\tau+1)\left(\widehat{M}(\tau+1) - M\tau+1 \right. \quad (28)$$

where $M(\tau+1)$ is a matrix of measurement vector.

(k) Estimation of error covariance matrix is given as

$$P(\tau+1) = P(\tau+1) - G(\tau+1) P_{yy} G^T(\tau+1) \quad (29)$$

3. SIMULATION AND RESULTS

This research paper assesses the performance of both algorithms by implementing in MATLAB PC environment. The measurements are assumed to be available continuously for every second. The observer is assumed to perform 'S' maneuvre in its course.

The target is assumed to be having different initial ranges, speeds and courses in different scenarios, which is given in Table 1. The target state vector's initial estimate for implementation of both algorithms is taken as

$$S_s(0,0) = [5 \quad 5 \quad 5000 \sin\beta_m \quad 5000\cos\beta_m]$$

The speed components of the target are each assumed as 5m/s. The target's initial position is calculated based on the Sonar Range of the Day (SRD), which is assumed to be 5000m. The initial state covariance matrix can be taken as a diagonal matrix if the uniform distribution of initial state estimate is considered and is given as

$$P(0,0) = diagonal \begin{bmatrix} 4v_x^2(0,0)/12 \\ 4v_y^2(0,0)/12 \\ 4r_x^2(0,0)/12 \\ 4r_y^2(0,0)/12 \end{bmatrix}$$

Table 1: Scenarios for the given algorithms

Scenarios	Initial Range (m)	Initial Bearing (deg)	Target Speed (m/s)	Observer speed (m/s)	Target course (deg)
1	3000	0	12	8	135
2	3500	0	12	10	110
3	4500	0	8	5	135

Table 2: Convergence time in seconds for 100 runs

Parameter convergence	Scenario 1		Scenario 2		Scenario 3	
	UKF	MGB EKF	UKF	MGB EKF	UKF	MGB EKF
Range	376	256	412	297	421	341
Course	367	280	448	355	455	419
Speed	400	262	438	302	463	373
Total Solution	400	280	448	355	455	419

The simulation and filtering for 100 Monte-Carlo runs are performed for the above mentioned scenarios using MATLAB [6] for both MGBEKF and UKF algorithms. The performance is assessed by using the Root-Mean-Squared (RMS) error of the target parameters and the solution is obtained based on the criteria of acceptance explained as follows and tabulated in table 2.

Range error estimate<=2.66% of the actual range
Course error estimate<=1°.
Speed error estimate<=0.33m/s.

It can be observed from the tables 2 that the solution convergence of MGBEKF algorithm is faster when compared to that of UKF algorithm for all the scenarios. Though the computational complexity of MGBEKF is a little higher than that of UKF, the convergence of the solution plays a key role in realistic scenarios.

Figure 2 showed the movements of the observer and target that were assumed for the scenario 2 and the estimated target path obtained using MGBEKF algorithm.

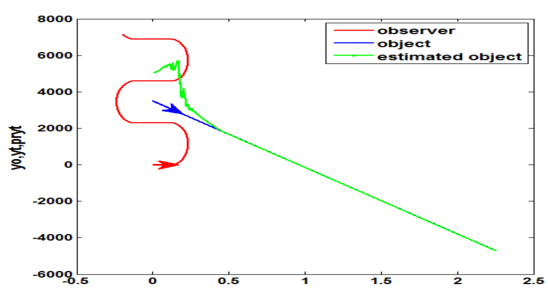

Figure 2: Observer and target movements

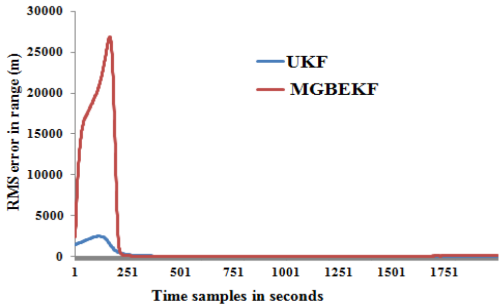

Figure 3: RMS errors in target range estimates

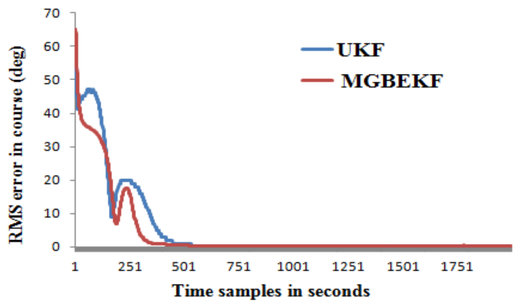

Figure 4: RMS errors in target course estimate

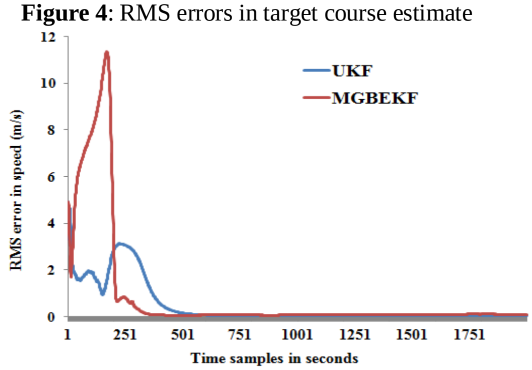

Figure 5: RMS error in estimates of speed

Figures 3-5 shows the comparison of RMS errors in estimates of range, estimates of course and estimates of speed of the target for both MGBEKF and UKF algorithms. It can be observed from the figures that MGBEKF algorithm attain low RMS error values faster than the UKF algorithm which leads to faster convergence of the solution.

4. CONCLUSION

In this paper, an attempt is made to compare two algorithms, MGBEKF and UKF. From the observations it can be said that the solution convergence of MGBEKF algorithm is faster when compared to that of UKF algorithm for all the scenarios. Though the computational complexity of MGBEKF is a little higher than that of UKF, the convergence of the solution plays a key role in realistic scenarios.

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