ISSN 2347 - 3983
Volume 12. No.6, June 2024
International Journal of Emerging Trends in Engineering Research
Available Online at http://www.warse.org/IJETER/static/pdf/file/ijeter011262024.pdf
https://doi.org/10.30534/ijeter/2024/011262024

Analytical Model of a New Acoustic Conductor Lined with Linear Increasing Perforated Area

Fathy Bakheet Shenoda[1], Mahmoud Younes El Aidy[2]
[1] Department of Acoustics, Mass and Force Metrology Division, National Institute of Standards, El-Sadat Street, El-Haram, El-Giza, P.O.B136 code 12211, Egypt, fathy.bakheet.shenoda@gmail.com
[2*] Department of Acoustics, Mass and Force Metrology Division, National Institute of Standards, El-Sadat Street, El-Haram, El-Giza, P.O.B136 code 12211, Egypt, melaidy@yahoo.com

Received Date: April 20, 2024 Accepted Date: May 21, 2024 Published Date : June 07, 2024

ABSTRACT

An acoustic conductor is a simple method of impedance-matching of plane sound waves in pipes. The present work offers a theoretical study of a new design conductor, this conductor is based on a constant cross-section pipe and treated with a linear gradually increasing perforated area along its length. In addition, the study outlines a design procedure for optimizing the conductor performance. The design parameters such as the conductor length, the graduality factor of the perforated area along the conductor pipe, and the conductor cross-section diameter are separately studied in detail for the proposed conductor. The wave equation describing the sound propagation through the conductor is considered, and the solution of this wave equation is derived using the modified Hankle function for a plane sound wave. In general, the frequency dependence of the computed parameters such as the reflection coefficient, the normalized wave resistance, and the normalized wave reactance at the input of the considered acoustic conductor shows a sharp drop frequency, at which the reflection coefficient is less than 0.1, as well as, the wave propagation resistance tends to unity while the reactance tends to zero. The study summarized all designed parameters in one design parameter $\boldsymbol{A_c}$, named the characteristic area of the conductor. The computed results show that increasing $\boldsymbol{Ac}$ leads to shifting the drop frequency towards low frequencies. This enables the designer to design the official sound conductor for specific applications under the available dimensions. A multiple treatment that gives small graduality and a large perforated area is computed, which provides good matching for the conductor. A reasonable agreement between the obtained results with a previous experimental study in our lab by Shenouda for this new conductor design.

Key words: Acoustic conductors, Perforated lining, Sound wave propagation in tubes, Sound reflection, Acoustic wave impedance.

1. INTRODUCTION

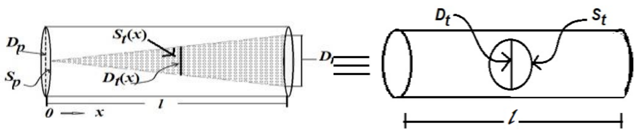
A large number of the sound-generating devices are tubular, with sound waves being set up inside the tube and some of this stored-up energy being radiated out to the open. A tube whose open end has a diameter smaller than the wavelength of sound sent out is a very inefficient radiator of sound, therefore, it is a bad sound conductor. To increase, the sound radiation efficiency, one method is to use a small diaphragm and to magnify its effective size by using a flaring tube (e.g. horn). This loudspeaker horn spreads out the concentrated sound waves coming from the diaphragm over a large enough area so that they can continue out from the mouth of the horn with very little reflection back to the diaphragm.

The present work deals with a study on an acoustic conductor relevant to the previously suggested by *Shenoda* [1], [2] and *Shenoda et al.* [3]. This type of conductor represents an effective and easy method of impedance matching for a plane sound wave incident at its entrance plane. This conductor consists of a tube of constant cross-section and treated at its side walls with an increasing perforated area along its length. *Albeer Shenouda* [4] carried out the experimental study of the design parameters of such conductors, which affect the matching performance of such conductors. He carried out experimental studies of different configurations to achieve optimum design goals. Also, he suggested an approach for such a conductor system

The present work aims to give the theoretical study of such acoustic conductors to establish and derive the wave equation describing the propagation of plane sound waves down the conductor. The sound reflection coefficient and both the normalized acoustic resistance and reactance at its inlet were determined and discussed.

2. SHENODA APPROACH

Shenouda, ***A***. approach [4] considers the linear increasing perforated area to consist of continuous orifices that are located close to each other (i.e. separated by a very small fraction of sound wavelength) and the action of the group is that of their equivalent parallel impedance. In his case, the attached mass of each orifice tends to zero [5] and all orifices can be taken together as one orifice with a total area "$\boldsymbol{S_t}$", which equals the sum of all orifice's area, Figure 1(b). If this orifice is merely a hole drilled in an infinitesimal thin wall of the main pipe (i.e. the thickness of the pipe wall $t \approx 0$) and because the radiation of sound takes place only at the outside of an unflagged aperture, the effective thickness of the pipe's wall $\boldsymbol{t} = 0.3\ \boldsymbol{D_t}$ to be considered.

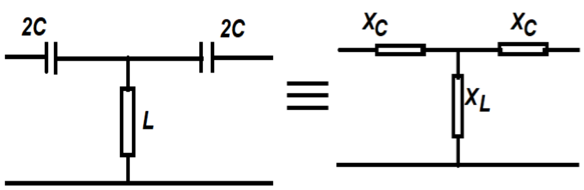


(a) (b)

Figure 1. The schematic of the conductor model

- $S_P = \pi \frac{D_P^2}{4}$, and
- $S_t(x) = \pi \frac{D_t^2(x)}{4}$,
- $D_t(x) = \frac{D_m}{l}.x$
- $\boldsymbol{S_t} = \boldsymbol{l}.\ \frac{D_m}{2} = \frac{\pi D_t^2}{4}$, and
- $\boldsymbol{D_t} = \sqrt{2\boldsymbol{l}.\frac{D_m}{\pi}}$, where $\boldsymbol{D_t}$ is the orifice's diameter

Shenouda, ***A*** employed the combination of resonators and orifices and constructed an acoustic filter by taking advantage of the analogy between acoustic and electrical filters, Figures 2(a) and (b). the sharpness of the cut-off of an electrical filter system that can be increased by using a combination reactance of one type of impedance in series with the line and reactance of another type of impedance shunted across the line [6].



(a) Electrical circuit of T-shaped high pass filter

(b) Acoustical analogy of this type of electrical filter

Figure 2. Equivalent of acoustic conductor

$X_C = \frac{1}{\omega_c C_a} = \frac{1}{2\omega_c C}$, then $C = \frac{C_a}{2} = \frac{S_P.l.}{4\rho\, c_o^2}$,

$X_L = \omega_c L_a \frac{\omega_c\, \rho.t'}{S_t} = \omega_c.L$, then $L = \frac{\rho.t'}{S_t}$

where $\boldsymbol{c_o}$ is the sound speed

He obtained the cut-off frequency $\boldsymbol{f_C}$ of such a sound conductor

$$f_C = \frac{1}{4\pi}.\frac{1}{\sqrt{LC}} = \frac{8825.3}{D_p}.\left(\frac{D_m}{l}\right)^{0.25}$$

where, $\boldsymbol{c_o}$ =34000 cm/sec, $\boldsymbol{D_P}$, is the pipe diameter of the conductor, $\boldsymbol{D_m}$, is the maximum width of the variable side perforated area at the output of the conductor and $\boldsymbol{l}$ is the length of the conductor and all are in cm.

He compared the measured cut-off frequency "$\boldsymbol{f_C}$" for several design models with those determined from his approach, which were in good agreement.

3. THEORY AND FORMULA

The treated device consists of a main cylindrical pipe of cross-section diameter "$\boldsymbol{D_P}$". The pipe is treated at its side wall and along the pipe length "l" with a linear increasing perforated area $\boldsymbol{S_t}(x)$ with orifices, that are very close to each other. The treated area has its maximum width "$\boldsymbol{D_m}$" at the end of the main pipe, Figure 1 (a). In our study, we consider the same suggestion of *Shenouda*, that the linear increasing perforated area can be taken as one orifice with the same total area "$\boldsymbol{S_t}$", Figure1(b).

For an orifice of radius, "$\boldsymbol{a}$=$\boldsymbol{D_t}$/$\boldsymbol{2}$" and effective thickness "$\boldsymbol{t'}$" both are small relative to the sound wavelength "λ", the flux normalized impedance "$\boldsymbol{Z_a}/\boldsymbol{\rho c}$" is [7]

$$\frac{Z_a}{\rho c} = \left[\frac{k_o^2}{4\pi} + j\ \frac{k_o\, t'}{4\pi a}\right].S_t,\ \ \left({D_t}/{2}\right) \ll \lambda \tag{1}$$

where $\boldsymbol{k_o}$ is the wave number, $t'$is the effective thickness of the pipe wall.

The first term of equation (1) results from the radiation of sound through the orifice into the surrounding medium, while the second term results from the entrance of the air in the orifice.

In our case as well as in the Shenouda approach, the orifice is considered as a hole drilled in a thin wall of the main pipe and hence the radiation of sound takes place only at the outside of an unflagged aperture and accordingly $t' = 0.3\ D_t$.

Considering Figure1(a), and (b) that $D_t(x)$ and $S_t(x)$ depend on $\boldsymbol{x}$ and substituting in the equation. 1 for $D_t(x)\frac{D_m}{l}.x$; $S_t(x) = \frac{\pi D_t^2(x)}{4} = \frac{\pi D_m^2}{4\, l^2}.\boldsymbol{x^2}$; $\boldsymbol{D_t} = \sqrt{2\boldsymbol{l}.\frac{D_m}{\pi}}$, and $t' = 0.3\ D_t$, we get

$$\frac{Z_a}{\rho c} = \left[\frac{k_o^2}{4\pi} + j\ \frac{k_o\, t'}{2\pi\, D_t\,(x)}\right].S_t(x),$$

$$= \frac{D_m^2\ \ k_o^2}{16\, l^2}.\ \ x^2 + j\ \frac{D_m\ k_o\, t'}{8\, l}\ .\ x \tag{2}$$

The normalized admittance of the increasing perforated area at the side wall of the treated conductor, ρc$\boldsymbol{Y}(x)$ is

$$\rho c\ Y(x) \cong 4\ - j\ \frac{8\, l}{k_o\, D_m\, t'}.\frac{1}{x} \tag{3}$$

The wave equation for the velocity potential, $\boldsymbol{u}(x)$, which represents the propagation of a plane sound wave down the treated conductor, can be obtained by applying the fundamental field equations (i.e. the force equation and the continuity equation) for periodic time dependence is given by,

$$\frac{d^2\, \boldsymbol{u}(x)}{dx^2} + \left[k_o^2 - j\ \left(\frac{k_o}{h_{eff}(x)}\right)\rho c\ \boldsymbol{Y}(x)\right].\boldsymbol{u}(x) = 0 \tag{4}$$

where $h_{eff}\,(x)$ is the effective pipe height, $h_{eff}\,(x)$= $\boldsymbol{S_p}/\boldsymbol{D_x}$, S_P is the free cross-section area of the main treated pipe, and $\boldsymbol{D_x}$ is the variable width of the side perforated area at a point $\boldsymbol{x}$.

Accordingly, the wave equation (4) becomes;

$$\frac{d^2\, \boldsymbol{u}(x)}{dx^2} + \left[k_o^2 - \frac{\boldsymbol{8}}{\boldsymbol{S_p\, t'}}\ - j\ \frac{\boldsymbol{4\, k_o\, D_m}}{\boldsymbol{S_p\, l}}.x\right].\boldsymbol{u}\ (x) = 0 \tag{5}$$

3.1. Solution of Wave Equation

The waveform equation (5) is nonlinear since the coefficient is a function in $\boldsymbol{x}$. Therefore, a suitable transformation can be applied to transfer this form to a well-known form equation. Using the following transformations, namely

$\boldsymbol{u}(x) = \boldsymbol{u}(z)$, and

$z = -\left(k_o^2 - \frac{8}{S_p\, t'}\right) \cdot \frac{S_p\, l}{4\, k_o\, D_m} + jx$, that means

$jx = z + \left(k_o^2 - \frac{8}{S_p\, t'}\right) \cdot \frac{S_p\, l}{4\, k_o\, D_m}$, we have

$$\frac{d^2\,\boldsymbol{u}(z)}{dz^2} + \left(\frac{k_o}{A_c}.z\right).\boldsymbol{u}\,(z) = 0 \qquad (6)$$

where, $A_C = \frac{S_p\, l}{4\, D_m}$, and defined as a conductor characteristic area

Using the following transformations, namely

$$T = \frac{2}{3}\sqrt{\frac{k_o}{A_c}}\,.\boldsymbol{z}^{\frac{3}{2}}, \text{ and } \boldsymbol{u}(z) = \sqrt{z}\,.\ \mathbf{W}(T),$$

Then equation (6) becomes

$$\frac{d^2\boldsymbol{W}(T)}{dT^2} + \frac{1}{T}.\frac{d\boldsymbol{W}(T)}{dT} + \left(1 - \frac{\left(\frac{1}{3}\right)^2}{\boldsymbol{T}^2}\right).\boldsymbol{W}(T) = 0 \quad (7)$$

This differential equation is a BESSEL differential equation and has a closed-form solution:

$$\boldsymbol{W}(T) = C_1\,.H^{(1)}_{1/3}\,(T) + C_2\,.H^{(2)}_{1/3}\,(T)$$

where $\boldsymbol{C_1}$ and $\boldsymbol{C_2}$ are arbitrary integration constants and $H^{(1)}_{1/3}(T)$ and $H^{(2)}_{1/3}(T)$ are Hankel functions of the first and second kind [8] of complex argument $\boldsymbol{T}$ and order 1/3. The two functions are triple-valued functions of T, with a branch point at the origin. The product, however, of the two multiple valued functions of $\boldsymbol{T}^{1/3}$ with $H^{(1\ or\ 2)}_{1/3}$ is a single-valued function and $\boldsymbol{h_1}$ or $\boldsymbol{h_2}$, represents the modified Hankel functions of order one-third. These functions $H^{(1)}_{1/3}$ or $H^{(2)}_{1/3}$ and $\boldsymbol{h_1}$ or $\boldsymbol{h_2}$ are related to each other by [9].

$$\left.\begin{array}{l} h_1\left(z.\sqrt[3]{\frac{k_o}{A_c}}\right) = T^{\frac{1}{3}}\,H^{(1)}_{1/3}(T) \\ h_2\left(z.\sqrt[3]{\frac{k_o}{A_c}}\right) = T^{\frac{1}{3}}\,H^{(2)}_{1/3}(T) \end{array}\right]$$

Accordingly, the complete solution of the differential equation (7) is given by,

$$\boldsymbol{W}(T) = C_1.T^{\frac{-1}{3}}\,.h_1(a.z) + C_2.T^{\frac{-1}{3}}\,.h_2(a.z) \qquad (8)$$

where, $a = \left(\frac{k_o}{A_c}\right)^{\frac{1}{3}}$. then

$$\boldsymbol{u}(z) \equiv \boldsymbol{u}(x) = \boldsymbol{u}_o.\sqrt{z}\,\boldsymbol{W}(T)$$

$$= \boldsymbol{u}_o.\sqrt{z}\ T^{-\frac{1}{3}}.[\,C_1\,h_1(a\,z) + C_2\,h_2(a\,z)] \qquad (9)$$

where, $\boldsymbol{u_o}$ the velocity potential at x=0, which can be determined by exciting the conductor at its inlet.

This solution (9) represents the incident as well as the reflected waves that propagate through the conductor. For harmonic time dependence of the sound field, the sound pressure $\boldsymbol{P}(x)$ and the sound particle velocity $\boldsymbol{v}(x)$ are given by,

$$\boldsymbol{P}(x) = -j\,\omega\,\rho\,\boldsymbol{u}(x)$$

$$= u_o d_1[\,C_1\,h_1(a\,z) + C_2\,h_2(a\,z)] \qquad (10)$$

$$\boldsymbol{v}\,(x) = \frac{d\,\boldsymbol{u}(x)}{dx}$$

$$= u_o d_2[\,C_1\,h_1'(a\,z) + C_2\,h_2'(a\,z)] \qquad (11)$$

where, h_1' and h_2' are the derivatives of the functions $\boldsymbol{h_1}$ and $\boldsymbol{h_2}$, respectively and $a = \sqrt[3]{\frac{k_o}{A_c}}$, $d_1 = -j\,\omega\rho\,(\frac{9\,A_c}{4\,k_o})^{\frac{1}{6}} = -j\,k_o\,\rho c\,(\frac{9\,A_c}{4\,k_o})^{\frac{1}{6}}$, and $d_2 = j\,(\frac{9\,k_o}{4\,A_c})^{\frac{1}{6}}$, $\omega = 2\pi f$, and $\boldsymbol{f}$ is the frequency of the incident wave at the conductor inlet.

3.2. One Dimensional Acoustic Conductor of Finite Length

Such an acoustic conductor of length "***l***" is usually connected at its input with a rigid pipe of equal cross-section and ends in the open air. Considering this arrangement, one can use the boundary conditions at the input of the conductor, namely the necessity of equal sound pressures and the continuity of sound flux to determine the constants $\boldsymbol{C_1}$ and $\boldsymbol{C_2}$ of equations (10) and (11) namely:

$$P_1(x)|_{x=0} = P_2(x)|_{x=0}\text{ , and } \boldsymbol{v}_1(x)|_{x=0} = \boldsymbol{v}_2(x)|_{x=0}$$

where, $P_2(x)$ and $\boldsymbol{v}_2(x)$ are those for the conductor at *x=0* according to equations (10) and (11), respectively.

On the other hand, the boundary conditions at the end of the conductor (i.e. at ***x****=l*), deliver the expression for the reflection coefficient $|r_i|$ |and the normalized wave impedance (Zo/ρc) at the input of the conductor, namely:

$$P_2(x)|_{x=l} = P_3(x)|_{x=l}\text{ , and } \boldsymbol{v}_2(x)|_{x=l} = \boldsymbol{v}_3(x)|_{x=l}$$

where $P_3(x)$ and $\boldsymbol{v}_3(x)$ are those for a plane sound wave or in the open air at *x=l*

Applying these boundary conditions, we obtain

$$\boldsymbol{r_i} = -\left\lfloor \frac{\boldsymbol{N8+N10(1-N4)}}{\boldsymbol{N10}} \right\rfloor \qquad (12)$$

where, $N1 = d_1\,h_1\,(a.z_o) + \rho c\,d_2\,h_1'\,(a.z_o)$,

$N2 = d_1\,h_2\,(a.z_o) + \rho c\,d_2\,h_2'\,(a.z_o)$,

$N3 = h_1\,(a.z_o) - \frac{N1}{N2}\,h_2\,(a.z_o)$,

$N4 = \frac{2d1}{N2}\,h_2\,(a.z_o)$,

$N5 = \frac{N1}{2d1N3}$,

$N6 = d_1\,h_1\,(a.z_l) - \rho c\,d_2\,h_1'\,(a.z_l)$,

$N7 = d_1\,h_2\,(a.z_l) - \rho c\,d_2\,h_2'\,(a.z_l)$,

$N8 = \frac{2N7}{N2}$, $N9 = \frac{N6}{d1N3}$, and

$N_{10} = N9 - N5N8$,

$$z_o = -\left(k_o^2 - \frac{8}{S_p\, t'}\right).\ \frac{A_C}{k_o};\ \boldsymbol{a} = \sqrt[3]{\frac{k_o}{A_C}};$$

$$t' = 0.3\sqrt{\frac{2\ l.D_m}{\pi}},\ \text{and}\ \ z_l = z_o + jl;$$

Accordingly, the wave impedance at the input of the conductor can be determined

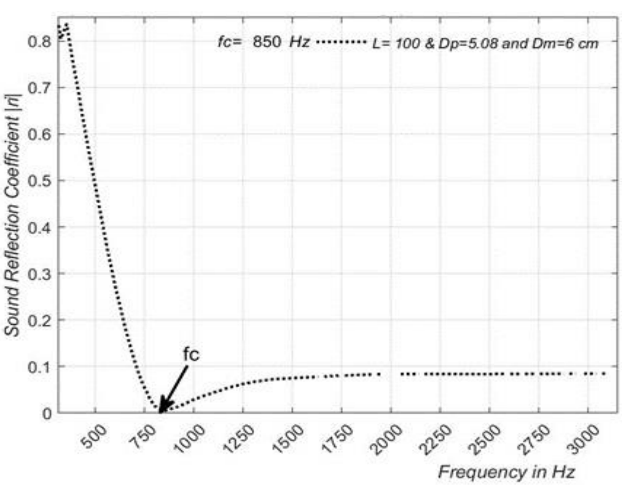
$$\frac{Z_O}{\rho c}\bigg|_{x=0} = \frac{1+\boldsymbol{ri}}{1-\boldsymbol{ri}} = \mathcal{R}_o + j\,\chi_o\ ,$$

where, $\mathcal{R}_o$ and χ_o are the normalized input wave resistance due to the sound radiation and the normalized mechanical reactance due to fluid inertia, respectively.

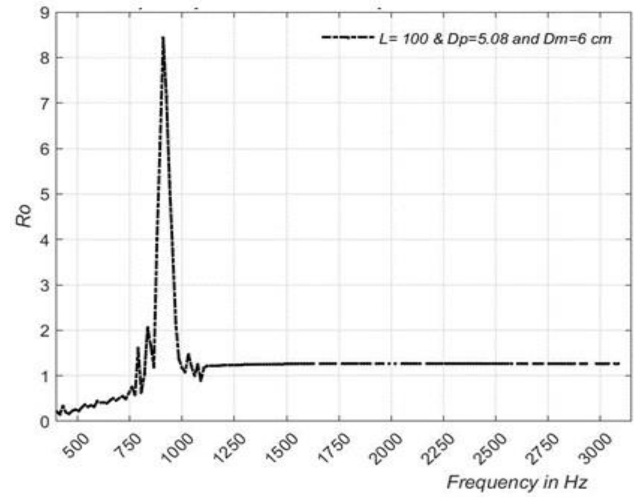
4. RESULTS AND DISCUSSIONS

The magnitude of the sound reflection coefficient, $|\boldsymbol{r_i}|$ and the real part $\mathcal{R}_o$, as well as, the imaginary part, χ_o of the normalized wave impedance at the conductor input were computed at different conductor lengths "$\boldsymbol{l}$" and cross-section diameter "$\boldsymbol{D_p}$" and also at different conductor characteristic areas "$\boldsymbol{A_C}$". For these purposes, a computer program in FORTRAN was prepared for estimating both sound reflection coefficient and sound impedances ($\mathcal{R}_o$ and χ_o) through estimating the values of $\boldsymbol{h}$'s and its derivative $\boldsymbol{h}$` values.

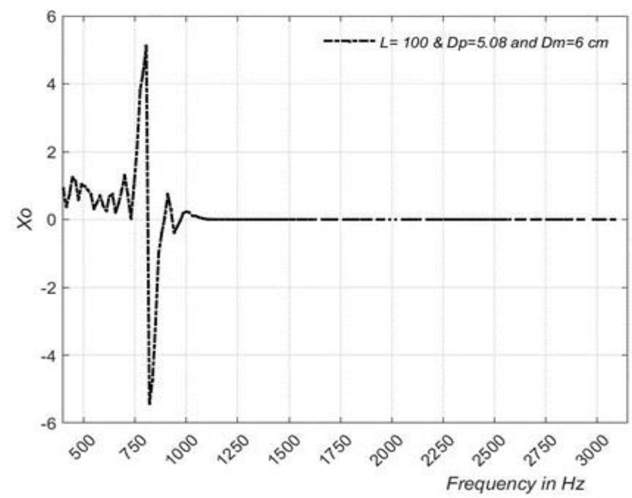
The sound reflection coefficient, $|\boldsymbol{r_i}|$ as well as $\mathcal{R}_o$ and χ_o were computed for a sound conductor of length $\boldsymbol{l}$ =100 cm and cross-section diameter $\boldsymbol{D_p}$ =5.08 cm (2 inch). This conductor is treated with linear increasing perforated area at its side wall which gives maximum width $\boldsymbol{D_m}$= 6 cm. The computed results are represented in Figures 3 (a) to (c). Figure 3(a) shows that the conductor has a sharp drop frequency, $\boldsymbol{f_C}$= 850 Hz after which the conductor has a good matching performance namely $|\boldsymbol{r_i}|$ is less than 0.1, which means that most of the sound energy incident at the input of the investigated conductor is transmitted. Figures 3(b) and (c) show that above $\boldsymbol{f_C}$ the acoustic impedance is nearly resistive and slightly more than 1 (i.e. $\mathcal{R}_o \approx 1$) the air characteristic impedance $\boldsymbol{\rho c}$ or that is for a plane sound wave, where χ_o is nearly zero, Figure 3(c). This is due to the very weak reflections at the conductor outlet. Therefore, the matching performance to a plane sound wave is improved.



(a) Computed reflection coefficient | $\mathbf{r_i}$ | at the duct inlet



(b) Computed normalized input wave resistance



(c) Computed normalized input wave reactance

Figure 3: The computed results at the inlet of the conductor of length $\boldsymbol{l}$=100 cm, $\boldsymbol{D_p}$=5.08 cm, and $\boldsymbol{D_m}$=6 cm.

4.1 Parametric Study of Conductor Performance

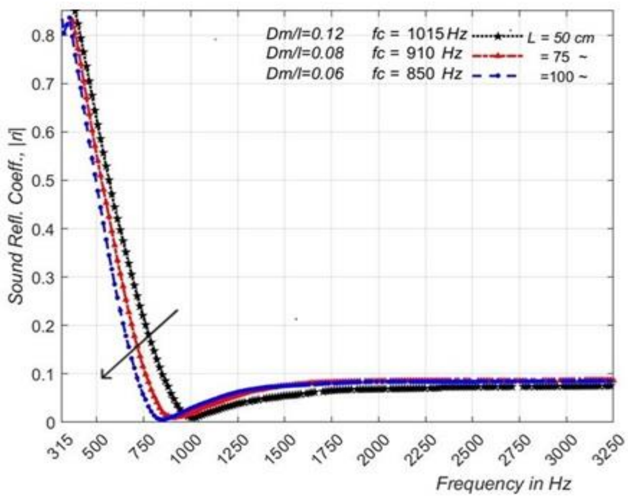
The design parameters of the conductor are the conductor length, "$\boldsymbol{l}$"; the graduation of varying the side perforated area "$\boldsymbol{D_m}$" and the conductor cross-diameter "$\boldsymbol{D_p}$". To optimize each design parameter, the matching performance, namely $|\boldsymbol{r_i}|$, $\boldsymbol{\mathcal{R}_o}$ and $\boldsymbol{\chi_o}$ are computed at the input of the conductor at different configurations when the other design parameters were fixed.

4.1.1 Effect of graduality

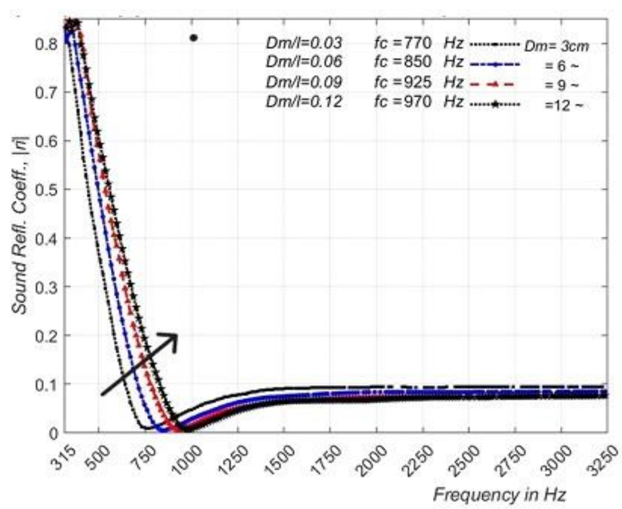
The graduality of the acoustic conductor is defined as $\boldsymbol{D_m/l}$, which can be varied by changing one of the two conductor parameters $\boldsymbol{D_m}$ or $\boldsymbol{l}$, or both. To study the effect of the graduality of the perforated side area, two groups of conductors were computed. In the first group, conductors have the same length $\boldsymbol{l}$=100 cm and the same cross-diameter $\boldsymbol{D_p}$=5.08 cm but have different $\boldsymbol{D_m}$ values; namely 3, 6, 9, and 12cm, which give the graduality $\boldsymbol{D_m}/l$= 0.03, 0.06, 0.09, and 0.12 were computed. The obtained results are represented in

Fig.4a. In the second group, three conductors of the same $\boldsymbol{D_P}$= 5.08 cm and same $\boldsymbol{D_m}$ = 6 cm but have different lengths $\boldsymbol{l}$ = 0.5, 0.75, and 1.0 m which gives graduality $\boldsymbol{D_m/l}$ = 0.12, 0.08 and 0.06 were computed. The obtained results are represented in Figure 4(b). Both figures show that in general, the matching performance of conductor to plane sound wave at constant $\boldsymbol{l}$ becomes better as the graduality factor $\boldsymbol{D_m/l}$ increases, Figure 4(a). For cross-diameters $\boldsymbol{D_p}$=10.16cm, the effect of graduality on the value of $|\boldsymbol{r_i}|$, and $\boldsymbol{f_c}$ is clear, Figures 4(c).

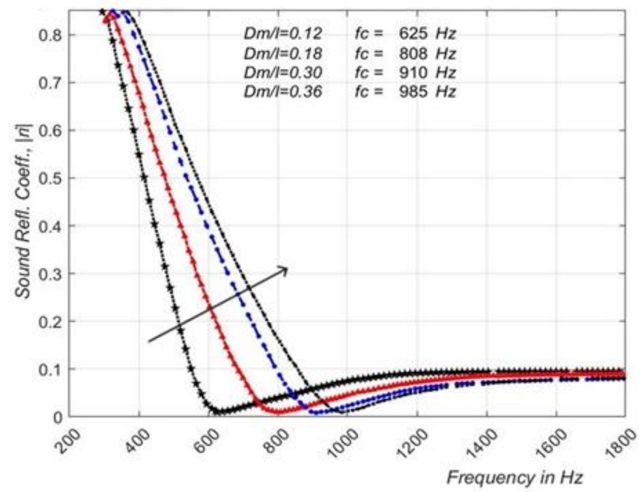
Figures 4(a) to (c) come up with the result, at constant $\boldsymbol{D_p}$, increasing $\boldsymbol{D_m}$ leads to improve the matching performance. Also, at constant $\boldsymbol{D_m/l}$ (e.g. 0.12), $\boldsymbol{f_c}$ decreases with increasing $\boldsymbol{D_p}$ (that is $\boldsymbol{f_c}$ α($\boldsymbol{1/D_p}$)).



(a) Computed |$\mathbf{r_i}$| for 3 conductors, $\boldsymbol{D_m}$=6cm and $\boldsymbol{D_p}$=5.08cm



(b) Computed |$\mathbf{r_i}$| for 4 conductors $\boldsymbol{L}$=100cm and $\boldsymbol{D_p}$=5.08cm



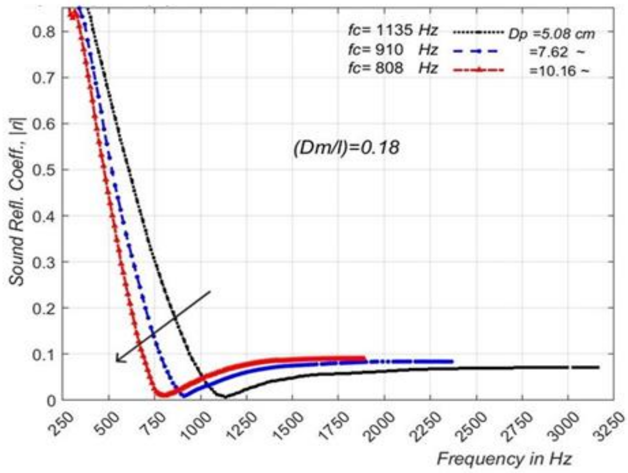
(c) Computed |$\mathbf{r_i}$| for 4 conductors, $\boldsymbol{L}$=50cm and $\boldsymbol{D_p}$=10.62cm

Figure 4: The computed results of conductors having different graduality, $\boldsymbol{D_m/l}$**.**

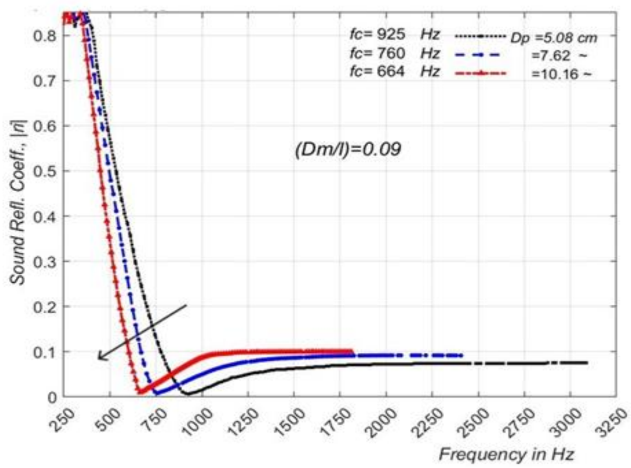
4.1.2 Effect of conductor cross-diameter, D_p

To estimate the effect of cross-diameter on the drop frequency $\boldsymbol{f_c}$, and consequentially the sound conductor performance, sound reflection coefficients |$\boldsymbol{r_i}$| of conductors having the same $\boldsymbol{l}$ and $\boldsymbol{D_m}$ but have different cross-diameters: $\boldsymbol{D_P}$ = 5.08, 7.62, and 10.16 cm were computed. Accordingly, the reflection coefficient, |$\boldsymbol{r_i}$| of two groups of conductors was considered. All conductors of the groups have the same $\boldsymbol{D_m}$=9 cm but each group of conductors represents a specific length e.g. $\boldsymbol{l}$ = 0.5m or 1.0m.

The first group consists of three conductors. They have the same length, $\boldsymbol{l}$= 0.5m, and are treated with a linear increasing perforated area of the same, $\boldsymbol{D_m}$= 9 cm, but they have different cross-diameter, $\boldsymbol{D_P}$ = 2, 3, and 4 inches. The computed results of this group are represented in Figure 5(a). The second group consists of three conductors. They have the same length, $\boldsymbol{l}$ =1.0m, and the same $\boldsymbol{D_m}$ = 9 cm and $\boldsymbol{D_P}$= 2, 3, and 4 inches. The computed results of this group are represented in Figure 5(b). Figures 5(a) and (c) represent the computed sound reflection coefficient, |$\boldsymbol{r_i}$| as a function of frequency at different cross-diameter and different graduality; $\boldsymbol{D_m/l}$ = 0.18, 0.12, and 0.09 for length, $\boldsymbol{l}$ = 0.5, and 1.0m, respectively. In general, increasing $\boldsymbol{D_P}$ leads to a decrease of $\boldsymbol{f_c}$ at constant $\boldsymbol{D_m/l}$ (i.e. $\boldsymbol{f_c}$ is inversely proportional to $\boldsymbol{D_P}$). In addition to that, it leads to a decrease in the high-frequency limit at which plane waves can be propagated down the conductor.



(a) Computed $|r_i|$ for 3 conductors, l =0.5m, D_m=9cm

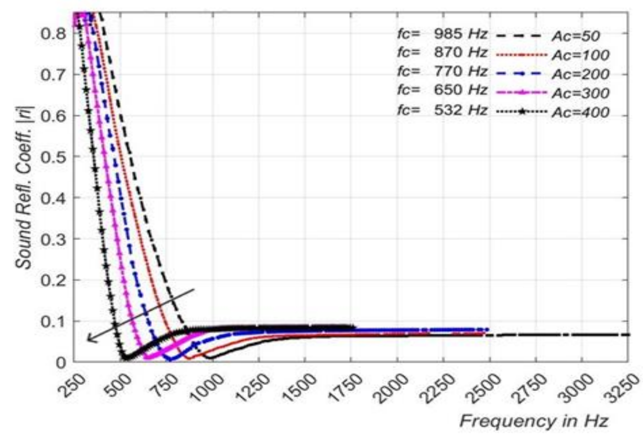


(b) Computed $|r_i|$ for 3 conductors, l=1.0m, D_m=9cm

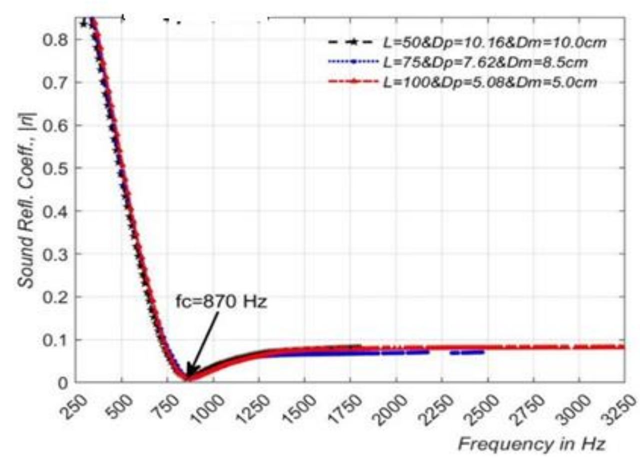
Figure 5: The computed sound reflection coefficient of sound conductors having different cross-diameters D_p

4.2 Sound Conductor Characteristic Area A_C

The conductor characteristic area A_C is a design parameter. Through this parameter, one can choose the suitable dimensions for the most efficient sound conductor according to the application and the available dimensions. This parameter is known as; $A_c = (S_p l/4D_m)$ which include all dimensional parameters. To study the effectiveness of this parameter A_c on the matching performance, five conductors have A_C = 50, 100, 200, 300, and 400 cm^2 were considered. These conductors are built up from different configurations of l, D_P, and D_m, the first conductor (A_C = 50 cm^2), can be built up from, l=*50*, D_P =5.08, and D_m =6.0 cm, the second conductor (A_C = 100 cm^2), from l=*75*, D_P= 7.62 and D_m =8.5 cm, the third conductor (A_C = 200 cm^2), from l=*50*, D_P=7.62 and D_m= 2.85 cm, the fourth conductor (A_C = 300 cm^2), from l=75, D_P=7.62 and D_m =6.75 cm, and the fifth conductor (A_C= 400 cm^2), from l=100, D_P=10.16 and D_m=5.0 cm. The sound reflection coefficient $|r_i|$ of the five conductors was computed. The results are represented in Figure 6(a). The figure shows that the increase of A_C values leads in general to a decrease in f_C and a very slight increase in $|r_i|$. Because $A_c = (S_p l/4D_m)$, it can be seen that: For a given conductor of cross-sectional area S_p, the increase of A_C at constant l is associated with a decrease of D_m. This means that the variation of the side perforated area takes place slowly and more gradually which leads in turn to a decrease of $|r_i|$. Also, the increase of A_C at constant D_m -value at the end of the conductor is associated with an increase of the conductor length l. This means that more sound energy is radiated from the treated side perforated area and the sound energy that reaches the end of the conductor becomes very weak, and accordingly the sound reflection at the end of the conductor has no influence and consequently, the sound reflection at the inlet, $|r_i|$ of the conductor decreases. Also, one of the most interesting results, is conductors having the same A_C value independent of D_P, D_m, and l give the same f_C value, keeping the inversely proportional to the increases of D_P, Figure 6(b).



(a) Computed $|r_i|$ of 5 conductors of different Ac



(b) Computed $|r_i|$ for Ac=100 cm^2 of different configurations of l, Dp, and D_m

Figure 6: The computed results of conductors having different characteristic areas, A_C.

4.3 MULTIPLE TREATMENT

The matching impedance can be improved at the conductor inlet, by applying the acoustic image principle. This can be implemented by two methods:

1- keeping the graduality (***D_m/l***) at its minimum value and the conductor is treated at its side wall and along its whole length with multiple equal and similar side variable perforated areas, (i.e. treated ***n*** times) which are distributed over the periphery of the conductor. In this case, $(S_P)_M$ for each treatment is considered as S_P/n and accordingly, $(D_P)_M$ is considered as $(D_P/\sqrt{n})$.

For this purpose, four conductors were computed. They have the same length ***l***=50 cm and the same cross diameter, D_P=5.08 cm but they are differently treated, keeping the graduality D_m/l = 0.03 for each treatment of the ***n***-times treatments. The four conductors are treated as follows:

- One of these conductors is treated one time (i.e. ***n***=1) with a variable linear increasing perforated area which leads to a maximum width of the perforated area at the conductor outlet, D_m = 1.5 cm.
- The second one is treated two times (i.e. ***n***=2) with variable linear increasing perforated areas each of them is equal and similar to that of the first one.
- The third one is treated three times (i.e. ***n***=3) with variable linear increasing perforated areas each of them is equal and similar to that of the first one.
- The fourth one is treated four times (i.e. ***n***=4) with variable linear increasing perforated areas each of them is equal and similar to that of the first one.

All the multiple treatments (i.e. for ***n***=2, 3, or 4) are distributed over the periphery of the main conductor's pipe. The obtained results are represented in Figure 7(a).

2- keeping the total linear increasing perforated area at the side wall of the conductor and along its whole length constant. For this purpose, 4 conductors were computed. The conductors have the same length l=50 cm and the same cross-diameter D_P= 5.08cm but they are differently treated. Keeping the sum of the total area of the multiple times linear increasing perforated areas is constant for all four conductors. The conductors are treated as follows:

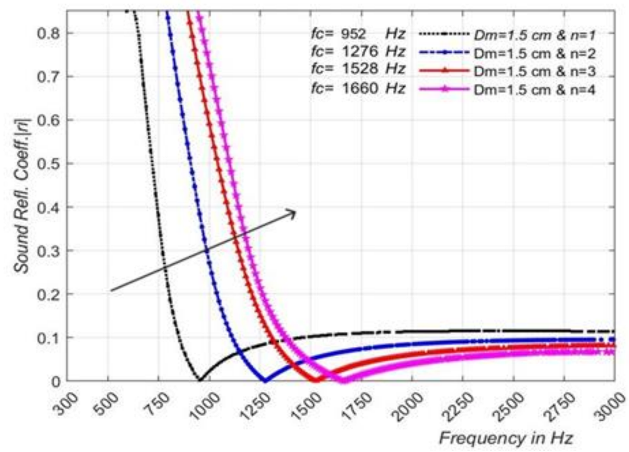
- The first one is treated one time (i.e. ***n***=1) with a linear increasing perforated area of graduality D_m/l = 0.12 which leads to a maximum width of the perforated area at the conductor outlet, D_m = 6.0 cm.
- The second one is treated two times (i.e. ***n***=2) each of them is treated with equal and similar linear increasing perforated areas, which leads to a maximum width of the perforated area at the conductor outlet, D_m = 3.0 cm for each (i.e. the graduality D_m/l = 0.06).
- The third one is treated three times (i.e. ***n***=3) each of them is treated with equal and similar linear increasing perforated areas, which leads to a maximum width of the perforated area at the conductor outlet, D_m = 2.0 cm for each (i.e. the graduality D_m/l = 0.04).
- The fourth one is treated four times (i.e. ***n***=4) each of them is treated with equal and similar linear increasing perforated areas, which leads to a maximum width of the perforated area at the conductor outlet, D_m = 1.5 cm for each (i.e. the graduality D_m/l = 0.03)

All the multiple treatments (i.e. for ***n***=2, 3, or 4) are distributed over the periphery of the main pipe of conductors. Also, the sum of the perforated areas for each of these conductors is the same. The computed results are represented in Figure 7(b).

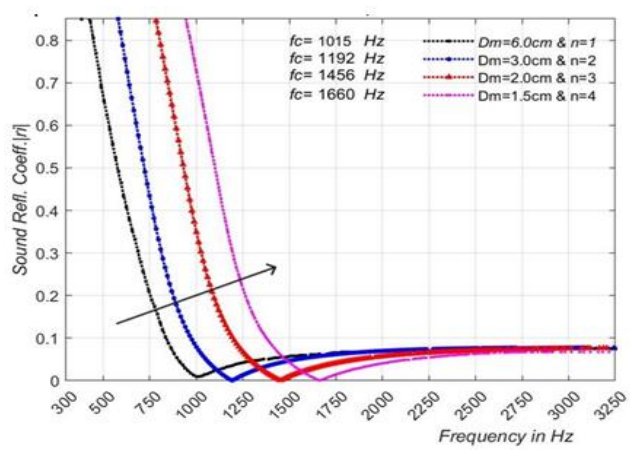
Figure 7(a) shows that by multiple treatments, the total perforated area increased and at the same time the graduality (***D_m/l***) is kept at its minimum value. This treatment realizes two conditions that help by optimizing the conductor design:

- minimum graduality (***D_m/l***) means better matching impedance at the conductor inlet and in turns leads to minimum reflected sound energy at the conductor inlet.
- Also, increasing the total perforated area by repeating the treatment several times, allows more sound energy to be released through it and accordingly, the sound energy reflected at the conductor outlet becomes very small and has no influence.

Accordingly, multiple treatments lead to a smooth $|r_i|$ - ***frequency*** curve and the acoustic wave impedance ***Z*** at the conductor inlet tends to be more resistive ($R_o \rightarrow 1$ and $X_o \rightarrow 0$).



(a) Computed $|\mathbf{r_i}|$ for conductors of ***l***=0.5m, ***Dp***=5.08cm, which treated with multi-slit areas of D_m=1.5cm



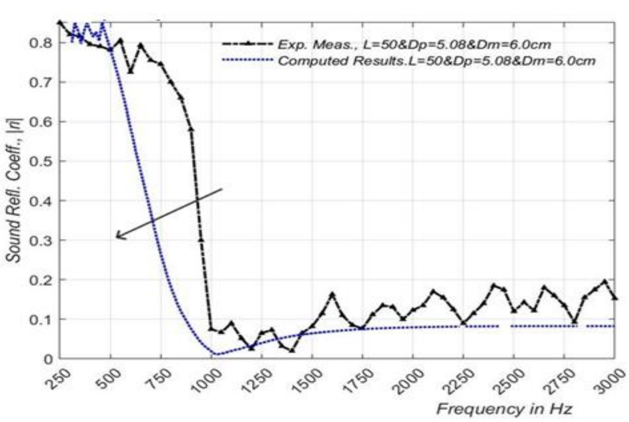
(b) Computed $|\mathbf{r_i}|$ for conductors of ***l***=0.5m, ***Dp***=5.08cm, and D_m=6cm divided into n slits

Figure 7: Multiple treatment perforated area.

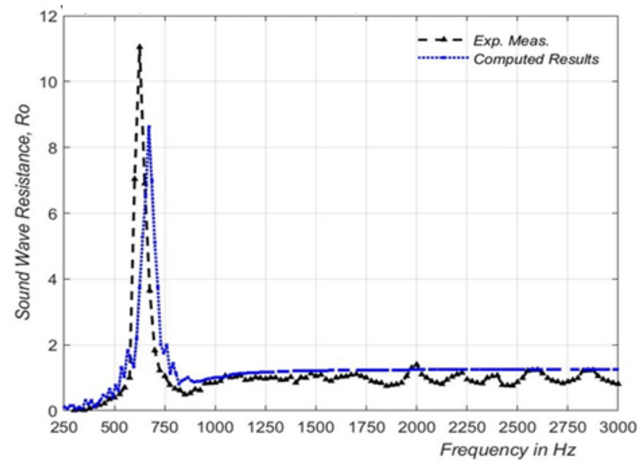
4 DISCUSSION

The computed results show that at very low frequencies namely, $f < f_C$, the matching of the conductor for a plane sound wave is very poor while it is very good at frequencies, $f_C \le f \le f_{1m}$. These two ranges of frequency are separated by the so-called, drop frequency f_C. At frequencies below f_C, the radiated acoustic power through the side open area is very low and the acoustics power reaches the conductor output is very strong. Therefore, the influence of the reflected acoustic energy at the conductor output is very significant. As a result, the dependence of $|r_i|$, $\mathcal{R}_o$ and χ_o on frequency in this range is not clear and sharp peaks occur. On the other hand, at frequencies equal to or above f_C, the radiated acoustic power through the side open area increases depending on the graduality factor (D_m/l) and length, l of the conductor. Accordingly, the acoustic power that reaches the conductor output becomes very small, and therefore the reflected acoustic energy there has little influence. As a result, the dependence of $|r_i|$, $\mathcal{R}_o$ and χ_o on frequency in this range is very poor and the peaks nearly disappear. In this frequency range (effective range "f_{eff}") which is within the range of inequality; $f_C \le f \le f_{1m}$. $|r_i|$ decreases towards a constant value of 0.1, whereas $\mathcal{R}_o$ becomes nearly constant and equal unity (i.e. equal ρc the characteristic impedance of a plane sound wave in air) and χ_o becomes nearly zero.

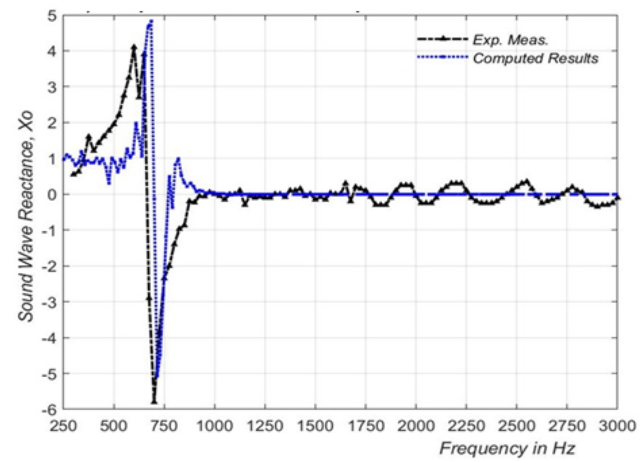
The design parameters such as length, size, and function of gradual variation of perforated area (D_m/l) can be optimized by the condition in the effective frequency range at which the maximum sound reflection coefficient $|r_i|$ at the conductor inlet does not exceed 0.1. An optimum design can be achieved with both minimum graduality (i.e. D_m optimum) and a minimum length of conductor. A better matching can be realized at the conductor inlet if the same total perforated area is divided into n times, which leads to better graduality of the variation of the perforated area. However, the drop frequency f_C is shifted to the higher frequency, Figure 8. A comparison between the computed $|r_i|$, $\mathcal{R}_o$ and χ_o with those measured by *Shenouda*, *A*., for a conductor of D_p=5.08 cm, l= 50 cm and graduality of the perforated area leads to D_m=6 cm gives good agreement for the value f_C and take the same trend of conductor exhibition for lower frequencies than f_C, Figure 8.



(a) Sound wave refl. Comp. between measured and computed



(b) Normalized input sound wave resistance Comp. between measured and computed



(c) Normalized input sound wave reactance Comp. between measured and computed

Figure 8: Comparison between measured and computed results for the conductor of D_p=5.08cm, l=0.5m, and D_m=6cm

5 CONCLUSION

The computed results came up with the following conclusions:

1- The frequency dependence of the computed $|r_i|$, $\mathcal{R}_o$ and χ_o for conductors of the same Dp and (D_m/l) but having different lengths l show that there is a sharp drop frequency f_C which separates between a sound reflection band $f \le f_C$ and a sound conduction band $f \ge f_C$. This drop frequency f_C is the same for all conductors in which the graduality factor D_m/l is kept constant.

2- The frequency dependence of the computed $|r_i|$, $\mathcal{R}_o$ and χ_o for conductors of the same length l and D_p but are treated with different graduality (D_m/l) which leads to different D_m showing an increase in f_C and improves the parameters $|r_i|$, $\mathcal{R}_o$ and χ_o (i.e. $|r_i| \to 0.1$, $\mathcal{R}_o \to 1$and $\chi_o \to 0$) by increasing D_m.

3- The frequency dependence of the computed $|r_i|$, for conductors of the same length, l and having different $D_p's$ and treated with different graduality show that f_C

decreases as $\boldsymbol{Dp}$ increases and the high-frequency limit f_{1m} decreases as D_p increases.

4- Optimum design is characterized by the values of the parameters, namely, $|r_i| \approx 0.1$, $\mathcal{R}_o \approx 1$and $\chi_o \approx 0$ in the effective range, $f_C \leq f \leq f_{1m}$ between the drop frequency f_C and the frequency f_{1m} at which the first high mode starts propagating down the conductor.

5- The conductor characteristic area A_C is a design parameter namely, ($A_C = S_p . l / 4 D_m$) and one can choose the suitable dimensions for the most official sound conductor according to the application and available dimensions.

6- It is very useful to design the conductor with multiple treatments with the smallest possible graduality (D_m/l). In this case, the matching impedance at the conductor inlet is improved and the increase of the total perforated areas allows more sound energy to be released through it and accordingly the sound reflected energy at the output becomes very small and has no influence.

7- Because very little energy is reflected at the entrance plane of the conductor in the effective frequency range, this type of conductor can be used to estimate the acoustic power radiated from ducted noise sources such as axial and radial ventilators.

Conflicts of Interest

The authors declare that they have no potential conflicts of interest with respect to the research and authorship

Funding

The author(s) received no financial support for the research, authorship, and/or publication of this article

REFERENCES

1. F. B. Shenoda, (1972) Acoustic termination for air duct" VDI-Gemeinshaftstagung Akustic and Schwingungen Technik. DAGA, Stuttgart, pp. 269-272.
2. F. B. Shenoda, (1982) A new silencer with good matching performance for air ducts and exhaust systems. Acustica, 50(2): 338-341.
3. F. B, Shenoda, R. N. Haroun and H. Silm, (1987) A study on a new silencer systemically lined with variable flow resistance. Acustica, 60(3): 94-101.
4. A. Shenouda, (1999) Study on a new acoustic conductor. M.Sc. Dissertation, Department of Electronics and Communication Eng., Cairo University, Egypt.
5. S. N. Rschviken, (1963) The theory of sound" Translated from Russian language by Blum, O. M. Translation edited by Doak, P. E. Pergemon press, Oxford, London, New York, Paris, pp. 207.
6. W. P.Mason, (1948), Electromechanical transducers and wave filter. D. Van Nostrand Co. pp. 28-31.
7. Lawrance, E. Kinder and Austin, R. Frey, Fundamental of acoustics. 2nd edition, John Wiley& Sons Inc., New York, London, Sydney, pp. 207.
8. Watson, G. N. (1944) A treatise on the theory of Bessel functions. MacMillan, New York.
9. The staff of the computation laboratory (1945) modified Hankel-functions of order one-third and their derivative. Voll. II Cambridge Mass. Harvard Uni. Press.