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An Optimized Newton-Raphson Algorithm for Approximating Internal Rate of Return

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ABSTRACT

The internal rate of return (IRR) is the most widely-used method in measuring the rate of return on investment (RROI), which helps investors decide whether an investment is viable or not. Iterative root-finding algorithms are the most efficient technique in calculating IRR, amongst which, the Newton-Raphson algorithm is the most popular and the fastest algorithm. However, when the primary unknown, which is provided by the user, is far from the actual root, the result of the algorithm, oftentimes, does not converge to the root. This problem is addressed by a midpoint-based Newton-Raphson algorithm. Nevertheless, said algorithm could further be improved in terms of proximity, speed, and accuracy. This study presents a novelty in estimating IRR using the centroid-based Newton-Raphson algorithm. The experimental results show that the presented algorithm is 32.76% faster than the midpoint approach. It also delivered a better average accuracy of 68.53% than the midpoint-based algorithm.

Keywords: root-finding algorithm, Newton-Raphson algorithm, IRR, convergence.

1. INTRODUCTION

The root-finding problem is one of the most relevant computational problems and it arises in a wide variety of practical applications in Physics, Chemistry, Biosciences, Engineering, et cetera [4]. It is a challenging task in scientific application problems to estimate the solution of non-linear equations [2], the function of IRR being one of them [17], as well as trial calculation, which may take numerous hours [18]. A more efficient way to estimate is by using a root-finding algorithm an iterative calculation scheme to approximate a single, isolated root of a function f(IRR), where the root IRR is a solution of the equation f(IRR) = 0 [12].

IRR is the most widely-used method in measuring the feasibility of a project or investment [4]–[6]. It is one of the tools that helps an intelligent enterprise in their decision making processes, which may help them in realizing certain goals [9]. However, IRR cannot be isolated in the equation and cannot be determined analytically, which led researchers to use iterative algorithms in estimating IRR [12], [17].

While there are many root-finding algorithms, such as bisection, false position, and secant, the Newton-Raphson algorithm is the most preferred algorithm due to its quick convergence, given a certain accuracy level [1], [3], [10], [16], by using the equation below [8].

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)} \qquad (1)$$

However, [19] stated that, though its convergence is very fast and the number of the digits doubles in every iteration, which helps us in reaching our tolerance of errors quickly, the guarantee of convergence is still ambiguous and there are instances when division by zero occurs, and it is a considerable disadvantage of Newton–Raphson method, and suggested that the initial guess must be chosen very close to the true root [6], [7], [11]. It also requires an initial guess value from the user, which gives a high probability of non-convergence when the user's guess is far from the true root. Insofar as IRR is concerned, the study of Pascual et al. [13] has addressed this issue by removing input of initial guess from the user and used the midpoint of cash flows as the initial IRR. Their study dramatically improved the convergence, speed, and accuracy of Newton-Raphson algorithm in determining IRR. However, the midpoint injected to the algorithm is static and the number of iterations, as well as the accuracy, can still be further enhanced using centroid.

Thus, this study proposes using the centroid of cash flows, as initial input, which further optimizes the newton-raphson algorithm in determining IRR. A comparison of the two algorithms based on proximity, speed, and accuracy is presented in this paper.

2. EXISTING NEWTON- RAPHSON ALGORITHM

Pascual et al. (2019) proposed a modified Newton-Raphson algorithm in approximating IRR which automatically generates the initial guess input by using the midpoint of time periods of cash flows equal to (((n-1)) ⁄ 2)+1. The modified equation is shown below [13].

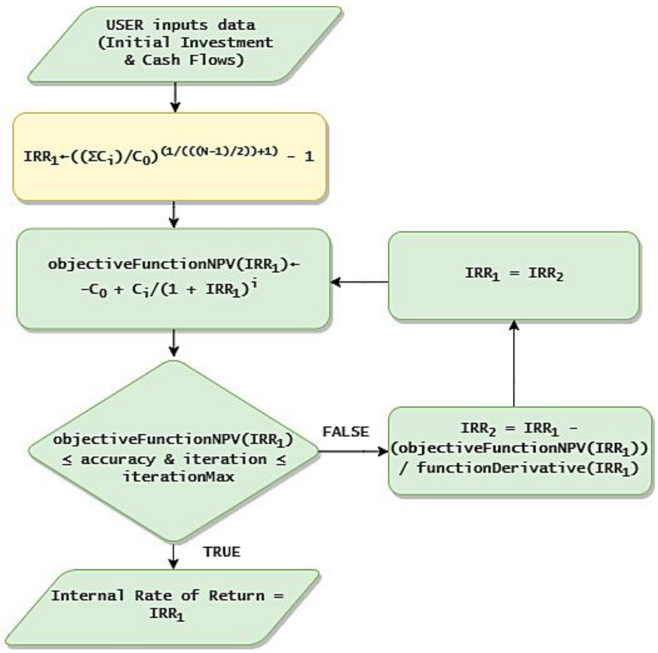

Figure 1: Midpoint-Based Newton-Raphson Algorithm[13]

$$IRR \leftarrow \frac{\sum_{i=1}^{n} C_i^{\frac{1}{((n-1)/2)+1}}}{|C_0|} - 1. \quad (2)$$

The process of the midpoint-based Newton Raphson algorithm is shown in Figure 1 above.

1. The user inputs the initial investment and cash flows;
2. The initial IRR is generated by using the midpoint of time periods of cash flows;
3. The net present value (NPV) of all cash flows discounted by IRR is then defined using function objectiveFunctionNPV(IRR_1);
4. If the condition is not met, a new IRR_1 is generated and the process goes back to step (3). Else, the final IRR is accepted.

This method is proven to improve the convergence, speed, and accuracy of the original algorithm, which gives excellent results that are close to the true solution without the need for a user's initial guessed values for the rate. This technique served as a reference point for the proposed improved algorithm of this study.

3. PROPOSED METHOD

The proposed method is presented in Figure 3, which is an enhancement of the midpoint-based Newton-Raphson process. The improvement consists of the replacement of the midpoint of cash flows of the midpoint-based IRR technique with the centroid of cash flows equal to ((C_1*x_1+ C_2*x_2+⋯+C_n*x_n)) ⁄ ((C_1+C_2+ ⋯ +C_n)).

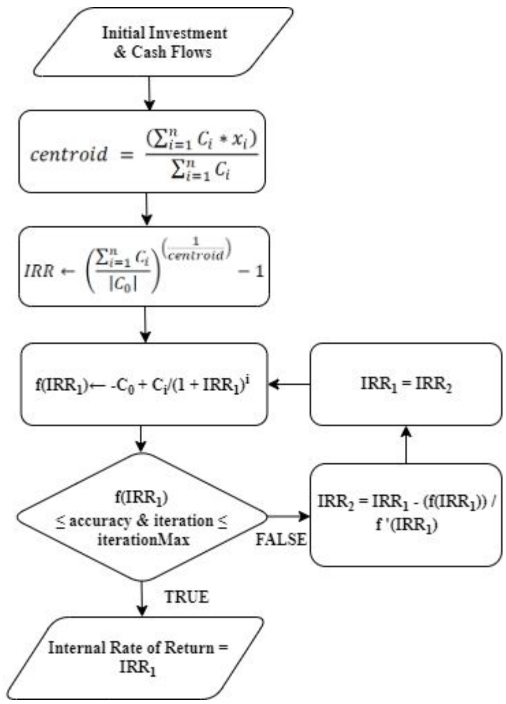

Figure 2: Proposed Enhanced Newton-Raphson Algorithm

A. *Centroid-Based Newton-Raphson Method*

1. Input initial investment and cash flows;
2. Calculate the centroid of time periods of cash flows with the equation

$$centroid \leftarrow \frac{(\sum_{i=1}^{n} C_i * x_i)}{\sum_{i=1}^{n} C_i} \quad (3)$$

where:

C_i = subsequent cash flows
x_i = distance of C_i from the time of first investment;

3. The initial IRR is generated using the equation:

$$IRR_1 \leftarrow \left(\frac{\sum_{i=1}^{n} C_i}{|C_0|}\right)^{\left(\frac{1}{centroid}\right)} - 1 \quad (4)$$

where:
C_0 = initial investment;

4. The non-linear function f(IRR_1) is then defined to get the net present value of all cash flows discounted by IRR;

5. If f(IRR_1) satisfies the condition, the last value of IRR is accepted, else a new IRR_1 is calculated, and the process goes back to step (4).

4. EXPERIMENTAL RESULTS

This section presents the simulation results of the proposed centroid-based Newton-Raphson algorithm by comparing it with the midpoint-based Newton-Raphson algorithm, in terms of proximity, speed, and accuracy.

4.1 Data Analysis

The test cases from the papers of Qiao and Zhang[15], Patrick and French[14], and Pascual et al. were used in the simulation of this study. Qiao and Zhang used the data of a hydraulic engineering project, which has the beginning investment in its four-year construction period of $70x10^6$ yuan each year. The annual benefit of the project is 45×10^6 yuan, and the yearly operation cost is 10×10^6 yuan. Then the IRR needs to be estimated for engineering economic analysis. The end of the fourth year is the discount base year. In other words, C_0, C_1, C_2 and C_3, respectively, equal -70 Million Yuan, C_4 equals 0, and each of C_5, C_6, … , C_{38} equals 35 Million Yuan.

Patrick and French made use of the following data: Initial investment, C_0=145; C_1=C_2=C_3=C_4=100 and C_5 = -275. While an installment plan for a Chevrolet Trailblazer was employed by Pascual et.al. It consists of cash flows C_0 = -₱1,438,888.00 and C_1,…,C_{60} = ₱32,269.00.

4.1 Numerical Analysis

As demonstrated in Table 1, the experiment shows that the centroid-based approach, an enhancement of the midpoint-based IRR technique, automatically computes an initial value which is more proximate to the real root IRR than that of the previous algorithm, thereby reducing the number of iterations and improving the accuracy of final IRR. This new algorithm simply needs the data consisting of the principal amount, also known as initial investment or outlay, C_0, and the subsequent cash flows, C_1, C_2, …, C_n. These data serve as the respective coefficients of the terms of the function to be differentiated by using Differential Calculus.

A. Result of Centroid Approach to the Automatic Initial Value Computation

Based on the simulation, the function of IRR generated by the centroid-based IRR technique for the three test cases, namely, Qiao-Zhang, Patrick-French, and Pascual et al., are 0.0000339345073427921, 0.00033580213556660, and 0.00000000093132257461547, respectively. These results are practically zero (0). Parenthetically, a function of IRR equal to zero (0) is a condition where the determined value of IRR is the exact value of IRR. Hence, the IRRs of the three test cases, which are 0.0946180588190897 (or 9.46180588190897%), 0.0878253038492212 (or 8.78253038492212%), and 0.01029926307 (or 1.029926307%) are almost exactly as f(IRR) is approximately zero.

This shows that the initial IRR of Qiao-Zhang test case using the centroid approach is 70.67% more proximate to the final IRR than the initial IRR of the midpoint-based IRR technique. The test case from Patrick & French also shows that the proposed method has a 71.31% initial IRR proximity to the final IRR than the initial IRR of the latter algorithm. On the other hand, the proposed technique, using the Pascual et al. test case, only garnered 23.37% proximity to the final IRR than the midpoint-based IRR technique. Still, this proves that the centroid-based IRR technique performed better than the midpoint-based IRR technique in terms of the proximity of the initial IRR to the actual root.

Table 1: Comparison Based on Proximity of Initial IRR

Method	Test case	IRR	f(IRR)
Midpoint-based IRR technique	Qiao-Zhang	0.0946180708797336000	2.5805889 E-08
	Patrick-French	0.08782827384976030	-4.5761992 E-06
	Pascual et al.	0.01029926307	9.313226 E-09
Centroid-based IRR technique	Qiao-Zhang	0.0946180588190897	3.39345073 E-05
	Patrick-French	0.0878253038492212	-3.35802136 E-04
	Pascual et al.	0.01029926307	9.3132257 E-10

B. Result on Speed and Accuracy

The enhanced algorithm's speed, as shown in Table 2 below, for the Qiao-Zhang test case is observed at only 3 iterations, compared to the midpoint approach of 6 iterations, or a reduction of fifty percent (50%) in the number of iterations. The same result holds for the test case Patrick- French.

Table 2: Comparison Based on Speed and Error Ratio

Method	Test case	Itera-tions	Runtime (in ms)	Error of Initial IRR
Midpoint-based IRR technique	Qiao-Zhang	6	115	0.49077797493
	Patrick-French	6	33	1.47273146112
	Pascual et. al	3	108	0.06585452372
Centroid-based IRR technique	Qiao-Zhang	3	83	0.14396269866
	Patrick-French	3	25	0.42251751744
	Pascual et. al	3	108	0.05046602346

By using different datasets in addition to the three test cases, the average iteration reduction percentage is thirty-two and seventy six-hundredth percent (32.76%). The average initial

IRR accuracy percentage is 68.53% better than that of the midpoint-based IRR technique.

5. CONCLUSION AND FUTURE WORK

The centroid-based Newton-Raphson algorithm, modifying the midpoint-based Newton-Raphson algorithm, by replacing its midpoint formula with the centroid formula, has performed better than the midpoint approach. The algorithm can automatically produce an initial value closer to a real root and dynamically takes into consideration the amounts of cash flows accounts for its low error, unlike the static midpoint technique, which does not respond to changes in cash flows. The centroid-based algorithm further ensures convergence as the initial IRR is closer to the real root by 55.11 % than the midpoint-based algorithm.

In future work, the proposed centroid-based approach is considered implementing in investment decision making and in determining realistic interest rates.

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