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The Degree Distance Index for Analysing the Structure of Social Networks

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ABSTRACT

Topological indices have been generally used for analyzing the structural properties of graphs and particularly for modeling the biological and the chemical properties of molecules in QSPR and QSAR studies. Recently, these invariants have been proposed as measures to analyze the whole structure of complex networks. In this paper, a new formula for the Degree Distance index using the $d_G^u(k)$ is obtained. Firstly, we discuss the use of this invariant to analyze the social networks. After that, we apply the new formula on some well-known simple connected graphs, such as Wheels, paths, stars and cycles.

Mathematics Subject Classification : 05C12, 05C05

Key words : Complex Networks, Degree Distance index, First Zagreb index, Molecular Descriptors, Topological indices, Social Networks, Wiener index,

I. INTRODUCTION

A particular attention is paid to graph invariants in recent research in mathematical chemistry. The main issue in this area of research is to calculate suitable numerical measures, and use them to quantify and understand the structural information of networks and graphs. Some such measures can serve as a standard for analyzing and exploring networks or for comparing and designing new networks. Over the last years, a large variety of measures or indices have been defined as descriptors for networks [12]. They are used as a universal language for describing the chemical structure of molecules, the chemical reaction networks, ecosystems, financial markets, the World Wide Web, and social networks [11, 19 - 20, 23 – 24].

Graph theory has an important effect on the development of the chemical, computer and biological sciences by using topological indices. A topological index is a numerical value mathematically derived from the graph structure and it does not depend on the labeling or pictorial representation of the graph. The topological indices of molecular graphs are widely used for establishing correlations between the structure of a molecular compound and its physico-chemical properties or biological activity. Moreover, they are considered as compulsory tools for analyzing some physico-chemical properties of molecules without performing any experiment [10]. Hundred of topological indices were defined by chemists and mathematicians after the first distance based topological index, called the Wiener index, which was proposed by H. Wiener (1947) for modeling the physical properties of Alcanes [9]. In 1971, Gutman & Trinajstić defined the Zagreb indices that are degree based topological indices [8] and were firstly proposed to be measures of branching of the Carbon-atom skeleton [7]. These indices and Randic index are the most used topological indices in chemical and mathematical literature so far. For more detailed discussions of the most well-known topological indices, we refer the interested reader to [1 – 6, 10] and the references therein.

In order to model real world situations from the basic sciences to engineering and social sciences, the graph theory uses graphs and there composed. A graph is an object G = (V, E) that consists of a nonempty set V and 2-element subsets of V namely E. The elements of V are called vertices and the elements of E are called edges. The distance d(u, v) between two vertices u and v is the length of the shortest path connecting these two vertices. The diameter D(G) of G denotes the maximum of the shortest path between all vertices of G . We denote by deg(v) the degree of the vertex v which is the number of edges incident to v.

2. THE DEGREE DISTANCE WITH $d_G^u(k)$

In this paper, we focus on the parameter $d_G(k)$, which is defined as the number of pairs of vertices of G that are at distance k, and the Wiener-Type invariant of a graph G that is mathematically defined as $W_\lambda(G) = \sum_{k\geq 1} d_G(k)k^\lambda$, where λ is a real number. Also, we use the so-called vertex transmission or simply the Wiener index of a vertex $v \in V(G)$, which equals to the sum of distances between a chosen vertex v and the other vertices in G, $w(v,G) = \sum_{u\in V(G)} d(u,v)$. Furthermore, we concentrate on the Degree Distance index that is defined as :

$DD(G) = \sum (deg(u) + deg(v))d(u,v) = \sum_{u\in V(G)} deg(u)w(u,G)$ [illegible]. The reader can see [13 - 18] and [21 - 22] for more details and results on the calculation of the Wiener index and the Degree Distance index, respectively.

Our work starts with the following proposed definitions.

Definition 2.1. Let G be a simple connected graph of diameter D(G) . For a given vertex v, $d_G^v(k)$ shows the number of pairs of vertices of G that are at distance k from v.

J. A. Rodrìguez-Velàzquez and all in [25] and S. Goel, and all in [26] were used the Wiener index and the Terminal Wiener index as effective measures to evaluate the efficiency of a network and to quantify the communication between persons in a social network. For us $d_G^v(k)$ plays an important role for analyzing a social network after representing his members by a set of vertices and its relations by a set of edges. To illustrate, each member v has exactly deg(v) neighbors that represent the $d_G^v(1)$, each friend of the vertex v has other neighbors which gives as a result $d_G^v(2)$ and we continue until the $D(G)^{th}$ level members. Following this technique, we can calculate the distance between each pair in a social network.

Lemma 2.2. Let G be a simple connected graph of diameter D(G). The vertex transmission of v can be rewritten as :

$$w(v,G) = 2(n-1) + \sum_{k=1}^{D(G)} (k-2)d_G^v(k). \quad (1)$$

Proof. By using the definition of $W_\lambda(G)$ and for λ = 1 . We have :

$$w(v,G) = \sum_{k\geq 1} k d_G^v(k) \quad (2)$$

For n vertices, we have :

$$n - 1 = \sum_{k\geq 1} d_G^v(k). \quad (3)$$

By using (3) in (2), we obtain :

$$w(v,G) = d_G^v(1) + 2(n-1-d_G^v(1) - \sum_{k\geq 3} d_G^v(k)) + \sum_{k\geq 3} k d_G^v(k).$$

Which yields to (1).

Definition 2.3. Let G be a simple connected graph of diameter D(G) . The Degree Distance dd(v, G) is defined as follows :

$$dd(v,G) = \sum_{k=1}^{D(G)} deg(v)(k-2)d_G^v(k). \quad (4)$$

Theorem 2.4. Let G be a simple connected undirected graph, with n vertices, m edges and D(G) ≥ 2 . Then :

$$DD(G) = 4m(n-1) + \sum_{v\in V(G)} dd(v,G). \quad (5)$$

Proof. Using the definition of DD(G) and **Lemma 2.2** :

$$\begin{aligned} DD(G) &= \sum_{v\in V(G)} deg(v)w(v,G), \\ &= \sum_{v\in V(G)} deg(v)(2(n-1) + \sum_{k=1}^{D(G)} (k-2)d_G^v(k)), \\ &= 4m(n-1) + \sum_{v\in V(G)} deg(v) \sum_{k=1}^{D(G)} (k-2)d_G^v(k). \end{aligned}$$

Such that $\sum_{v\in V(G)} deg(v) = 2m$ for any graph with n vertices and m edges.

Corollary 2.5. Let G be a graph with n vertices, m edges and D(G) = 2 . Then :

$$DD(G) = 4(n-1)m - M_1(G). \quad (6)$$

Proof. By using **Theorem 2.4** and **Lemma 2.2** for k ≤ 2 , where $M_1(G) = \sum_{v\in V(G)} (deg(v))^2$ is the seconde Zegrib index. □

In order to clarify the novel formula of DD(G), we present in the following section its application to analyze some well-known graphs like : Path, Cycle, Wheel and Star graphs.

3. APPLICATION

In this section, We apply the previous corollaries and theorems on some graphs with difierent diameters.

3.1. Graphs of diameter two (D(G) = 2)

3.1.1. The Star planar graph ($\mathcal{S}_n$)

Lemma 3.1. We denote by $\mathcal{S}_n$ a Star planar graph of n vertices and m = n − 1 edges. Then :

- $M_1(\mathcal{S}_n) = (n-1)n, \quad for \quad (n \geq 3),$
- $d^{v_1}_{\mathcal{S}_n}(1) = deg(v_1) = n-1,$
- $d^{v_i}_{\mathcal{S}_n}(1) = deg(v_i) = 1, \; for \; i \neq 1,$
- $d^{v_i}_{\mathcal{S}_n}(2) = n-2, \; for \; i \neq 1,$

Proof. By calculation.

Corollary 3.2. Let $\mathcal{S}_n$ be a Star planar graph with n ≥ 3 vertices. Then :

$$DD(\mathcal{S}_n) = (n-1)(3n-4), \qquad for \quad (n \geq 3) \qquad (7)$$

Proof. We simply apply **Theorem 2.4** with **Lemma 3.1 :**

$$\begin{aligned} DD(\mathcal{S}_n) &= 4m(n-1) + \sum_{v \in V(\mathcal{S}_n)} dd(v, \mathcal{S}_n) \\ &= 4(n-1)(n-1) - (n-1)^2 - (n-1) \end{aligned}$$

□

3.1.2. The Weel planar graph ($\mathcal{W}_n$)

Lemma 3.3. We denote by $\mathcal{W}_n$ a Wheel planar graph of n vertices and m = 2n - 2 edges (see Figure 1). Then :

- $M_1(\mathcal{W}_n) = n^2 + 7n - 8, \quad for \quad (n \geq 3),$
- $d^{v_1}_{\mathcal{W}_n}(1) = deg(v_1) = n-1,$
- $d^{v_i}_{\mathcal{W}_n}(1) = deg(v_i) = 3, \; for \; i \neq 1,$
- $d^{v_i}_{\mathcal{W}_n}(2) = n-4, \; for \; i \neq 1,$

Proof. By calculation. □

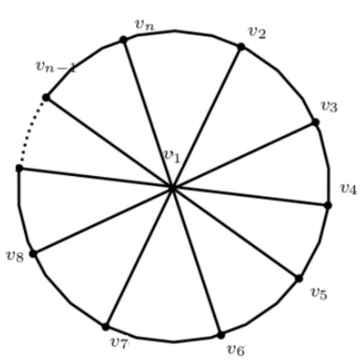

Figure 1 : The Wheel planar graph $\mathcal{W}_n$

Corollary 3.4. Let $\mathcal{W}_n$ be a Wheel planar graph with n ≥ 4 vertices. Then :

$$DD(W_n) = 7n^2 - 23n + 16, \qquad for \quad (n \geq 4) \qquad (8)$$

Proof. We apply **Theorem 2.4** and **Lemma 3.3** :

$$\begin{aligned} DD(\mathcal{W}_n) &= 4m(n-1) + \sum_{v \in V(\mathcal{W}_n)} dd(v, \mathcal{W}_n) \\ &= 4m(n-1) - (n - \mathcal{C}_n + 3(n-1)(-3) \end{aligned}$$

□

Lemma 3.5. The Cycle planar graph, noted $\mathcal{C}_n$, is a graph of n ≥ 2 vertices and m = n edges (see Figure 2). $\forall v_i \in V(\mathcal{C}_n)$:

- $d^{v_i}_{\mathcal{C}_n}(k) = \begin{cases} 2, & if \; n \; is \; even \; and \; 1 \leq k \leq \frac{n-2}{2} \\ 1, & if \; n \; is \; even \; and \; k = \frac{n}{2} \\ 2, & if \; n \; is \; odd \; and \; 1 \leq k \leq \frac{n-1}{2} \end{cases},$
- $deg(v_i) = 2$

Proof. By calculation. □

Theorem 3.6. Let $\mathcal{C}_n$ be a Cycle planar graph of n ≥ 2 vertices and m edges. Then :

$$DD(\mathcal{C}_n) = \begin{cases} \frac{1}{2}n^3, & if \; n \; is \; even \\ \frac{1}{2}n^3 - \frac{1}{2}n, & if \; n \; is \; odd \end{cases} \qquad (9)$$

Proof. We use **Theorem 2.4** and **Lemma 3.5** :

$$\begin{aligned} DD(\mathcal{C}_n) &= 4m(n-1) + \sum_{v \in V(\mathcal{C}_n)} dd(v, \mathcal{C}_n) \\ &= 4m(n-1) + 2n(\sum_{k=1}^{\frac{n}{2}-1}(k-2)2 + (\frac{n}{2}-2)1) \\ &= n(5n-8) + 2n(\frac{n(n-2)}{4} - 2(n-2)) \end{aligned}$$

The seem if n is odd. □

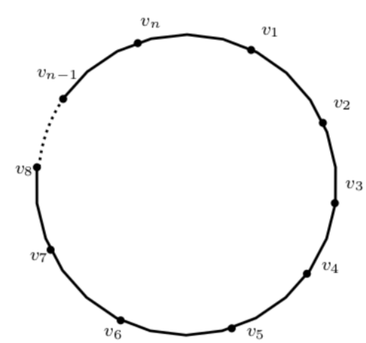

Figure 2 : The Cycle planar graph $\mathcal{C}_n$

3.2.2. The Path planar graph ($\mathcal{P}_n$)

Lemma 3.7. The Path planar graph, noted $\mathcal{P}_n$, is a graph of n ≥ 2 vertices and m = n − 1 edges (see Figure 3). $\forall v_i \in V(\mathcal{P}_n)$:

- $d_{\mathcal{P}_n}^{v_i}(k) = d_{\mathcal{P}_n}^{v_{n-(i-1)}}(k) = \begin{cases} 1, & \text{if } n \text{ is even}, i \leq k \leq n-i \text{ and } 1 \leq i \leq \frac{n}{2} \\ 2, & \text{if } n \text{ is even}, i > k \geq 1 \text{ and } 1 \leq i \leq \frac{n}{2} \\ 1, & \text{if } n \text{ is odd}, i \leq k \leq n-i \text{ and } 1 \leq i \leq \frac{n-1}{2} \\ 2, & \text{if } n \text{ is odd}, i > k \geq 1 \text{ and } 1 \leq i \leq \frac{n-1}{2} \\ 2, & \text{if } n \text{ is odd}, 1 \leq k \leq \frac{n-1}{2} \text{ and } i = \frac{n+1}{2} \end{cases}$,
- $deg(v_1) = deg(v_n) = 1$,
- $deg(v_i) = 2$, if $i \neq \{1, n\}$.

Proof. By calculation. □

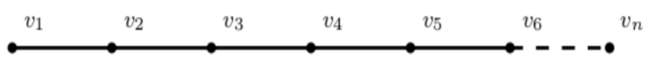

Figure 3 : The Path planar graph $\mathcal{P}_n$

Theorem 3.8. Let $\mathcal{P}_n$ be a Path planar graph of n ≥ 2 vertices and m edges. Then :

$$DD(\mathcal{P}_n) = \frac{1}{3}n(n-1)(2n-1). \quad (10)$$

Proof. We use **Theorem 2.4** and **Lemma 3.7** :

$$DD(\mathcal{P}_n) = 4m(n-1) + \sum_{v_i \in V(G)} dd(v_i, G)$$

$$= 4m(n-1) + \Big[\sum_{i=1}^{\frac{n}{2}}(\sum_{k=i}^{n-i} deg(v_i)(k-2)d_{\mathcal{P}_n}^{v_i}(k) + \sum_{k=1}^{i-1} deg(v_i)(k-2)d_{\mathcal{P}_n}^{v_i}(k))2\Big](\frac{(-1)^n+1}{2})$$

$$+\Big[\sum_{i=1}^{\frac{n-1}{2}}(\sum_{k=i}^{n-i} deg(v_i)(k-2)d_{\mathcal{P}_n}^{v_i}(k) + \sum_{k=1}^{i-1} deg(v_i)(k-2)d_{\mathcal{P}_n}^{v_i}(k))2 + \sum_{k=1}^{\frac{n-1}{2}} deg(v_{\frac{n+1}{2}})(k-2)d_{\mathcal{P}_n}^{v_{\frac{n+1}{2}}}(k)\Big]$$

$$(\frac{-(-1)^n+1}{2})$$

$$= 4m(n-1) + ((-1)^n+1)\Big[\sum_{k=1}^{n-1} deg(v_1)(k-2)d_{\mathcal{P}_n}^{v_1}(k) + \sum_{k=1}^{1-1} deg(v_1)(k-2)d_{\mathcal{P}_n}^{v_1}(k) + \sum_{i=2}^{\frac{n}{2}}(\sum_{k=i}^{n-i} deg(v_i)(k$$

$$-2)d_{\mathcal{P}_n}^{v_i}(k) + \sum_{k=1}^{i-1} deg(v_i)(k-2)d_{\mathcal{P}_n}^{v_i}(k))\Big] + (\frac{-(-1)^n+1}{2})\Big[2(\sum_{k=1}^{n-1} deg(v_1)(k-2)d_{\mathcal{P}_n}^{v_1}(k) + \sum_{k=1}^{1-1} deg(v_1)$$

$$(k-2)d_{\mathcal{P}_n}^{v_1}(k)) + 2\sum_{i=2}^{\frac{n-1}{2}}(\sum_{k=i}^{n-i} deg(v_i)(k-2)d_{\mathcal{P}_n}^{v_i}(k) + \sum_{k=1}^{i-1} deg(v_i)(k-2)d_{\mathcal{P}_n}^{v_i}(k)) + \sum_{k=1}^{\frac{n-1}{2}} deg(v_{\frac{n+1}{2}})(k$$

$$-2)d_{\mathcal{P}_n}^{v_{\frac{n+1}{2}}}(k)\Big]$$

$$= 4m(n-1) + ((-1)^n+1)\Big[\sum_{k=1}^{n-1} 1(k-2)1 + \sum_{i=2}^{\frac{n}{2}}(\sum_{k=i}^{n-i} 2(k-2)1 + \sum_{k=1}^{i-1} 2(k-2)2)\Big] + (\frac{-(-1)^n+1}{2})$$

$$\Big[2(\sum_{k=1}^{n-1} 1(k-2)1) + 2\sum_{i=2}^{\frac{n-1}{2}}(\sum_{k=i}^{n-i} 2(k-2)1 + \sum_{k=1}^{i-1} 2(k-2)2) + \sum_{k=1}^{\frac{n-1}{2}} 2(k-2)2\Big]$$

$$= 4(n-1)^2 + ((-1)^n+1)\Big[\frac{(n-1)(n-4)}{2} + \sum_{i=2}^{\frac{n}{2}}((n-2i+1)(n-4) + 2(i-4)(i-1))\Big]$$

$$+(\frac{-(-1)^n+1}{2})\Big[(n-1)(n-4) + 2\sum_{i=2}^{\frac{n-1}{2}}((n-2i+1)(n-4) + 2(i-4)(i-1)) + \frac{(n-1)(n-7)}{2}\Big]$$

$$= 4(n-1)^2 + ((-1)^n+1)\Big[\frac{(n-1)(n-4)}{2} + \frac{n}{2}(n-2)((n+1)(n-4)+8) - \frac{2(n+1)(n(n+2)-8)}{8}$$

$$+\frac{2}{24}(n(n+2)(n+1)-24)\Big] + (\frac{-(-1)^n+1}{2})\Big[(n-1)(n-4) + 2\sum_{i=2}^{\frac{n-1}{2}}((n-2i+1)(n-4) + 2(i-4)(i-1))$$

$$+\frac{(n-1)(n-7)}{2}\Big]4(n-1)^2 + ((-1)^n+1)\Big[\frac{1}{3}n^3 - 5n^2\frac{25}{6} - 2\Big] + (\frac{-(-1)^n+1}{2})\Big[\frac{2}{3}n^3 - 5n^2 + \frac{25}{3} - 4\Big]$$

□

4. CONCLUSION

In this paper we have mentioned some theoretical results about the Degree Distance index DD(G) relating to the $d_G^u(k)$ and the diameter of a graph G . We have also discussed the use of $d_G^u(k)$ to analyze social networks and particularly for accounting the distances between the social networks' members. We finished the paper with an application on some graphs like Wheel planar graph $\mathcal{W}_n$, Star planar graph $\mathcal{S}_n$, Cycle planar graph $\mathcal{C}_n$, and the Path planar graph $\mathcal{P}_n$ using the proposed theorem.

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