



Modeling in the Frequency Domain of Control Systems

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ABSTRACT

Modeling in the frequency domain for control systems are made up of many sub-topics such as modeling of electric/electronic circuits, modeling of translational mechanical systems, modeling of rotational mechanical systems, modeling of rotational mechanical systems with gears, modeling of electromechanical systems, nonlinear systems, linearization techniques, concepts of poles and zeros of a system, and the pole-map zero. These topics, as well as other topics related to modeling in the frequency domain, will be discussed in this paper.

Key words: Frequency Domain, Control Systems, Mechanical Systems, Rotational Systems.

1. INTRODUCTION

Frequency Domain analysis studies signal with respect to frequency, rather than with respect to time. Information being studied is plotted having frequency on the x-axis and the amplitude on the y-axis. The said plot displays how the energy of the signal is distributed as a function of frequency.

Time and frequency domain representations are both present in electrical signals. Both voltage and current measurements are expressed as functions of time in the time domain. Furthermore, digital information is usually displayed by a voltage as a function of time as well as the signals being measured on an oscilloscope.

On the other hand, frequency-domain representations are commonly used in linear system analysis. Most interference in and source of signals are defined in the time domain, however, the frequency domains offer more convenient system behavior and signal transformations. The use of a linear frequency response function in linear frequency domain analysis allows the evaluation to have a fundamental basis in learning system configurations like the control bandwidth and sensitivity function [1].

When performing a measurement of the frequency of a signal, it is also the analysis of the signal in the frequency domain. Some oscilloscopes are used in the analysis of periodic signals

in the frequency domain, however, a more effective tool for this analysis is a spectrum analyzer. A spectrum analyzer displays 2D graph of the power of the signal at the x-axis and its frequency at the y-axis. The said graph is referred to as the frequency spectrum of a signal since it indicates how strong a signal is at all values of frequencies. The simplest and most common example in the frequency domain is the sine wave, which is a perfectly periodic signal. The frequency spectrum of a 100-Hz sine wave is composed of only one frequency, and so the frequency spectrum is illustrated below.

Several research studies which involve identification procedures are developed in the frequency domain. According to one study, methods which are based on the frequency domain use concepts such as storage modulus and loss factor. Sometimes, equations presented in the time domain must be transformed to the frequency domain, the common tool for this transformation is known as the Fourier transform [2].

The approach of the frequency domain for linear systems is where the control, processing, and communication dependent on. This approach has been popular in the fields of science and engineering. Key principles connected to frequency analysis is the Frequency Response Function (FRF), which is the basis of the concepts such as Nyquist stability, Bode diagrams, filter designs, and many more [3].

2. LITERATURE REVIEW

In one particular study, the grey box system identification method was pertained to address the issue of fractional-order model identification from existing frequency domain data. Due to linked expenses, this was the only possible and feasible method or approach for industrial cases of system identification wherein the capability of conducting experiments involving time domain were limited. This study evaluates the potential of the said approach and examines the model parameter estimation method depending on the existing frequency domain data. For this study, there are three optimization algorithms used: Trust Region Reflective (TRR) nonlinear least-squares estimation method, Levenberg-Marquardt (LM) nonlinear least-squares estimation method, and the Nelder-Mead (NM) direct search simplex method. This research suggested a grey box model identification method for fractional-order systems relying on

the frequency domain data available. The said identification approach was verified using an academic sample where a lot of optimization algorithms under the identification process were put to test. Based on the scope of this study, the Trust Region Reflective (TRR) method has obtained the most exemplary results among the different optimization algorithms. On the other hand, the Nelder-Mead simplex method had shown almost the same quality of performance, therefore, it could generally be utilized to solve the problems where bounds are either hard to estimate or unknown. For future research work, the researchers recommend the assessment of the current approach based on real-life industrial information and re-evaluation of the Levenberg-Marquardt algorithm using the approach of coordinate transformation to permit the application of search parameter bounds [4].

In another study, a frequency domain with a three-phase transformer model was implemented using nodal method. The model was able to show and demonstrate effectiveness and ease through its involvement on basic distribution networks. Also taken into account was the angular displacement parallel to winding connections, which verified and proved the versatility of the model. Furthermore, this paper also described the function of frequency-domain transformer modeling processes for electromagnetic transient analysis of networks. The said was dependent on the utilization of admittance matrices which can be incorporated with bigger networks later on. For the actual experiments, the researchers have used the model to compute the transient over voltages which are results of switching and the Numerical Laplace Transform (NLT) to obtain the time domain response. According to the researchers, the model is easy to modify in order to try and consider winding connections and the appropriate angle displacement inherent to them. The accuracy and effectiveness of the model were evaluated through having the results compared with EMTP-RV or Electromagnetic Transients Program [5].

The study by Gocheva et.al. focused on proposing a novel approach with regard to using generalized nets in the modeling of electronic circuits. Generalized nets are utilized in choosing components, connecting, modifying, and simulating electronic circuits. The proposed matrix calculus was done with respect to time discretization. It takes into account the performance capacity of generalized nets as an aid in parallel processing. Systematization of electronic components is one of the most essential elements and considerations of the proposed approach. Furthermore, it provides opportunities to manage complex electronic circuits using basic methods like matrix equations instead of solving differential equations [6].

3. THEORETICAL CONSIDERATION

One of the important concepts in modeling in the frequency domain in control systems is electrical circuits. One of the

most basic, if not the most basic concept that is known in electrical circuits is the Resistance Inductance Capacitance circuit or the RLC circuit. The RLC circuit is a big help in understanding behaviors that are evident in control systems. RLC circuits may be considered as series or parallel. The main difference of these two circuits are obvious from the names itself, series RLC circuits have a series connection for the resistors, inductors, and capacitors, while for parallel RLC circuits, resistors, inductors, and capacitors are connected in parallel with respect to one another [7].

The next topic for modeling in the frequency domain is about translational mechanical systems. One of the first things to know about translational mechanical systems is that it moves along a straight line and that it is made up of three basic elements, which are mass, spring, and dashpot or damper. When a force is applied to a given translational mechanical system, an opposing force is present which is caused by the mass, elasticity, and the friction which is present in a given system. It is also known that since the force is applied and the force which is present in the system which may be considered as opposing forces are of opposite directions, the algebraic sum of the forces in the system and is applied to the system is zero.

The first of the three is mass (figure 1), it may be defined as the body's property which is able to store kinetic energy. It was also mentioned in the previous part that when a force is applied to a body that has a mass, it will be opposed by a force which is due to the mass of the system. It should also be noted that this force that opposes is proportional with respect to the acceleration of a given body.

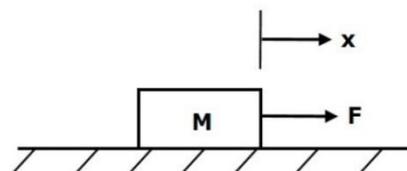


Figure 1: Mass Force Diagram

The next is spring (figure 2), it may be defined as an element which is able to store potential energy. For a spring, when a force is applied on a spring, it will be opposed by a force which is caused by the elasticity of a spring. This opposing force is proportional with respect to the displacement of the spring.

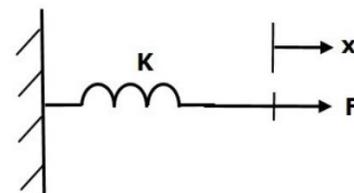


Figure 2: Spring Diagram

Lastly, for translational mechanical systems is the dashpot, it may be explained by having a force applied to a dashpot and the opposing force is caused by the friction of the dashpot (figure 3). This opposing force is proportional with respect to the velocity of the body.

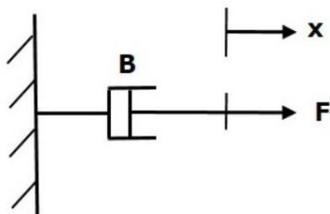


Figure 3: Damper Diagram

Next for modeling in the frequency domain is rotational mechanical systems (figure 4), these systems move in a fixed axis and are made up of three elements, which are the moment of inertia, torsional spring, and dashpot. It should also be noted that the application of torque to a rotational mechanical system is opposed by opposing torque which is caused by the moment of inertia, elasticity, and friction of the system. Similar to the algebraic sum in translational mechanical systems, the torque applied in rotational mechanical systems and the opposing torque are in opposing directions, thus the torques acting on the system is zero.

The first in rotational mechanical systems is the moment of inertia. It is similar to mass in translational mechanical systems in a sense that it is the one which stores kinetic energy. It should be noted that when a torque is applied on a body with a moment of inertia, an opposing torque is caused by the moment of inertia in the body itself and that opposing torque is proportional with respect to the angular acceleration of the body.

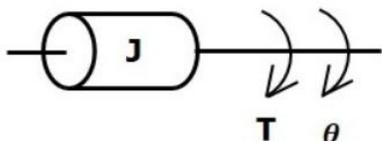


Figure 4: Inertia Diagram

The next is torsional spring (figure 5), again similar to the spring in translational mechanical systems, it is the one which stores the potential energy. When a torque is applied to a torsional spring, an opposing torque which is caused by the elasticity of torsional spring opposes it giving the torsional spring a relationship of being proportional to the angular displacement of the torsional spring.

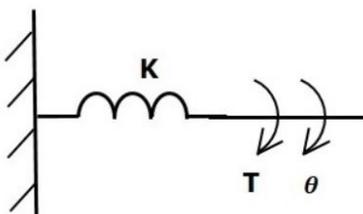


Figure 5: Rotation of K Diagram

Lastly, dashpot can be described as having a torque applied to it with an opposing torque which is caused by the rotational friction (figure 6) of the dashpot itself. This is also proportional with respect to the velocity of the body [8].

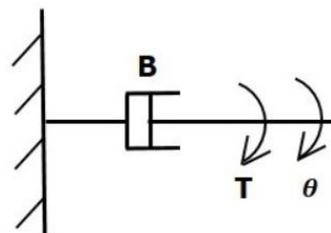


Figure 6: Rotation of B Diagram

The next topic which is under modeling in the frequency domain is also rotational mechanical systems, but this time with gears. If one thinks about it, it is not common that we see or know of mechanical systems that have no gear trains in them. It should be noted that gears are responsible in allowing the trade-off of speed for torque. For practice problems and for the basics of gears, it is assumed that gears fit perfectly, but this is not what really happens since gears exhibit backlash, this means that one of the gears will move through a small angle at a time before it even meets or connects with the teeth of the other gears. The image below shows two gears with their teeth connected to one another [9].

This next topic is about modeling of electromechanical systems, this system is made up of an electrical subsystem as well as a mechanical subsystem, thus the name electromechanical system. Some previous concepts like mass, elasticity, and damping are also related to electromechanical systems. There are cases for some devices like motors and speakers where the mass is driven with the use of a force that is generated by the electrical subsystem and there are also times when in some devices like microphones, the motion of the mass is the one which generates either a voltage or current in the electrical subsystem. One of the most important concepts in electromechanical systems is an electric motor. There are various types of electric motors, however, the two main categories that are considered in most systems are direct current motors (DC) and alternating current motors (AC) and under the DC motor category, the two considered are armature-controlled motor and field-controlled motor [10].

The last topic to be discussed in this part is the concept of poles and zeros. Poles and zeros are used in analyzing the performance of a system as well as to check the stability of it. It may also be defined as it is the one responsible for controlling the system whether it will work or not. It should be noted that most of the time, the numbers of poles and zeros are equal with one another in a given system, but there are also cases when the number of poles is greater than the number of zeros.

Poles may be defined as the roots of the denominator of a given transfer function. In the equation shown below, poles are the roots of $D(s)$ and may be evaluated by taking $D(s) = 0$ and s are solved. In obtaining the response for a system, the location of poles is analyzed with the real and imaginary value

parts of each of the poles. The real part is the one which gives the exponential, while the imaginary part is the one which gives the sinusoidal values.

$$H(s) = \frac{N(s)}{D(s)}$$

Zeros may be defined as the roots of the nominator of a transfer function. In the same equation that was given for poles, the zeros may be obtained through taking $N(s) = 0$ and s are solved. As mentioned earlier, zeros are less or equal to the number of poles. Zeros also show that the output at those given frequencies are zero [11].

When poles and zeros have been found, plotting them is possible. Poles and zeros are plotted in the S-Plane (figure 7), it is a complex plane that contains a real and an imaginary axis which refers to the complex-valued variable z . When mapping poles in the plane, it is denoted by "x", while an "o" for zeros. The figure below shows a sample of plotting the poles and zeros [12].

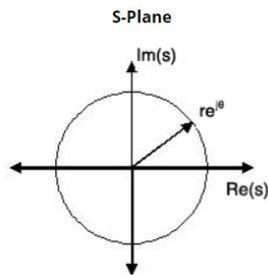


Figure 7: The S plane

4. DESIGN CONSIDERATION

The series RLC circuit (figure 8) shown below can be used to analyze and understand the performance of the control system [13].

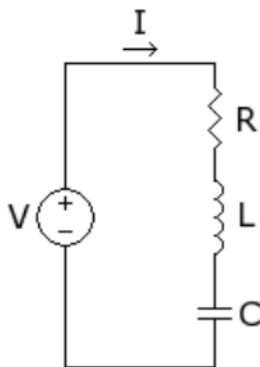


Figure 8: RLC Circuit

From the figure, the input of the system is given by

$$v_i(t) = Ri(t) + L \frac{di(t)}{dt} + \frac{1}{C} \int_0^t i(\tau) d\tau.$$

The output equation can then be defined as

$$v_o(t) = \frac{1}{C} \int_0^t i(\tau) d\tau.$$

and in order to obtain the transfer function, it is required to perform a Laplace transform which then yields

$$V_o(s) = \frac{1}{Cs} I(s).$$

Shown below (figure 9) is the block diagram for the RLC series circuit.

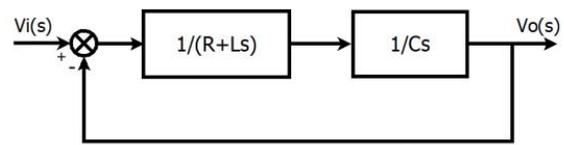


Figure 9: RLC Series Circuit

For the parallel RLC circuit (figure 10), the equation is shown by

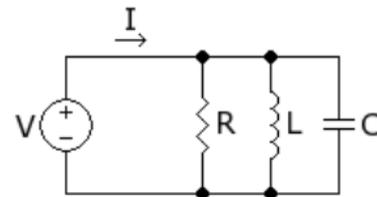


Figure 10: RLC Parallel Circuit

$$i(t) = \frac{v_R(t)}{R} + \frac{1}{L} \int_0^t v_L(\tau) d\tau + C \frac{dv_C(t)}{dt}.$$

and the output can be represented through the capacitor and when combined will yield

$$v_o(t) = v_R(t) = v_L(t) = v_C(t),$$

$$i(t) = \frac{v_o(t)}{R} + \frac{1}{L} \int_0^t v_o(\tau) d\tau + C \frac{dv_o(t)}{dt}.$$

The Laplace transform is then given by [14]

$$I(s) = \frac{V_o(s)}{R} + \frac{1}{Ls} V_o(s) + Cs V_o(s).$$

and the resulting transfer function will be

$$\frac{V_o(s)}{I(s)} = \frac{RLs}{RLCs^2 + Ls + R}.$$

The block diagram for the parallel RLC circuit is seen below (figure 11) [14].

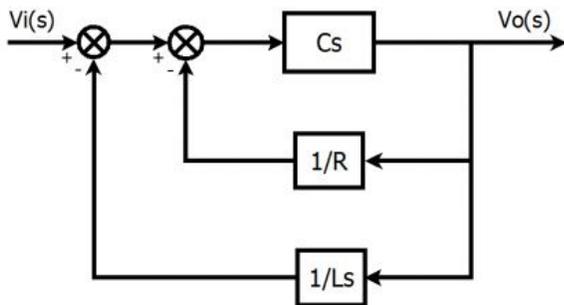


Figure 11: Parallel RLC Block Diagram

5. CONCLUSION

Modeling in the frequency domain can help in analyzing the behavior of a certain system. For example, in a study entitled Grey Box Identification of Fractional-order System Models from Frequency Domain Data by Aleksei Tepljakov, Eduard Petlenkov, and Juri Belikov in 2018, it was used in presenting and processing the data using different algorithms. The System can be programmed using the database pattern of [15,16]. The researches from [17,18,19] can be used as a pattern to make the program codes. Using the frequency domain data, is the most feasible approach for the type of research that they are conducting. For the researchers, it was considerably more economical compared to other approaches. The same case can be said for the other research. Modeling in the Frequency Domain not only provides a more detailed approach in understanding the system but also proposes a mathematical approach to carefully interpret the system.

REFERENCES

- [1] Y. Yoon, Z. Sun, and H. Du, "Inverse modeling approach for parametric frequency domain analysis of an electrohydraulic system," *Mechanical Systems and Signal Processing*, Vol. 121, pp. 412-425, 2019. <https://doi.org/10.1016/j.ymsp.2018.11.041>
- [2] S. Zhang, H. Wang, J. Gao, and C. Xing, "Frequency domain point cloud registration based on the Fourier transform," *Journal of Visual Communication and Image Representation*, Vol. 61, pp. 170-177, 2019. <https://doi.org/10.1016/j.jvcir.2019.03.005>
- [3] R. Bayma, Y. Zhu, and Z. Lang, "The analysis of nonlinear systems in the frequency domain using nonlinear output frequency response functions," *Automatica*, Vol. 94, pp. 452-457, 2018. <https://doi.org/10.1016/j.automatica.2018.04.030>
- [4] A. Tepljakov, E. Petlenkov and J. Belikov, "Grey Box Identification of Fractional-order System Models from Frequency Domain Data," 2018 41st International Conference on Telecommunications and Signal Processing (TSP), pp. 1-4, 2018. <https://doi.org/10.1109/TSP.2018.8441247>
- [5] G. Bilal, J. M. Villanueva-Ramirez, and P. Gomez, "Frequency domain transformer model for electromagnetic transient analysis of networks," 2017 North American Power Symposium (NAPS), pp. 1-5, 2017. <https://doi.org/10.1109/NAPS.2017.8107183>
- [6] P. V. Gocheva, N. L. Hinov and V. P. Gochev, "Modeling of Electronic Circuits with Generalized Nets," 2018 IX National Conference with International Participation (ELECTRONICA), pp. 1-4, 2018. <https://doi.org/10.1109/ELECTRONICA.2018.8439168>
- [7] P. D. Love, J. Zhou, and J. Matthews, "Project controls for electrical, instrumentation and control systems: Enabling role of digital system information modelling," *Automation in Construction*, Vol. 103, pp. 202-212, 2019. <https://doi.org/10.1016/j.autcon.2019.03.010>
- [8] K.A. Lazopoulos, D. Karaoulanis, and A.K. Lazopoulos, "On fractional modelling of viscoelastic mechanical systems," *Mechanics Research Communications*, Vol. 98, pp. 54-56, 2019. <https://doi.org/10.1016/j.mechrescom.2017.08.006>
- [9] W. Liang, X. Yang, and J. Yao, "Rotation effects of force transducer on the output of the build-up system," *Measurement*, Vol. 138, pp. 659-671, 2019. <https://doi.org/10.1016/j.measurement.2019.01.071>
- [10] A. Kefal, C. Maruccio, G. Quaranta, and E. Oterkus, "Modelling and parameter identification of electromechanical systems for energy harvesting and sensing," *Mechanical Systems and Signal Processing*, Vol. 121, pp. 890-912, 2019. <https://doi.org/10.1016/j.ymsp.2018.10.042>
- [11] M. Dorosti, R.H.B. Fey, M.F. Heertjes, M.M.J. van de Wal, and H. Nijmeijer, "Iterative pole-zero finite element model updating using generic parameters," *Mechatronics*, Vol. 55, pp. 180-193, 2018. <https://doi.org/10.1016/j.mechatronics.2018.06.012>
- [12] G. Blanco, "Poles of the complex zeta function of a plane curve," *Advances in Mathematics*, Vol. 350, pp. 396-439, 2019. <https://doi.org/10.1016/j.aim.2019.04.048>
- [13] A. Yazdani, and R. Iravani, "Voltage-Sourced Converters in Power Systems-Modeling," *Control and Applications*, 2010. <https://doi.org/10.1002/9780470551578>
- [14] A. Ramirez, M. Abdel-Rahaman, and T. Noda, "Frequency-domain modeling of time-periodic switched electrical networks: A review," *Ain Shams Engineering Journal*, 2017. <https://doi.org/10.1016/j.asej.2017.07.002>
- [15] A. Africa, C. Alcantara, M. Lagula, A. Latina Jr. and C. Te, "Mobile phone graphical user interface (GUI) for appliance remote control: An SMS-Based electronic appliance monitoring and control system." *International Journal of Advanced Trends in Computer Science and Engineering (IJATCSE)*, Vol. 8, No. 3, pp. 487-494, 2019. <https://doi.org/10.30534/ijatcse/2019/23832019>
- [16] A. Africa, and C. Uy, "Development of a cost-efficient waste bin management system with mobile monitoring

and tracking.” International Journal of Advanced Trends in Computer Science and Engineering (IJATCSE). Vol. 8, No. 2, pp. 319-327, 2019.

<https://doi.org/10.30534/ijatcse/2019/35822019>

- [17] A. Africa, “A Rough Set Based Solar Powered Flood Water Purification System with a Fuzzy Logic Model.” ARPN Journal of Engineering and Applied Sciences. Vol. 12, No. 3, pp.638-647, 2017.
- [18] A. Africa and J. Velasco, “Development of a Urine Strip Analyzer using Artificial Neural Network using an Android Phone.” ARPN Journal of Engineering and Applied Sciences. Vol. 12, No. 6, pp. 1706-1712, 2017.
- [19] A. Africa, S. Bautista, F. Lardizabal, J. Patron, and A. Santos, “Minimizing Passenger Congestion in Train Stations through Radio Frequency Identification (RFID) coupled with Database Monitoring System.” ARPN Journal of Engineering and Applied Sciences. Vol. 12, No. 9, pp. 2863-2869, 2017.