



Time Response Analysis of Control Systems

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ABSTRACT

In time response analysis, there are many various sub-topics that are included. Some of these topics which will be discussed in this research paper are the different types of test signals, transient analysis and steady-state analysis concepts, first-order transient response analysis, second-order transient response analysis, higher-order LTI system transient response analysis, and under high order LTI system transient response analysis are the effects of adding poles and zeros and non-minimum phase for zeros, and lastly under systems is the effect of nonlinearities.

Key words: Control Systems, Time Response, System Analysis, LTI Systems.

1. INTRODUCTION

Several industrial methods are composed of operations and system components having different purposes and connections with other components. Process control applications including real-time optimizations utilize nonlinear steady-state process modeling to calculate or estimate the optimal set point at steady-state operation [1].

According to one study, the designs in the time domain can decrease the calculations compared to the designs in the frequency domain. Time-domain designs are significant to applications which need short delays. Example of these is real-audio communications. Time-domain processes bypass the edge effects among consecutive snapshots of input signals which are usually implemented in the frequency domain. The use of time-domain filters is mathematically more effective than its corresponding frequency-domain filters in some cases. This occurs when there is enough number of short delays [2].

In the topic of control systems, predicting and controlling unmeasurable performance variables in huge, complex scale systems is essential. One solution to this is having a low-order standard plant model. In one study, an iterative method is used to modify the Finite Element (FE) model using poles and zeros. This method corresponds to poles and zeros in sequence in an iterative way. Damping systems have an effect on the imaginary parts of the poles and zeros [3].

2. LITERATURE REVIEW

In one study, a method for modeling production systems is proposed and presented. Fundamental principles of production processes are suggested by PSM or Production System Modeling. Several research studies for PSM concentrate on the most basic structure in production systems which are serial production lines. Frequently, decomposition methods are used in examining more complicated production lines. However, the said methods have only the capability of managing steady-state situations. Existing closed-form transient modeling are not directly applicable to those production systems the contain rework loops. Alongside the modeling method, a so-called “Self-View” method is proposed for determining the transient and steady-state results. PSM transient analysis is extended in this study to more complex production lines. Furthermore, this study addresses the issues of limitations of machine performance and buffer capacity rework loops. With these, the number of iterations, settling time, and other transient performance parameters are evaluated. Several processes are also conducted in this research such as studies insensitivity of blockage, production rate, starvation, and work-in-process on rework rate. As a conclusion, the presented “Self-View” method used in this research study demonstrates the potential for the modeling of other complex production systems [4].

Electric vehicles are projected to become dominant modes of transportation in the future. Moreover, these are assumed to serve another part in power systems with respect to load balancing and electrical storage. One paper considers this additional role of electric vehicles and their effect on the power systems’ steady-state stability, specifically in terms of large-scale electric vehicle integration. Furthermore, this research builds a model framework that evaluates four essential aspects: electric vehicle capacity forecasting, optimization of an object function, electric vehicle station sitting and sizing, and steady-state stability. Included also in this paper is a numerical study that utilizes projected United Kingdom 2020 power system data wherein the results demonstrate that the electric vehicle capacity forecasting model presented in the study performs effectively to present

electric vehicle charging and discharging profiles. Determining the steady-state stability with an actual model of a small-scale city power system and establishing criteria for electric vehicle station sitting and sizing are the functions of the proposed model in this research paper [5].

Among the different types of nonlinear system models, the one that received the widest acknowledgment and recognition are block-oriented models because of the system identification and automatic control community. Generally, these models are designed by combining linear dynamic parts together with static nonlinear parts in a different configuration of interconnections. The Hammerstein model and the Wiener model are two fundamental nonlinear block-oriented models. The Hammerstein model has a static nonlinear part with a linear dynamic part. On the other hand, the Wiener model has the exact inverse process with that of the Hammerstein model. In one particular study, proposed is an identification technique of Hammerstein-Wiener nonlinear system combined with special test signals. Those special test signals which consist of distinct signals and uniformly random multi-step signals are used for the division of identification conflicts of the linear dynamic and output static parts, and eventually uses correlation analysis method for estimation of parameters. Also included is a fixed filter used to construct an extension of the Hammerstein-Wiener model for noise correlation computations through poles and zeros of the extension. Hence, the model parameters estimated can be derived using error compensation based recursive least square method. Based on the evaluation of simulation results, the proposed method can determine the parameters of Hammerstein-Wiener nonlinear system effectively even with the existence of process noise [6].

3. THEORETICAL CONSIDERATION

The first to be discussed is the different types of test signals, there are four types of signals, namely Impulse Signal, Step Signal, Ramp Signal, Parabolic Signal [7].

Impulse signal (figure 1) is considered as a signal which imitates the sudden shock quality of the actual input signal. It is also the obtained output of a given system for the times when it is given a small input. Another thing to remember about impulse signal is that it gives emphasis to the change in the system when it reacts to a given external change, which is also somewhat the reply of the said system to the direct delta input. Shown below is a sample of an impulse signal [8].

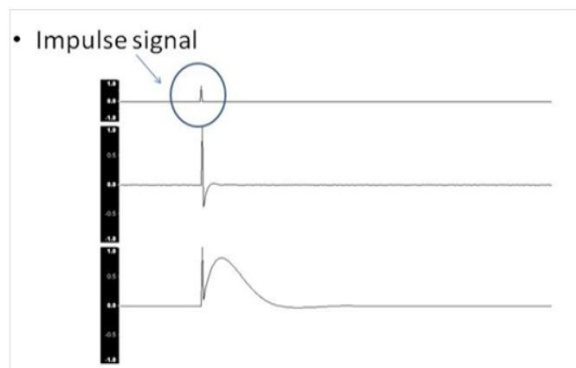


Figure 1: Impulse Signal

The second is step signal, it may be defined as the sudden change in the properties of the actual signal. This signal is the signal which is used to see the transient response for a given system since it can give the how the system will be replying to both the interruptions to the systems as well as the stability of the system.

Third is the ramp signal (figure 2), it may be defined as the signal with constant velocity attribute of the actual input signals. It is also used for the function of seeing the behavior of a given system with the velocity factor. Shown below is a sample of ramp signals.

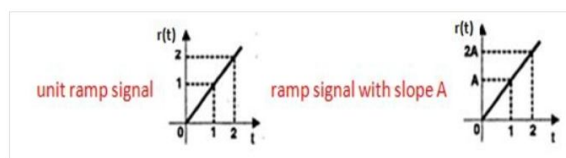


Figure 2: Ramp Signal

Lastly, there is the parabolic signal (figure 3), it may be defined as the signal which gives the constant acceleration distinction of actual input signals. The main function of this signal is to give the idea that the system can respond along together with acceleration. Shown below is a sample of a parabolic signal.

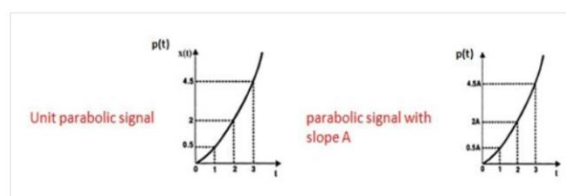


Figure 3: Parabolic Signal

The next topic to be discussed is steady-state analysis and transient state. When there is a disturbance of a given system and the said disturbance happens in the input, in the output, or in both input and output, then the changing of the state from one state to another would take a longer time. This time that it takes to change from one state to another is called transient time and the value of the current as well as the voltage for this said time is called transient response. Depending on the

parameters of the given system, the transient is allowed to have oscillations that are either sustained in nature or decaying in nature, thus the control system is divided into transient response analysis and steady-state analysis. The main difference of the two is that transient state response deals with the nature of the response itself for a given system for the time when it is subjected to an input, on the other hand, steady-state analysis is the one which deals with estimating the magnitude for a steady-state error among the input and the output [9].

The next topic is the first order of transient response. In this part, it is important to go back to capacitors and inductors, it should be noted that when a circuit contains a capacitor or an inductor, the circuit becomes dynamic meaning, the behavior is a function of time, the behavior is described with the use of differential equations, and it has a transient response and a steady-state. It should also be noted that resistive circuits have no transient. For first-order, solving it also does not need any equation, finding the starting point of capacitor voltage or inductor current is needed as well as its ending point and the time constant [10].

Next is the higher-order LTI systems, in the first-order LTI systems that contains constant step or zero inputs, they have simple exponential responses which are characterized by using time constants, however for higher-order LTI systems, these become more complicated, but are as equally important as well as more interesting since, for these, there is no simpler or easier way such as a shortcut to obtaining the desired solutions. In higher-order LTI systems, Laplace transform is usually what is used [11].

The next topic to be discussed is the effects of adding poles and zeros. The general effect of this is the tendency of having a shift of the locus towards the right side of the s-plane and there is also a huge effect on the stability which makes it lower [12].

4. DESIGN CONSIDERATIONS

There are two kinds of circuits that can be observed using transient analysis which is first-order and second-order circuits. The order of the circuit is determined by the energy storage elements present [13]. First-order circuits have only one storage element and can be defined using by applying Kirchhoff's Laws which will then yield a first-order differential equation. On the other hand, second other circuits will have a second-order differential equations since it has two storage elements present [14,15]. When programming the system, the studies of [16,17,18] can be used.

Evaluating the circuit will require two steps, first is to analyze the circuit prior to switching and the second is to analyze after switching. When placing the system in a database the studies of [19,20] can be used as a pattern.

The first step helps to understand the conditions of the storage elements. To show the process, an example (figure 4) is shown below.

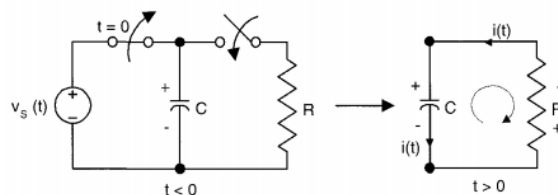


Figure 4: First Order RC Circuit

From the circuit, we can deduce the initial conditions of the circuit to be:

$$v_C(t) = v_s(t), \quad (t \leq 0).$$

This equation is obtained by applying Kirchhoff's Law with V_C being the initial voltage across the capacitor and V_s as the source. At $t < 0$ we know the value of V_s and V_C to be equal and since V_C is a continuous function of time, the equation can be expressed as:

$$v_C(0_+) = v_C(0_-) = V_0.$$

For the second step, the circuit will be observed after switching. Kirchhoff's law is again applied and will yield the equation,

$$v_R(t) + v_C(t) = 0,$$

V_r is the voltage across the resistor and V_C is the voltage across the capacitor after switching. By applying Ohm's law, the equation can then be expressed as:

$$i(t)R + v_C(t) = 0.$$

By using the relationship of $i(t)$ between the elements, the equation can be expressed by the equation below.

$$RC \frac{dv_C(t)}{dt} + v_C(t) = 0$$

The second equation is expressed as

$$v_C(t) = Ae^{st}$$

And then combined with the previous equation to arrive at the equation

$$s = -\frac{1}{RC}$$

Which is also known as the characteristic equation. The general equation which is also the solution to the initial value problem will then appear as

$$v_C(t) = V_0 e^{-t/RC}$$

Over time, it is expected that circuit variables will decay exponentially which is represented by:

$$\tau = RC$$

For the circuit in observation, the relationship between the resistance R and the time constant is:

$$p(t) = \frac{v_C(t)^2}{R}$$

The figure 5 below shows the decay of the voltage of the capacitor with respect to time [7]

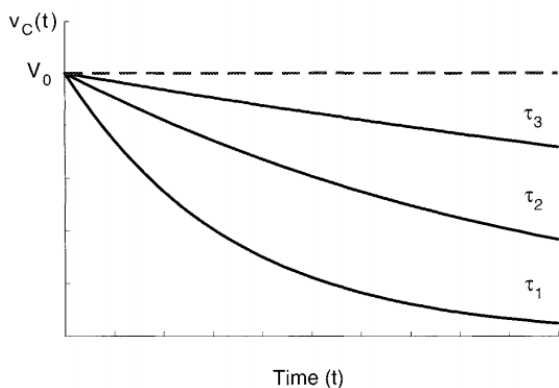


Figure 5: Voltage Decay

5. CONCLUSION

The transient analysis is important in observing electric circuits to show the inner workings of the circuit as the switching occurs. Storage elements are what makes transient analysis important since there is decay in the energy stored in the element as time increases. This was shown in the previous

chapter using first order differential equations. The storage element charges and discharges as the switching occurs. The analysis can be represented by a graph which helps the researcher in understanding the circuit.

On the other hand, steady state analysis is involved in observing the behavior of the circuit as t approaches infinity. Unlike the transient analysis, which is responsible for the initial to the final state. Both of these are imperative in designing a control system to know how it behaves and responds.

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