

## Linear System Interconnections, Steady-State Analysis and Stability Theory



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### ABSTRACT

There are three main topics to be discussed in this paper, the first topic is about Linear System Interconnections. Some of the sub-topics under this topic are Series Systems, Parallel Systems, Feedback Systems, Feedforward Systems, Block Diagram Representation of Systems, Signal Flow Graph Representation of Systems, and Bond Graphs. The second topic is about Steady-State Analysis and under this topic are three sub-topics, namely, The Final Value Theorem, Static Error Constants, and Steady-State Error Due to Noise. The last topic is on Stability Theory and some of the sub-topics are the Concept of Stability and the Necessary Conditions for Stability. This paper aims to provide the definitions and explanations for the principle of steady-state analysis and stability theory. Furthermore, this study discusses the function of each and its relationship with linear systems and other types of systems.

**Key words:** LTI Systems, Block Diagrams, Feedback Systems, Feedforward Systems.

### 1. INTRODUCTION

One of the most fundamental problems in systems and theory of control is the construction of a controller wherein the closed-loop system performs exactly like a provided specification system. Controller systems have a tendency to be “highly-dimensional”, giving more issues in the implementation. One of the considered solutions for this complex problem is the approximation of a linear plant system by a lower-dimensional system. One study focuses on linear plant systems with two kinds of inputs  $f$  and  $u$ , and two types of outputs  $z$  and  $y$  [1].

The steady-state analysis is an analyzing process used in alternating current or ac circuits with the same method of solution of direct current or dc circuits. In relation, steady-state stability refers to the capability of a system or machine to go back or return in its initial state.

The complex dynamical system with finite-time medium obtains stability within a finite amount of time in the so-called finite-time control method. A lot of fields and disciplines in

engineering use this method. One study improved a method for adaptive control for chaotic power systems with the utilization of finite-time stability theory. Another alternative for it is the passivity-based control approach. These tools are used for the analysis of the stability of “integer-order” nonlinear systems. Passivity is associated with the external input and output of the system. Passive systems are types of dynamic systems that pay attention to the exchange of energy between the system and the exterior [2]. This paper discusses the individual concepts of steady-state analysis and stability theory as well as the relationship of these two with linear system interconnections.

### 2. LITERATURE REVIEW

Switched systems are hybrid systems which are composed of families of subsystems and a governing rule that maintains the switching between them. Until today, the stability theory of switched systems is widely known by researchers from various fields and disciplines. There were already several published studies on the stability and stabilizability of switched linear systems in the past years. One study focuses on the stabilization of continuous-time switched linear system. From the results obtained in the study, the researchers have established a method for the stabilizing switching law. In this paper, the researchers presented a necessary condition for the stabilization of the said switched linear systems. The problem of stability in continuous-time switched linear systems have been successfully evaluated in this study. There were derivations presented on the sufficient conditions for the presence of a stabilizing switching law. For future research studies, it is recommended to work on how to provide an estimation of the number of steps required to construct the stabilization. Furthermore, future researchers may also consider Lyapunov functions as solutions instead of using Euclidean distance [3].

The process of multiplexing emerges in situations where in a range of various tasks are to be served by a central server. In the case that this server is unable of parallel processing and at the same time has the assignment of manipulating  $N$  different systems, the server must serially process the range of tasks, which eventually leads to multiplexing. As an alternative, if the  $N$  controllers use only one communication channel, a similar problem of multiplexing emerges. This paper studies

two different control problems on the processes of sparse and optimal multiplexing. In this study, sparse and optimal multiplexing is treated as a “finite ensemble” of linear control systems. Multiplexing of controllers involves an algorithm that chooses one from the ensemble of linear systems at each time is manipulated provided an ensemble of linear control systems. Pontryagin maximum principle has been the basis of the methods developed in the study [4].

The use of linear methods in the analysis of steady-state operations of distribution systems is starting to receive more recognition and relevance in the aspect of planning and operating because of the growth of distributed energy resources or DERs. Some examples of the applications of linear processes are power system optimization studies, power losses estimation, power flow analysis, and sensitivity analysis. This paper presents a novel linear method which relies on a Jacobian approach to be used for the radial distribution network. This also includes lateral deviations and distributed energy resources. First presented is the set of linear equations which models the distribution system, then it is solved in a closed-form. High levels of accuracy and low computation times are obtained by adopting a particular set of variables for modeling and the network’s radial topology. With the use of a 24-nodes and a 237-nodes network, the efficiency of the said approach was verified. The proposed method has been proven to display a good performance with respect to the accuracy of results and the efficiency of the algorithm [5].

### 3. THEORETICAL CONSIDERATIONS

There are many sub-topics that are part of the first topic which is about Linear System Interconnections and each of the said topics are able to show different phenomena and other things that are related to it, some examples of this are the hidden modes as well as the scarcity in the number of control variables. Some of the features that were mentioned are not capable of being taken into account with the use of the transfer function approach are further explained with the use of language modules. Some of the keywords to be noted when dealing with some of the sub-topics like Series Systems, Parallel Systems, Feedback Systems, etc., are interconnection, transfer functions, controllability, and behavioral approach [6].

Another sub-topic under this topic is the Feedforward System, in the journal “Mixed feedback and feedforward control design for multi-axis vibration isolation systems”, the feedforward control is utilized in order to have a reaction with the effect of floor vibrations. With this research as well as the findings and results that were said in the journal, it can be said that this is an example of an application of linear system interconnections [7].

In this next journal by Massimiliano Veronesi and Antonio Visioli, the sub-topic Final Value Theorem from the next topic which is Steady-State Analysis is discussed. In the

journal, the theorem was used as a basis for the process parameters estimation as well as for the performance assessment and controller retuning. With the use of the final value theorem, some new techniques were proposed to be applied for the said parameters. It was also mentioned that the biggest feature of the said ways is the application to closed-loop systems as well as for routine operational data [8].

For the next sub-topic under Steady-State Analysis, an application for the Steady-State Error Due to Noise is seen in a journal titled “Transient and steady-state readout of nanowire gas sensors in the presence of low-frequency noise”. Here, some of the highlights of the journal are that there are comparisons between transient and steady-state gas sensing, and this is in the presence of  $1/f$  noise, another is the theoretical analysis of the signal-to-noise ratio of the sensing of chemical and again it is in the presence of  $1/f$  noise. Another is that there are criteria which are obtained and used for transient or steady-state sensing and the most suitable is presented. Shown in this journal is an example of a possible application of steady-state analysis [9].

For the last topic, which is about Stability Theory, a journal by Cong Wang, Hongli Zhang, Wenhui Fan, and Ping Ma discusses this and is titled “Adaptive control method for chaotic power systems based on finite-time stability theory and passivity-based control approach”. Some of the main things that were done in this journal are the adaptive control method which is used for chaotic power systems with the use of the finite-time theory as well as with the passivity-based control approach. It was also stated here that with the use of the finite-time stability theory, it is possible to achieve the chaotic power system stability for a finite-time. The controller mentioned in this journal is also used in performing extensive studies that deal with the fourth-order power system. In this journal, the implementation of the stability theory may be seen [10].

### 4. DESIGN CONSIDERATIONS

Block diagrams are used to visualize the process of control systems. Series Systems can be represented in a linear manner which signifies that the previous process must be completed before it moves on to the next process. Parallel systems, on the other hand, are processes that execute simultaneously. Below is the block representation (figure 1) of the series and parallel systems.

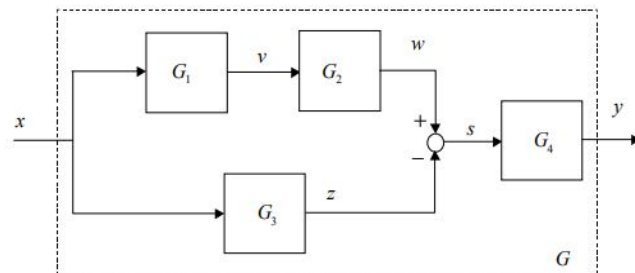


Figure 1: System Block Diagram

The process G2 comes after G1. These processes are connected in series while G3 is connected in parallel to G1 and G2.

The feedback interconnection is represented by the output of the system being connected to a previous part of the control system. This means that the output of the previous process will affect the output of the next iteration. The figure 2 below shows an example of the feedback block diagram [11].

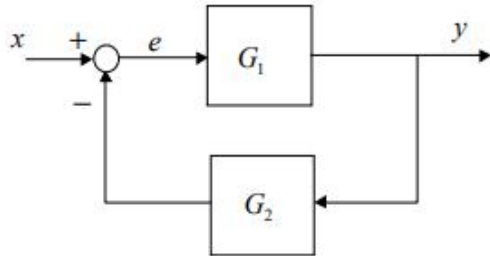


Figure 2: Feedback Block Diagram

These block diagram representation of control systems may become too complicated and difficult to understand. There are rules or formulae that may be applied to simplify the block diagram and represent it in a single transfer function [12].

Signal Flow Graphs are much like block diagram representations. However, instead of processes, signal flow graphs represent the flow of power in a microwave network. Both signal flow graph technique and algebraic manipulation always give the same results so it can be used to double-check the output by using the other technique. However, it is much easier to simplify by using the signal flow graph technique. Shown below is an example (figure 3, figure 4 and figure 5) of the step by step process of signal flow analysis.

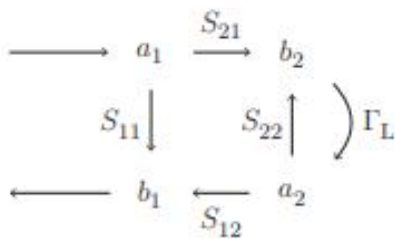


Figure 3: Process 1

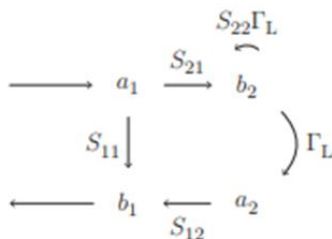


Figure 4: Process 2

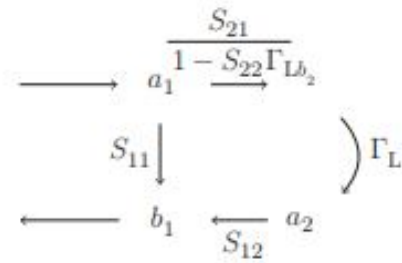


Figure 5: Process 3

The simplified equation for the signal flow graph is shown below [13].

$$\Gamma_{in} = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L}$$

Equation 1.

Stability theory tackles the stability of the output of different control systems. It is involved in using differential equations to represent the stability of the system despite having some perturbation due to external or internal disturbances [14,15].

## 5. CONCLUSION

The different approaches discussed in this paper are used to observe and improve different systems that are already being implemented. Systems that are still under development also undergoes some of the analysis techniques that are tackled in this paper. This ensures the efficiency and the effectiveness of the system. When making it to a program, the system structures of [16,17,18] can be used while the database structure of [19] can be patterned.

Using these techniques will help the researchers further understand the system from different points of view and make sure that the behavior of the study. For example, in laser machining, it is important that the researchers understand completely the way a certain material behaves when subjected to intense heat. They can use steady-state analysis to understand the material as it changes by heating. This way, they can predict the outcome of the research and verify it through performing the actual experiment.

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