



Frequency Response, Root Locus and Sensitivity Analysis

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ABSTRACT

There are three main topics to be discussed in this paper, namely Frequency Response Analysis, Root Locus Analysis and Design, and Sensitivity Analysis. Under Frequency Response Analysis some of the sub-topics are about Frequency Response Concepts, Magnitude, and Phase Spectra, Bode Plots, Sketching the Magnitude and Phase Spectra, Controller Synthesis Using Frequency Response Method, Nyquist Diagrams, and Nichols Charts. Under the topic of Root Locus Analysis and Design is the sub-topics Root Locus Concepts as well as its Sketching, Controller Synthesis Using Root Locus, and Analog Circuit Implementations of P, PD, PI, PID, Lag, Lead, and Lag-Lead Compensators. Lastly, for the topic of Sensitivity Analysis, the sub-topics are Sensitivity to Changes in a Parameter and Sensitivity to Disturbance Signals.

Key words: Frequency Response, Phase Spectra, Root Locus, Nyquist Controller

1. INTRODUCTION

Frequency Response Analysis or FRA is a widely known, distinctive method used for mechanical fault detection in power transformers. In spite of the accreditation of FRA processes, several researchers are still studying the assessment of results. This is due to the fact that the analyses of frequency responses vary depending on the case. Hence, in the interpretation of transformer FRA results, one should know the effects of different mechanical configurations on the FRA trace of the specific transformer. The best way to understand the effects of mechanical configurations is to conduct experiments or replace modeling methods. It is a primary challenge for FRA evaluation that no single general rule can be made since every transformer varies in construction and performance. Although it is a commonly used method in mechanical fault detection, the diagnosis of FRA is not limited to mechanical examinations only. However, advance detection of faults can save transformers from catastrophic failures [1].

The conventional Root-Locus process is utilized for the plot and analysis of roots of a real polynomial using an adjustable parameter inserting the coefficients of the polynomial relatively. This method emerged in 1948 and developed by Walter R. Evans and Kazimir F. Theodorichik. The Root-Locus method was widely known and utilized for designing and analyzing control systems and servomechanisms with generally rational transfer functions. The design of the Root-Locus method includes a set of rules enabling an accurate and rapid outline of root loci for low-order systems with rational transfer functions. Over the years, numerous modifications and numerical procedures for Root-Locus plotting were proposed. There were also several attempts to expand the method and apply it to a wider range of systems [2].

One of the most fundamental parts of any applied analysis methods is the Sensitivity Analysis. This analysis is brought out to verify the establishment of a “probabilistic graphical model” and to assess its sturdiness to the misspecification of its probabilities. Most research studies focused on Bayesian Network or BN models. However, results of sensitivity are also present in Markov networks and chain event graphs. Typically, there are two phases in the sensitivity analyses in BN models. For the first phase, some of the model parameters vary and the results of the variations on output probabilities of interest are examined. In the second phase, variations in parameters, then, their results are wrapped up by measurement of distance or divergence between the initial and varied or modified distributions in the BN model. Generally, the first phase of sensitivity analysis involves a basic mathematical function, also called the sensitivity function, which outlines an output probability of interest as a function of Bayesian Network parameters. In the second phase, parameters like Chan-Darwiche distance, Kullback-Leibler divergence, and ϕ -divergences are utilized for the measurement of the total effect of the parameter variations. One significant research focused on the identification of parameter covariations. These are methods of parameter adjustment with respect to the sum to a condition followed by a parameter variation. Majority of the stated results are particularly derived for BNs, however, these can also hold several various models as long as the atomic probabilities of these models can be presented as multilinear polynomials.

2. LITERATURE REVIEW

Due to faults of short circuits and several environmental factors, power transformers are prone to deformations. Most of these faults are because of progressive nature and must be repaired immediately as soon as the condition arises. A widely utilized tool for the detection of such faults is the Frequency Response Analysis or FRA. Researchers work hard to examine the effects of different mechanical deformations on the transformer FRA signature since credible interpretation codes for these signatures have not been fully established [3]. Moreover, the majority of the studies were performed on a transformer electric circuit model with a high frequency. This is because of the invasive nature of the faults when putting on an actual transformer. One way to have reliable FRA interpretation codes is to construct equivalent electric circuit models which can generate FRA signatures as similar as possible to the actual FRA signature trend of an actual transformer. This study presents a transformer electric circuit level of high frequency complete with details but no considerations on mutual inductances. Included in the paper are the computational details of the model parameters. The proposed model was evaluated through comparison of its FRA signature with an equivalent transformer model during different conditions which are obtained by simulations and measurements. According to numerical indices, there is a good relationship between the trend of the simulation and the trend of the FRA signatures. Therefore, the proposed model can be utilized to learn the effects of different internal transformer faults on FRA signatures [4].

The lac operon or lactose operon of the bacterium *E. coli* is a gene regulatory network which controls lactose metabolism. This paper utilizes the root-locus method to perform an assessment on the parameter regions that maintain the bistability in the lac operon. Compared to previous research studies, bistability conditions studied in this paper includes a complete range of the required conditions. The application of the root-locus method to the polynomial equilibrium equation of the lac operon model is where the mentioned conditions were derived. The derivation aims to identify the parameter values which produces the multiple real roots required for bistability. Furthermore, the used lac operon model was presented as a basic differential equation system in the form of a state equation and having a rational value in the right-hand side. The designed root-locus method can be utilized to understand the steady-state performance of a convergent biological system model of any kind. The presented bistability analysis using root-locus method may be used to study the impact of a particular mechanism which is not included in the basis minimal lac operon model. Moreover, this can also be applied to understand the lac operon's bistable behavior on various natural conditions [5].

Usage of renewable forms of energy like solar energy is one feasible approach to minimize the energy consumed by a refrigeration system or device. Among the types of

refrigeration systems, the one that stands out is the ejector refrigeration system because of its requirement of low-level temperature. Theoretical studies and obtained experimental results demonstrate that the ejector refrigeration system carries huge potential in terms of conservation of energy and preservation of the environment. This paper presents the issues of sensitivity analysis and thermodynamic modeling of the ejector. For the first part, a novel thermodynamic model is presented which was dependent on the inferences of consistent pressure and area mixing. Unlike conventional ejector models, this thermodynamic model has fewer parameters and more plain construction. Eventually, the ejector's sensitivity analysis is implemented with the adjoint sensitivity method as a basis. To show the connection between the performance of the ejector and the system parameters, three different sensitivity coefficients were provided. The researchers have concluded that both operating conditions and geometric parameters have an enormous impact on the COP or coefficient of performance of the ejector refrigeration system [6].

3. THEORETICAL CONSIDERATIONS

One of the sub-topics relating to frequency response analysis is its concepts and its magnitude and phase spectra. In a journal by Zhao, X, et al. entitled High-Frequency Electric Circuit Modeling for Transformer Frequency Response Analysis Studies, some concepts of frequency response are discussed. Some of which are improved the model for power transformer frequency response analysis. In this journal, a proposed model for the impact of different mechanical fault types as well as the power transformer frequency response and its levels [7].

Another sub-topic is about Gain Stability Margin, in a journal by Siddharth, et al. entitled Phase and Gain Stability Margins for a class of Nonlinear Systems. In this journal, there are new definitions for Phase and Gain Stability Margins for systems which are nonlinear. Also, in this journal, the gain and phase stability margins are said to be the maximum increase for both the gain as well as the maximum phase shift which can be added to the characteristics of the frequency. There are also algorithms that are presented in this paper [8].

The next topic to be discussed is the root locus, in a journal that was made by Toosi, root locus was also defined as the location of the closed-loop poles of a given system. In this journal, it was also discussed how to find this root locus with no explicit computations needed and there are discussions on simple rules in drawing the root locus of the said system directly taken from the given information of the open-loop poles and zeros. Another thing to be noted from this journal is the concept of the appropriate value for the gain taken from the root locus in obtaining the performance which is satisfactory for it [9].

The last topic to be discussed is about Sensitivity Analysis, in

a journal by Leonelli, titled Sensitivity Analysis Beyond Linearity, there are many different graphical models which are possible to parameterize in order to have probabilities that are atomic and can be symbolized using monomial functions. Aside from this, derivation of the many different divergences as well as the distances that are dealt with unrelated co variation schemes. The journal is relatively easy to understand since the methodology was shown in a way that it is easy to understand and in an educational application [10].

4. DESIGN CONSIDERATIONS

Frequency response analysis is commonly used due to the fact that is very responsive to the many changes in the system that is being tested. Sweep frequency response analysis is done by applying a sinusoidal voltage to the input using a sweep generator. For example, in a power transformer, there are two parameters that can be measured using SFRA, transfer function and the impedance. There are three facets required in the testing. First is the connection of the terminals which include tested and non-tested terminals. Next is the measurement type which can be classified into transferred and non-transferred measurements. Transferred measurements can be used to obtain the impedance and the other measurement type can be used for the transfer function. The last on the list is the number of measurements which depends on multiple factors, one of which is the type of measurement that is considered in performing the testing [11]. The figure 1 below displays an illustration of the plot of an average frequency response function, which, in a particular study, an accelerant.

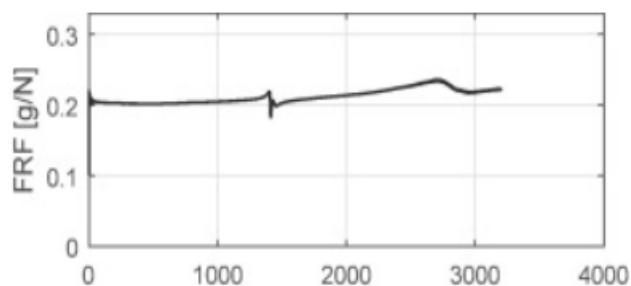


Figure 1: Average Frequency Response Function [12]

Another way to analyze a system is by using the Root locus method. This method can be used to observe the behavior of the system after having some changes. It is a graphical method that is used to obtain the zero of a closed-loop system in relation to the zeros of the open-loop system [13]. This will enable the evaluation of the behavior of the stability and the dynamic response [14]. Root Locus method can also be applied in fractional-order or FO systems. Fractional-order systems have been utilized in several research studies and activities both in modeling and control. Fractional-order differential equations are proven to offer clearer and more skeptical representations of both natural and man-made systems. Compared to the integer-order equivalent values, fractional-order controllers were able to demonstrate a greater

closed-loop performance. Root Locus method can be utilized for the evaluation of the stability of a given closed-loop or CL system (figure 2).

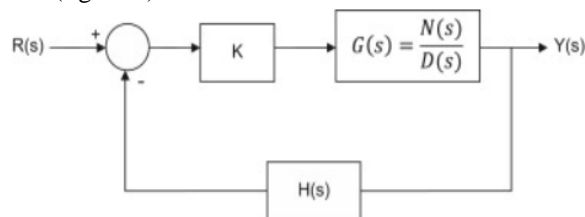


Figure 2: Block Diagram Representation of Closed-Loop Linear FO System [15]

Sensitivity analysis is a type of analysis that focuses on the changes that can occur in altering previous assumptions in the study. This kind of analysis is for the testing of the stability or robustness of the system under testing. It questions the prior assumptions if it is accurate and acceptable [16]. The model outputs are the parameters that are being analyzed and needed by the researcher. These data are what determines the results of the sensitivity analysis. The system may or may not need some improvements based on these results [17].

5. CONCLUSION

The different analysis procedures discussed in the paper are important in evaluating the behavior of the system under testing. These analysis techniques have different advantages over other techniques as discussed in the paper. In frequency response analysis, the advantage is the sensitivity of the procedure which is imperative to obtain a more accurate reading for the system.[18,19] Throughout the years, there have been several research studies and advances, and applications utilizing the Root Locus approach such as delayed systems and parameter systems. Root locus analysis focuses on the behavior of the system after having some changes in the process. Moreover, it is proven to be a useful and powerful technique in the design and analysis of single-input single-output systems (SISO) which are types of linear time-invariant or LTI systems. Stability and design are the two primary applications of Root Locus method. It is used to achieve adequate conditions on real parameters where the closed-system is stable. In addition, the Root Locus method also provides a tool for designing lead-lag compensators. Lastly, the Sensitivity analysis tests the assumptions that are the basis of the system and tests whether these assumptions are accurate. It also determines the impact of changing those initial conditions on the system. When programming the system, the data structures from the researches of [20,21,22] can be used while it can be patterned in the database of [23].

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