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1-Fair Dominating Sets in Some Class of Graphs

Jayasree T G [1], Radha Rajamani Iyer [2]

[1]Adi Shankara Institute of Engineering and Technology, Kalady, Kerala, India. jayasreetg@gmail.com

[2]Department of Mathematics, Amrita School of Engineering, Coimbatore, India. r_radhaiyer@cb.amrita.edu

ABSTRACT

A set D of vertices in a graph G (V, E) is a dominating set of G, if every vertex of V not in D is adjacent to at least one vertex in D. A dominating set D of G (V, E) is a k - fair dominating set of G, for k ≥ 1, if every vertex in V - D is adjacent to exactly k vertices in D. The k - fair domination number γ_{kfd} (D) of G is the minimum cardinality of a k - fair dominating set. In this article, we determine the k-fair domination number of some class of graphs for k = 1.

Key words: Efficient Domination, Cartesian Product of Graphs, Strong Product of Graphs.

1. INTRODUCTION

Let G (V, E) be a simple graph with vertex set V and edge set E. The order and size of G are denoted by n and m respectively. For graph theoretic terminology we refer to Gary Chartrand and Ping Zhang [15] and [16, 17] Haynes et al. For any vertex $v \in V$, the open neighbourhood N (v) is the set $\{v \in V : uv \in E\}$, and the closed neighbourhood N[v] is the set $N(v) \cup \{v\}$. For any $S \subseteq V$, $N(S) = \cup_{v \in S} N(v)$ and $N[S] = N(S) \cup S$.

A fair dominating set in graph G(V, E) is a dominating set D such that all vertices not in D are dominated by same number of vertices from D, that is, every two vertices not in D has same number of neighbours in D. The fair domination number γ_{fd} (G) of G is the minimum cardinality of an fd-set.

A dominating set $D \subseteq V(G)$ is a 1-fd set in G, if for every two distinct vertices, $u, v \in (V - D)$, $|N(u) \cap D| = |N(v) \cap D| = 1$. That is, every two distinct vertices not in D have exactly one neighbour from D. But we know that dominating set D of G (V, E) is an efficient dominating set of G if every vertex in V - D is adjacent to exactly one vertex in D. The efficient domination number $\gamma_e(G)$ of G is the minimum cardinality of an efficient dominating set. Hence $\gamma_{1fd}(G) = \gamma_e(G)$. Therefore, in this note we use the notation $\gamma_e(G)$ instead of $\gamma_{1fd}(G)$.

The domination in graphs is one of the vital area in graph theory which has attracted many researchers because of its potentiality to solve and address many real life situations like in the communication, social network and in defense purpose to name a few. In a communication network, let D denote the set of transmitting stations so that every station not belonging to D has a link with at least one station in D. We have to protect these set of stations from faults at any cost and hence the number of such sets should be minimum. This leads to the definition of 1-fd sets in any graphs.

1.1 Paths P_n and Cycles C_n

We refer to Gary Chartrand and Ping Zhang [15] for the following results.

Result: 1: Let P_n be a path of length (n - 1), then $\gamma(P_n) = \left\lceil \frac{n}{3} \right\rceil$, for $n \geq 2$.

Result: 2: Let C_n be a cycle of length n, then $\gamma(C_n) = \left\lceil \frac{n}{3} \right\rceil$, for $n \geq 3$.

1.1.1.Theorem: Let P_n be a path of length (n - 1), then $\gamma_e(P_n) = \left\lceil \frac{n}{3} \right\rceil$, for $n \geq 2$.

Proof: The case is obvious for n = 1, 2 and 3, since one vertex is enough to dominate the entire graph as 1-fair domination. Hence $\gamma_e(P_1) = \gamma_e(P_2) = \gamma_e(P_3) = 1$.

Case 1: If $n \equiv 0 (mod\ 3)$. Or if the graph is $P_3, P_6, P_9, \ldots,$. In a path one vertex can dominate as 1-fd for maximum of two vertices, P_n needs $\frac{n}{3}$ vertices to dominate the graph. Hence $\gamma_e(P_n) = \frac{n}{3}$.

Case 2: If $n \equiv 1 (mod\ 3)$. Or if the graph is $P_4, P_7, P_{10}, \ldots,$. The path can be considered as the combination of $\frac{n-4}{3}$, number of P_3 and two P_2 graphs. Hence $\gamma_e(P_n) = \frac{n+2}{3}$.

Case 3: If $n \equiv 2 (mod\ 3)$. Or if the graph is $P_5, P_8, P_{11}, \ldots,$. The path can be considered as the combination of $\frac{n-2}{3}$, number of P_3 and one P_2 graphs. Hence $\gamma_e(P_n) = \frac{n+1}{3}$. Hence the result can be generalised as: $\gamma_e(P_n) = \left\lceil \frac{n}{3} \right\rceil$, for $n \geq 2$.

1.1.2.Theorem: Let C_n be a cycle of length n, then: $\gamma_e(C_n) = \begin{cases} \frac{n}{3}, \ if\ n \equiv 0\ (mod\ 3) \\ \left\lceil \frac{n}{3} \right\rceil, if\ n \equiv 1 (mod\ 3) \\ \left\lceil \frac{n}{3} \right\rceil + 1, if\ n \equiv 2 (mod\ 3) \end{cases}$.

Proof: The result can be established by the following cases.

Case 1: If $n \equiv 0 (mod 3)$. Or if the graph is $C_3, C_6, C_9, \ldots,$. In this case, C_n needs $\frac{n}{3}$ vertices to dominate the graph as 1-fd. Hence $\gamma_e(C_n) = \frac{n}{3}$.

Case 2: If $n \equiv 1 (mod 3)$. Or if the graph is $C_4, C_7, C_{10}, \ldots,$. The graph can be considered as the combination of $\frac{n-4}{3}$, number of P_3 along with two self dominating vertices. Hence $\gamma_e(C_n) = \frac{n+2}{3}$.

Case 3: If $n \equiv 2(mod3)$. Or if the graph is $C_5, C_8, C_{11}, \dots,$. Then the graph can be considered as the combination of $\frac{n-5}{3}$, number of P_3 along with three self dominating vertices. Hence $\gamma_e(C_n) = \frac{n+4}{3}$. Or can be generalised as:

$$\gamma_e(C_n) = \begin{cases} \frac{n}{3}, & if\ n \equiv 0\ (mod\ 3) \\ \left\lceil \frac{n}{3} \right\rceil, & if\ n \equiv 1(mod\ 3) \\ \left\lceil \frac{n}{3} \right\rceil + 1, & if\ n \equiv 2(mod\ 3) \end{cases}$$

. This completes the proof.

2.PRODUCT GRAPHS

The Cartesian product of two graphs G and H, is a graph $G \boxdot H$, [16] with vertex set $V(G \boxdot H) = V(G) \times V(H)$ and edge set $E(G \boxdot H) = \{((x_1, y_1), (x_2, y_2)) :$ $: x1,x2 \in EG\ with\ y1 = y2\ or\ y1,y2 \in EG\ with\ x1 = x2$ Hamed Hatami and Pooya Hatami [4] have studied the perfect dominating sets in the Cartesian product of cycles of prime order. A necessary and sufficient condition for the existence of an efficient dominating set in the Cartesian product of two cycles has explained by T.Tamiz Chelvam and Sivagnanam M [13]. In this paper, we try to find the 1-fd sets in the Cartesian product of P_n and C_n with K_2.

Already we have the following results.

Result: 1: $\gamma(K_2 \boxdot P_n) = \left\lceil \frac{n+1}{2} \right\rceil$, for $n \geq 2$.

Result: 2: $\gamma(K_2 \boxdot C_n) = \left\lceil \frac{n}{2} \right\rceil$, for $n \geq 3$.

2.1.Theorem: Let P_n be a path of length (n - 1) and K_2 be the complete graph of two vertices then the 1-fd sets in the Cartesian product of these graphs can be determined by: $\gamma_e(K_2 \boxdot P_n) = \left\lceil \frac{n+1}{2} \right\rceil$, for $n \geq 2$.

Proof: The product graph $(K_2 \boxdot P_n)$ is the grid graph $(P_2 \boxdot P_n)$ and the graph consisting of two copies of P_n graph with corresponding vertices are joined by an edge. Now consider the following cases.

Case 1: When n = 3. Let the vertices of the product graph are v_1, v_2, v_3 and u_1, u_2, u_3. Here we need only two vertices say v_1 and u_3 are enough to dominate the entire graph as 1-fd. Hence $\gamma_e(K_2 \boxdot P_n) = 2$.

Case 2: When n is an odd number. That is when n = 5, 7, 9, ...,. As the given graph is product graph each increment in the value of n contributes two in the number of vertices. These four vertices can be treated as the internal vertices in the graph and these four vertices contributes additional one in the domination number. Hence $\gamma_e(K_2 \boxdot P_n) = \frac{n+1}{2}$.

Case 3: When n is an even number. That is when n = 4, 6, 8, ...,. The graph is an extension to the above case as it has two more vertices and one more vertex is needed to dominate the additional two vertices. Hence $\gamma_e(K_2 \boxdot P_n) = \left\lceil \frac{n+1}{2} \right\rceil$. This completes the proof.

2.2.Theorem: Let C_n be a cycle of length n, and K_2 be the complete graph of two vertices then the 1-fd sets of the Cartesian product of these graphs can be determined by:

$$\gamma_e(K_2 \boxdot C_n) = \begin{cases} 2 \times \left\lceil \frac{n}{4} \right\rceil - 1, & if\ n \equiv 1\ (mod\ 4) \\ 2 \times \left\lceil \frac{n}{4} \right\rceil, & if\ n \equiv 0, 2\ (mod\ 4) \\ 2 \times \left\lceil \frac{n}{4} \right\rceil + 1, & if\ n \equiv 3(mod\ 4) \end{cases}.$$

Proof: The product graph $K_2 \boxdot C_n$ is the graph consisting of two copies of C_n whose corresponding vertices are joined by an edge. In such structure one vertex can dominate maximum of three vertices and hence the result can be established using the following cases.

Case 1: Let $n \equiv 0\ (mod\ 4)$. Then total number of vertices in the graph will be N = 8, 16, 24, ..., . In such cases a dominating set of $2 \times \frac{n}{4}$ vertices dominate the entire graph as 1-fd.

Case 2: Let $n \equiv 1\ (mod\ 4)$. Then total number of vertices in the graph will be N = 10, 18, 26,..., . Here the graph can be considered as the distinct union of two graph say G_1 and G_2 where G_1 is dominated as 1-fd using $2 \times \frac{n}{4}$ vertices and G_2 is a graph with two vertices which add on only one in the domination number. Hence $\gamma_e(K_2 \boxdot C_n) = 2 \times \left\lceil \frac{n}{4} \right\rceil - 1$.

Case 3: Let $n \equiv 2\ (mod\ 4)$. Then total number of vertices in the graph will be N = 12, 20, 28,..., . The graph can be considered as the distinct union of two graph say G_1 and G_2 where G_1 dominated as 1-fd sets using $\frac{n}{4}$ vertices and G_2 is a graph in which one vertex will stand independent. Hence we need one more vertex to dominate this graph. Hence $\gamma_e(K_2 \boxdot C_n) = 2 \times \left\lceil \frac{n}{4} \right\rceil$.

Case 4: Let $n \equiv 3\ (mod\ 4)$. Then total number of vertices in the graph will be N = 14, 22, 30,..., . The graph can be considered as the distinct union of two graph say G_1 and G_2 where G_1 dominated as 1-fd sets using $\frac{n}{4}$ vertices and G_2 is a graph which need two vertices to dominate this graph. Hence $\gamma_e(K_2 \boxdot C_n) = 2 \times \left\lceil \frac{n}{4} \right\rceil + 1$. Hence we can generalise the result as:

$$\gamma_e(K_2 \boxdot C_n) = \begin{cases} 2 \times \left\lceil \frac{n}{4} \right\rceil - 1, & if\ n \equiv 1\ (mod\ 4) \\ 2 \times \left\lceil \frac{n}{4} \right\rceil, & if\ n \equiv 0, 2\ (mod\ 4) \\ 2 \times \left\lceil \frac{n}{4} \right\rceil + 1, & if\ n \equiv 3(mod\ 4) \end{cases}.$$

3.SIERPINSKI GRAPHS

Graph of Sierpinski type play an important role in different areas of Mathematics as well as in several other scientific fields. The graphs S (n, 3), which are isomorphic to Hanoi graphs modeling the Tower of Hanoi with 3 pegs and n discs. The generalized Sierpinski graphs,[2] S (n, k) is defined as follows:

For $n \geq 1$ and $k \geq 1$, the vertex set of S (n, k) consists of all n - tuples of integers 1, 2, 3, ... , k, that is, $V(S(n, k)) = [1, k]^n$. Two different vertices $u = (u_1, u_2, u_3, \dots, u_n)$ and $v = (v_1, v_2, v_3, \dots, v_n)$ are adjacent if and only if there exists an $h \in [1, n]$ such that:

1. $u_t = v_t$ for $t = 1, 2, 3, \dots, h - 1$;
2. $u_h \neq v_h$;

3. $u_t = v_h$ and $v_t = u_h$ for $t = h+1, h+2, \dots, n$.

The vertices $\langle iii \dots i \rangle$, $i \in [1,k]$, are called the extreme vertices of S (n, k). For $i \in [1,k]$, let S_i (n+1, k) denote the sub graph of S (n+1, k) induced by the vertices of the form $\langle iii \dots i \rangle$. Also, S_i (n+1, k) is isomorphic to S (n, k), where S (n, k) consists of k copies of S (n-1, k) for n > 1 with S (1, k) is a complete graph of k vertices.
By definition of S (n, k), the cardinality $|V(S(n,k))| = k^n$, $|E(S(n,k))| = \frac{k(k^n - 1)}{2}$ and $\Delta(S(n,k)) = k$ for $n \geq 2$.

3.1.Theorem: Let C_n be a cycle of length n, then the 1-fd sets can be determined by the following rules:

$$\gamma_e(S(2, C_n)) = \begin{cases} n \times \gamma_e(C_n), & if\ n \equiv 0\ (mod\ 3) \\ n \times [\gamma_e(C_n) - 1], & if\ n \equiv 1 (mod\ 3) \\ (n \times [\gamma_e(C_n) - 1]) + 1, & if\ n \equiv 2 (mod\ 3) \end{cases}.$$

Proof: The graph S (2, C_n) is a cycle with each vertex is again a cycle. Refer the following figure for S (2, C_6). As $\gamma_e\ (C_n)$ is already found, the result follows.

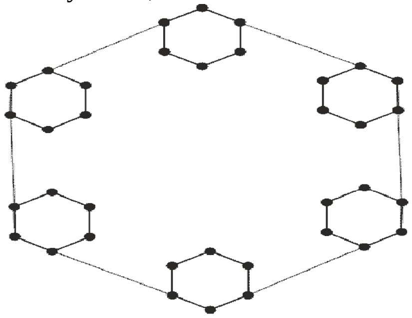

Figure 1: S (2, C_6)

3.2.Theorem: Let K_n be the complete graph with 'n' vertices, then $\gamma_e(S(2, K_n)) = n$.
Proof: The graph S (2, K_n) is a complete graph with each vertex is a K_n. The graph S (2, K_5) is given in the following figure.

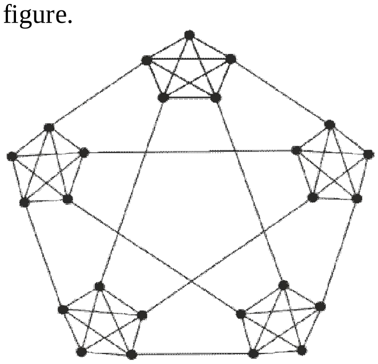

Figure 2: S (2, K_5).

From the figure it is clear that there exist only one vertex which is adjacent to the other vertices of K_n, while all other vertices are adjacent to the vertex of one more K_n. So if we take the vertex which is independent of other K_n to the dominating set, each vertex in the dominating set efficiently dominate each K_n, and hence $\gamma_e(S(2, K_n)) = n$.

4.GRAPHS WITH $\gamma(G) = \gamma_{1fd}(G)$

It is clear from the definition that, $\gamma(G) \leq \gamma_{1fd}(G)$. Due to the importance of the dominating set, we always prefer $\gamma_{1fd}(G)$ to be the minimum. Now consider some graphs with minimum $\gamma_{1fd}(G)$, that is, consider graphs with $\gamma(G) = \gamma 1fdG$. The following results are obvious.

1. For complete graph K_n, $\gamma_e(K_n) = 1$.
2. For wheel graph $W_{1,n}$, $\gamma_e(W_{1,n}) = 1$.
3. For star graph $S_{1,n}$, $\gamma_e(S_{1,n}) = 1$.
4. For complete bipartite graph $K_{m,n}$, $\gamma_e(K_{m,n}) = 2$.

4.1.Strong Product Graphs

Let G and H be two graphs with the set of vertices $u = \{u_1, u_2, u_3, \dots, u_n\}$ and $v = \{v_1, v_2, v_3, \dots, v_m\}$ respectively. The strong product of G and H is the graph $G \boxtimes H$ formed by the vertices $V = \{(u_i, v_j): 1 \leq i \leq n,\ 1 \leq j \leq m\}$ and two vertices (u_i, v_j) and (u_s, v_t) are adjacent in $G \boxtimes H$ if and only if $(u_i = u_s\ and\ v_j\ adjacent\ to\ v_t)$, $(u_i\ adjacent\ to\ u_s\ and\ v_j = v_t)$ or $(u_i\ adjacent\ to\ u_s\ and\ v_j\ adjacent\ to\ v_t)$.
In this section we establish the efficient domination number $\gamma_e(G)$ or $\gamma_{1fd}(G)$ for the strong product graphs $P_m \boxtimes P_n$. We also try to find $\gamma_{2fd}(G)$ for some m and n.

4.1.1.Proposition: For any $n \geq 2$, $\gamma_e(P_1 \boxtimes P_n) = \left\lceil \frac{n}{3} \right\rceil$. Also $\gamma_{2fd}(P_1 \boxtimes P_n) = \left\lceil \frac{n+1}{2} \right\rceil$.
Since $[P_1 \boxtimes P_n] = P_n$, the result is obvious.

4.1.2.Lemma: $\gamma_e(P_2 \boxtimes P_n) = \begin{cases} 1 & if\ n = 2,3 \\ 2 & if\ n = 4,5,6 \end{cases}$.

Proof: To establish the result, first prove the result for the given values for n taking m = 2.
Case: 1: Let n = 2. Then one among the four vertices of $P_2 \boxtimes P_2$ efficiently dominates the entire graph and hence $\gamma_e(P_2 \boxtimes P_2) = 1$.

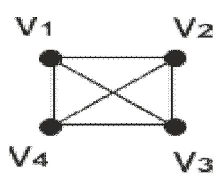

Figure 3: B_1

For future references let us name this graph as B_1 with $\gamma_e[B_1] = 1$.
Case: 2: Let n = 3. Then one of the centre vertices in the graph of $P_2 \boxtimes P_3$ efficiently dominates the entire graph. Hence in this case also $\gamma_e(P_2 \boxtimes P_3) = 1$.

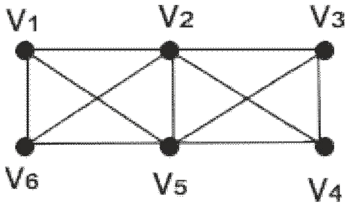

Figure 4: B_2

Let us denote this graph as B_2 with $\gamma_e[B_2] = 1$.
Case: 3: Let n = 4. In this case the graph $P_2 \boxtimes P_4$ has three columns C_1, C_2, C_3.

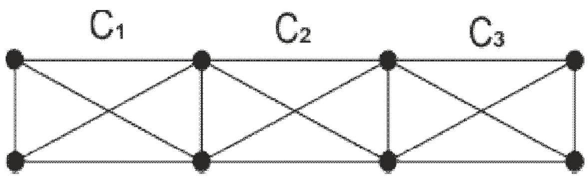

Figure 5: $P_2 \boxtimes P_4$

Here the first and last columns can be considered as two B_1 graphs which efficiently dominates the entire graph with $\gamma_e(P_2 \boxtimes P_4) = 2$.
Case: 4: Let n = 5. In this case the graph $P_2 \boxtimes P_5$ has four columns C_1, C_2, C_3, C_4.

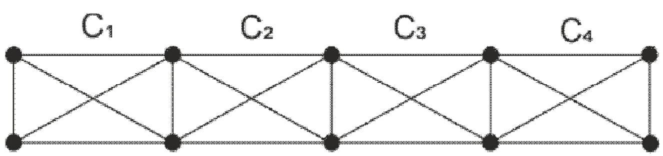

Figure 6: $P_2 \boxtimes P_5$

Here the first two columns can be treated as one B_2 graph of $\gamma_e[B_2] = 1$and last column as one B_1 graphs of $\gamma_e[B_1] = 1$. These two sub graphs can be considered as two distinct blocks which efficiently dominates the entire graph with $\gamma_e(P_2 \boxtimes$ $P5=2$.
Case:5: Let n = 6. In this case the graph $P_2 \boxtimes P_6$ can be partitioned in to two B_2 graph of $\gamma_e[B_2] = 1$.
Hence $\gamma_e(P_2 \boxtimes P_6) = 2$. This completes the proof.

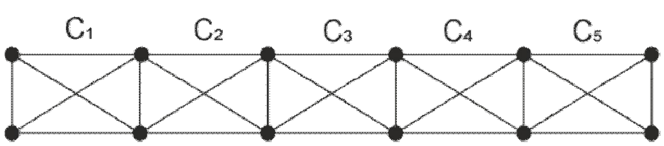

Figure 7: $P_2 \boxtimes P_6$

4.1.3.Lemma: $\gamma_{2fd}(P_2 \boxtimes P_n) = \begin{cases} 2 \ if \ n = 2, 3 \\ 4 \ if \ n = 4, 5, 6 \end{cases}$

Proof: To obtain 2-fd set, we need two vertices from B_1 and from B_2 with γ_{2fd} is 2. To establish the result, the entire graph should be partitioned into sub graphs of B_1 and B_2 as above. Each sub graph contributes 2 to the domination number. Hence the proof.

4.1.4.Proposition: For any $n \geq 6$, $\gamma_e(P_2 \boxtimes P_n) = \left\lceil \frac{n}{3} \right\rceil$.

Proof: Let E be the efficient dominating set of $P_2 \boxtimes P_n$. To describe the efficient dominating set, consider the sub graphs B_1 and B_2.

If $n \equiv 0 \ (mod\ 3)$, the graph $P_2 \boxtimes P_n$ can be partitioned efficiently by $\frac{n}{3}$. number of B_2 graphs.
If $n \equiv 1 \ (mod\ 3)$,, the graph $P_2 \boxtimes P_n$ can be partitioned efficiently by $\frac{n-5}{3}$ number of B_2 graphs along with two B_1 sub graphs.
If $n \equiv 2 \ (mod\ 3)$,, the graph $P_2 \boxtimes P_n$ can be partitioned efficiently by $\frac{n-2}{3}$ number of B_2 graphs along with one B_1 sub graph. This completes the proof.

4.1.5.Proposition: For any $n \geq 6$, $\gamma_{2fd}(P_2 \boxtimes P_n) = 2 \times \left\lceil \frac{n}{3} \right\rceil$.

Proof: Since each sub graph contributes 2 to the 2-fd set, the result follows.

4.1.6.Observation: For $n \geq 2$, $\gamma_{2fd}(P_2 \boxtimes P_n) = 2 \times \gamma_e(P_2 \boxtimes P_n)$.

4.1.7.Theorem: For any $m, n \geq 2$, $\gamma_e(P_m \boxtimes P_n) = \left\lceil \frac{m}{3} \right\rceil \times \left\lceil \frac{n}{3} \right\rceil$.

Proof: To establish the result, first prove the result for some values for m.
Case: 1: For m = 1 and m = 2 we have the results. $\gamma_e(P_m \boxtimes$ $Pn= n3$.

Case: 2: For m = 3. Here we can have one more sub graph B_3 as $P_3 \boxtimes P_3$ with efficient domination number $\gamma_e[B_3] = 1$.

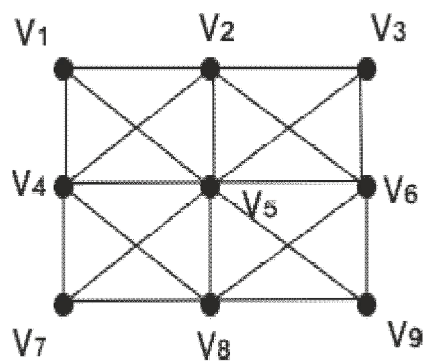

Figure 8: B_3

Now if $n \equiv 0 \ (mod\ 3)$, the graph $P_3 \boxtimes P_n$ can be partitioned efficiently by $\frac{n}{3}$ number of B_3 graphs.
If $n \equiv 1 \ (mod\ 3)$, the graph $P_3 \boxtimes P_n$ can be partitioned efficiently by $\frac{n-5}{3}$ number of B_3 graphs along with two B_2 sub graphs.
If $n \equiv 2 \ (mod\ 3)$, the graph $P_3 \boxtimes P_n$ can be partitioned efficiently by $\frac{n-2}{3}$ number of B_3 graphs along with one B_2 sub graph. Hence $\gamma_e(P_3 \boxtimes P_n) = \left\lceil \frac{n}{3} \right\rceil$.
Case:3: Now for $m \geq 4$ and for $n \geq 2$ the graph can be partitioned as blocks of $P_1 \boxtimes P_n$, $P_2 \boxtimes P_n$ or $P_3 \boxtimes P_n$. Hence for any $, n \geq 2$, $\gamma_e(P_m \boxtimes P_n) = \left\lceil \frac{m}{3} \right\rceil \times \left\lceil \frac{n}{3} \right\rceil$, which completes the proof.

4.2.Sierpinski Graphs S (r, K_n)

We already defined and find $\gamma_e[S(2, K_n)]$ for Sierpinski Graphs please refer [Sierpinski Graphs]. The same can be extended for any r.

$\gamma_e[S(r, K_n)] = n^{(r-1)}$, for $r \geq 1$ and $n \geq 3$.

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