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Estimation of the effect of the composition of the wall on the comfort of a building

Ivan Amaya-Ruiz[1], Carlos A. Fuentes-Perez[1], Gerardo J. Arista-Gonzalez[2], Maria T. Sanchez-Medrano[1], Elena F. Izquierdo Kulich[3], Edgardo J. Suarez-Dominguez[1*]
[1.] FADU. Universidad Autónoma de Tamaulipas, Tampico, Tamaulipas, Mexico. *Edgardo.suarez@uat.edu.mx
[2.] Habitat Faculty. UASLP. San Luis Potosi, SLP. México.
[3.] Department of Chemistry-Physics, Faculty of Chemistry, University of Havana, Havana, Cuba.

ABSTRACT

Temperature is one of the parameters that most significantly influences the comfort of living quarters, requiring the use of air conditioning systems, depending on weather conditions, to maintain this parameter within an appropriate range of values. The use of construction materials with relatively low amounts of thermal conductivity contributes to keeping the interiors at a comfortable temperature with a relatively low energy cost. In this work, a mathematical model is presented describing temporal behavior of the temperature profile through a wall composed of three different types of materials, which can be used to evaluate the effect of the composition of the wall on the comfort of a building.

Key words: vernacular house walls, heath transfer in walls, heat model.

1. INTRODUCTION

Currently, buildings need to be efficient and reduce emissions and reduce energy. To carry out an adequate design of the building walls, a set of experimental methods are used to recognize the thermal behaviours [1] The calculation is necessary because it allows for efficient design of buildings and envelopes [2] and are related to the comfort of the spaces. Some eco-efficient home design techniques may be a responsible answer to environmental construction problems [3,4].

Some of the modern materials are aimed at being obtained from nature or in places around the buildings that we call vernacular architecture [5], there are other more elaborate forms of improvement such as intelligent modification of the components [6].

Since the main current components imply concrete, structural elements of different materials have been evaluated, highlighting that derivatives of cement or concrete mixtures transmit higher heat than others as shown by [7].

Other techniques are related to ecological surface alternatives such as efficient roofs [8] characterized by a significant reduction in heat transfer. Simulation analyzes are therefore more relevant to select the appropriate materials [9] where even more sophisticated transfer processes can occur that can increase the comfort of the building where it is applied [10]. Thermal analysis is important not only in materials but also in cities monitoring [11] and it has many other uses as in fluid dynamics [12].

2. METHOD

In the present work, we develop a theoretical model for the determination of the temperature changes of heat applied to a wall that can present up to three different material components using Euclidean coordinate geometry.

We determined thermal diffusivity of three materials, commonly used in the elaboration of walls for buildings. To determine the thermal conductivity of the walls, a thermal diffusivity measuring device model: KD2 Pro Thermal Properties Analyzer from Decagon Devices, Inc. was used, evaluating 10 samples for each material.

2.1 Obtaining the model

The system considered is a wall made up of three different materials [13], through which heat flow is produced, generated by the temperature difference between both sides of the wall, as illustrated in Figure 1.

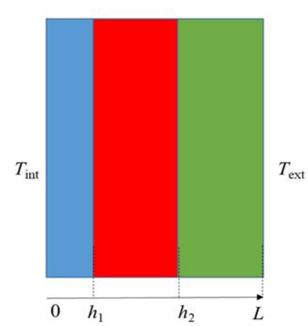

Figure 1: Diagram of the wall of a room made up of three different construction materials, where Tint is the temperature inside, and Text is the temperature outside

From the equations of temperature change in the Cartesian coordinate system and considering that the heat transfer mechanism is by conduction in the x-direction, we obtain:

$$\begin{cases} \frac{\partial T_1(t,x)}{\partial t} = \alpha_1 \frac{\partial^2 T_1(t,x)}{\partial x^2} \ : \ 0 < x < h_1 \\ \frac{\partial T_2(t,x)}{\partial t} = \alpha_2 \frac{\partial^2 T_2(t,x)}{\partial x^2} \ : \ h_1 < x < h_2 \\ \frac{\partial T_3(t,x)}{\partial t} = \alpha_3 \frac{\partial^2 T_3(t,x)}{\partial x^2} \ : \ h_2 < x < L \end{cases} \tag{1}$$

where h_1 is the thickness of material 1, $h_2 - h_1$ the depth of material 2, $L-(h_2+h_1)$ the thickness of material 3, and the coefficient α_j represents the thermal diffusivity of the material, defined as:

$$\alpha_j = \frac{k_j}{\rho_j C_{p,j}} \tag{2}$$

where $C_{p,j}$ is the heat capacity, ρ_j the density and k_j is the coefficient of thermal conductivity of material j.

It was considered that the system is initially in steady-state, with a constant temperature difference between the exterior and interior of the wall. A *step-jump* type change in the exterior temperature is established, such that temperature profile evolves in time until it reaches a new steady state.

In steady-state, the time derivative that appears on the left side of each of the equations becomes zero, in such a way that the system of ordinary differential equations is obtained:

$$\begin{cases} 0 = \alpha_1 \frac{\partial^2 T_1(x)}{\partial x^2} \ : \ 0 < x < h_1 \\ 0 = \alpha_2 \frac{\partial^2 T_2(x)}{\partial x^2} \ : \ h_1 < x < h_2 \\ 0 = \alpha_3 \frac{\partial^2 T_3(x)}{\partial x^2} \ : \ h_2 < x < L \end{cases} \tag{3}$$

whose exact analytical solution is given by:

$$\begin{cases} T_1(x) = C_{10} + C_{11}x \ : \ 0 < x < h_1 \\ T_2(x) = C_{20} + C_{21}x \ : \ h_1 < x < h_2 \\ T_3(x) = C_{30} + C_{31}x \ : \ h_2 < x < L \end{cases} \tag{4}$$

where C_{jk} are the integration coefficients, the value of which is determined from the boundary conditions:

$$\begin{aligned} &T_1(0) = T_0; T_1(h_1) = T_2(h_1) \\ &J_1(h_1) = J_2(h_1); T_2(h_2) = T_3(h_3) \\ &J_2(h_2) = J_3(h_2); T_3(L) = T_L \end{aligned} \tag{5}$$

where J_k is the heat flux:

$$J_k = -\alpha_k \frac{\partial T_k}{\partial x} \tag{6}$$

Determining the integration constants and appropriately substituting the steady state temperature profile is obtained:

$$\begin{cases} T_{1,s}(x) = T_0 + \left(\frac{T_L - T_0}{h_1\left(1-\frac{\alpha_1}{\alpha_2}\right) + h_2\left(\frac{\alpha_1}{\alpha_2} - \frac{\alpha_1}{\alpha_3}\right) + L\frac{\alpha_1}{\alpha_3}} \right) x \ : \ 0 < x < h_1 \\ T_{2,s}(x) = \left(\frac{\left(h_2\left(\frac{\alpha_1}{\alpha_3} - \frac{\alpha_1}{\alpha_2}\right) - L\frac{\alpha_1}{\alpha_3}\right)(T_L - T_0)}{h_1\left(1-\frac{\alpha_1}{\alpha_2}\right) + h_2\left(\frac{\alpha_1}{\alpha_2} - \frac{\alpha_1}{\alpha_3}\right) + L\frac{\alpha_1}{\alpha_3}} + T_1 \right) + \left(\frac{\frac{\alpha_1}{\alpha_2}(T_L - T_0)}{h_1\left(1-\frac{\alpha_1}{\alpha_2}\right) + h_2\left(\frac{\alpha_1}{\alpha_2} - \frac{\alpha_1}{\alpha_3}\right) + L\frac{\alpha_1}{\alpha_3}} \right) x \ : \ h_1 < x < h_2 \\ T_{3,s}(x) = \left(T_1 - \frac{\frac{\alpha_1}{\alpha_3}(T_L - T_0)}{h_1\left(1-\frac{\alpha_1}{\alpha_2}\right) + h_2\left(\frac{\alpha_1}{\alpha_2} - \frac{\alpha_1}{\alpha_3}\right) + L\frac{\alpha_1}{\alpha_3}} L \right) + \left(\frac{\frac{\alpha_1}{\alpha_3}(T_L - T_0)}{h_1\left(1-\frac{\alpha_1}{\alpha_2}\right) + h_2\left(\frac{\alpha_1}{\alpha_2} - \frac{\alpha_1}{\alpha_3}\right) + L\frac{\alpha_1}{\alpha_3}} \right) x \ : \ h_1 < x < h_2 \end{cases} \tag{7}$$

For non-steady state behaviour, the deviation variable is:

$$u_k(t,x) = T_k(t,x) - T_{k,i}(x) \tag{8}$$

which represents the difference between the non-steady value and the initial steady-state value. The system of partial differential equations gives the non-steady state behavior of the deviation variable:

$$\begin{cases} \frac{\partial u_1(t,x)}{\partial t} = \alpha_1 \frac{\partial^2 u_1(t,x)}{\partial x^2} \ : \ 0 < x < h_1 \\ \frac{\partial u_2(t,x)}{\partial t} = \alpha_2 \frac{\partial^2 u_2(t,x)}{\partial x^2} \ : \ h_1 < x < h_2 \\ \frac{\partial u_3(t,x)}{\partial t} = \alpha_3 \frac{\partial^2 u_3(t,x)}{\partial x^2} \ : \ h_2 < x < L \end{cases} \tag{9}$$

the Laplace transform is applied to find a solution, considering time as an independent variable, such that the system of partial differential equations (9) transformed into the system of ordinary differential equations:

$$\begin{cases} su_1(s,x) = \alpha_1 \frac{\partial^2 u_1(s,x)}{\partial x^2} \ : \ 0 < x < h_1 \\ su_2(s,x) = \alpha_2 \frac{\partial^2 u_2(s,x)}{\partial x^2} \ : \ h_1 < x < h_2 \\ su_3(s,x) = \alpha_3 \frac{\partial^2 u_3(s,x)}{\partial x^2} \ : \ h_2 < x < L \end{cases} \tag{10}$$

whose exact analytical solution is given by:

$$\begin{cases} u_1(s,x) = C_{12} e^{\frac{x}{\alpha_1}\sqrt{s\alpha_1}} + C_{11} e^{-\frac{x}{\alpha_1}\sqrt{s\alpha_1}} \ : \ 0 < x < h_1 \\ u_2(s,x) = C_{22} e^{\frac{x}{\alpha_2}\sqrt{s\alpha_2}} + C_{21} e^{-\frac{x}{\alpha_2}\sqrt{s\alpha_2}} \ : \ h_1 < x < h_2 \\ u_3(s,x) = C_{32} e^{\frac{x}{\alpha_3}\sqrt{s\alpha_3}} + C_{31} e^{-\frac{x}{\alpha_3}\sqrt{s\alpha_3}} \ : \ h_2 < x < L \end{cases} \tag{11}$$

the initial value theorem of the Laplace transform allows the integration constants:

$$\lim_{t \to 0} f(t) = \lim_{s \to \infty} sF(s) \tag{12}$$

such that, taking into account that for a time equal to zero $u_k = 0$, we obtain:

$$C_{12} = C_{22} = C_{32} = 0 \tag{13}$$

and, therefore, the system of equations (11) is reduced to:

$$\begin{cases} u_1(s,x) = C_{11} e^{-\frac{x}{\alpha_1}\sqrt{s\alpha_1}} \ : \ 0 < x < h_1 \\ u_2(s,x) = C_{21} e^{-\frac{x}{\alpha_2}\sqrt{s\alpha_2}} \ : \ h_1 < x < h_2 \\ u_3(s,x) = C_{31} e^{-\frac{x}{\alpha_3}\sqrt{s\alpha_3}} \ : \ h_2 < x < L \end{cases} \tag{14}$$

the boundary conditions are:

$$\begin{aligned} C_{11} e^{-\frac{h_1}{\alpha_1}\sqrt{s\alpha_1}} &= C_{21} e^{-\frac{h_1}{\alpha_2}\sqrt{s\alpha_2}} \\ C_{21} e^{-\frac{h_2}{\alpha_2}\sqrt{s\alpha_2}} &= C_{31} e^{-\frac{h_2}{\alpha_3}\sqrt{s\alpha_3}} \\ C_{31} e^{-\frac{L}{\alpha_3}\sqrt{s\alpha_3}} &= \frac{\left(T_{2,f}(L) - T_{2,i}(L)\right)}{s} \end{aligned} \tag{15}$$

such that the Laplace transform of the temperature profile is:

$$\begin{cases} u_1(s,x)=\frac{(T_{2,f}(x)-T_{2,i}(x))}{s}\exp\left(\left(\left(\frac{\sqrt{\alpha_2}-\sqrt{\alpha_1}}{\sqrt{\alpha_1}\sqrt{\alpha_2}}\right)h_1+\left(\frac{\sqrt{\alpha_3}-\sqrt{\alpha_2}}{\sqrt{\alpha_2}\sqrt{\alpha_3}}\right)h_2+\frac{L\sqrt{\alpha_1}-x\sqrt{\alpha_3}}{\sqrt{\alpha_1}\sqrt{\alpha_3}}\right)\sqrt{s}\right) : h_2<x<L \\ u_2(s,x)=\frac{(T_{2,f}(x)-T_{2,i}(x))}{s}\exp\left(\left(\left(\frac{\sqrt{\alpha_3}-\sqrt{\alpha_2}}{\sqrt{\alpha_2}\sqrt{\alpha_3}}\right)h_2+\frac{L\sqrt{\alpha_2}-x\sqrt{\alpha_3}}{\sqrt{\alpha_2}\sqrt{\alpha_3}}\right)\sqrt{s}\right) : h_1<x<h_2 \\ u_3(s,x)=\frac{(T_{2,f}(x)-T_{2,i}(x))}{s}\exp\left(\left(\frac{L-x}{\sqrt{\alpha_3}}\right)\sqrt{s}\right) : 0<x<h_1 \end{cases} \quad (16)$$

Applying the inverse of the Laplace transform we obtain:

$$\begin{cases} T_1(x,t)=(T_{1,f}(x)-T_{1,i}(x))\operatorname{erfc}\left(\frac{\left(\left(\frac{\sqrt{\alpha_2}-\sqrt{\alpha_1}}{\sqrt{\alpha_1}\sqrt{\alpha_2}}\right)h_1+\left(\frac{\sqrt{\alpha_3}-\sqrt{\alpha_2}}{\sqrt{\alpha_2}\sqrt{\alpha_3}}\right)h_2+\frac{L\sqrt{\alpha_1}-x\sqrt{\alpha_3}}{\sqrt{\alpha_1}\sqrt{\alpha_3}}\right)}{2\sqrt{t}}\right)+T_{1,i}(x) : 0<x<h_1 \\ T_2(x,t)=(T_{2,f}(x)-T_{2,i}(x))\operatorname{erfc}\left(\frac{\left(\left(\frac{\sqrt{\alpha_3}-\sqrt{\alpha_2}}{\sqrt{\alpha_2}\sqrt{\alpha_3}}\right)h_2+\frac{L\sqrt{\alpha_2}-x\sqrt{\alpha_3}}{\sqrt{\alpha_2}\sqrt{\alpha_3}}\right)}{2\sqrt{t}}\right)+T_{2,i}(x) : h_1<x<h_2 \\ T_3(x,t)=(T_{3,f}(x)-T_{3,i}(x))\operatorname{erfc}\left(\frac{\left(\frac{L-x}{\sqrt{\alpha_3}}\right)}{2\sqrt{t}}\right)+T_{3,i}(x) : h_2<x<L \end{cases} \quad (17)$$

Considering the system in which:

$$\alpha_1=0.3;\alpha_2=0.1;\alpha_3=0.5$$
$$h_1=0.4;h_2=0.5;L=1$$

where the initial and final initial states are defined by:

$$T_{L,i}=50;T_{L,f}=60$$
$$T_{0,i}=30;T_{L,f}=40$$

3. RESULTS AND DISCUSSION

The model predicts the temporal evolution of the temperature profile shown in Figure 2.

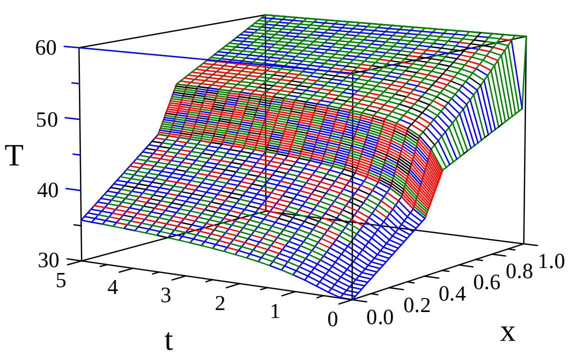

Figure 2: Temporal behaviour of the temperature profile

The effect of the materials that make up the wall on its thermal insulation capacity is quantified through a time constant that represents the time required for the interior temperature of the wall to reach 50% of its final value when it has been produced. A change in the outside temperature, so that as this time increases it is possible to keep the room within a more comfortable temperature range.

The temporal constant is estimated from the model, considering in this case, the behavior of temperature $T_1(x,t)$ for $x=0$, so:

$$\tau=1.1\left(\left(\frac{\sqrt{\alpha_2}-\sqrt{\alpha_1}}{\sqrt{\alpha_1}\sqrt{\alpha_2}}\right)h_1+\left(\frac{\sqrt{\alpha_3}-\sqrt{\alpha_2}}{\sqrt{\alpha_2}\sqrt{\alpha_3}}\right)h_2+\frac{L\sqrt{\alpha_1}}{\sqrt{\alpha_1}\sqrt{\alpha_3}}\right)^2 \quad (18)$$

If values of h_1, h_2 and L are expressed like a function of the thickness ε_j of each of the material or layers, equation 18 is rewritten:

$$\tau=1.1\left(\frac{\varepsilon_1}{\sqrt{\alpha_1}}+\frac{\varepsilon_2}{\sqrt{\alpha_2}}+\frac{\varepsilon_3}{\sqrt{\alpha_3}}\right)^2 \quad (19)$$

material 1 is the one found inside the room and material 3 being in contact with the outside. Note that the value of this time constant increases with the thickness of the material and the parameter α_j. decreases.

Table 1 shows the thermal diffusivity of three materials, used for the construction of a dividing wall 10 cm thick. Figure 3 shows the behavior of the time constant for the thickness of the wood (interior) keeping the same thickness of concrete and the total one; we see the increase in the time consistent with the depth of these materials, where that most significant influence is wood allowing the reduction of heat transfer.

Table 1: Physical properties and thickness of each material on the wall (Results with a maximum error of 2%)

Material	position	Thermal diffusivity cm2/h	Thickness cm
Plaster	outside	16.935	5 - *x*
Wood	Inside	3.958	5
concrete	means	17.901	*x*

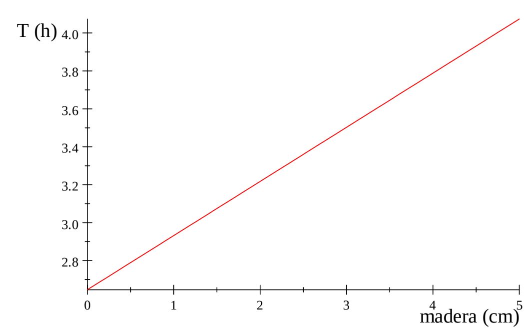

Figure 3: The behavior of the time constant for the thickness of the wood, keeping steady the width of the cement and the total thickness of 10cm.

5. CONCLUSION

From the law of conservation of energy, a system of temporal partial differential equations was obtained expressing temperature changes spatially and temporally in a system made for three solids with different thermal properties.

Three layers next to each other correspond to construction systems in the Mexican vernacular house walls; the system of partial differential equations was solved considering a temperature difference between the interior and the exterior of the wall, where the latter varies as a function of time. The equations obtained allow estimating the behaviour of the spatial temperature profile for a time. A dynamic parameter was determined, in this case, the time constant, which allows quantifying the comfort of the house based on the composition of the wall. This constant expresses the time necessary for the interior temperature of the wall to take a stationary value in a time when a change in the external temperature has occurred. Such that the home is considered more comfortable in temperatures around 25°C, it implies that time constant increases whenever the outside temperature is higher than the inside temperature of the studied element.

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