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# Mathematical and Instrumental Methods for Calculating the Optimal Scheme of the Electrical Network

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# ABSTRACT

This article discusses the optimal location of power lines. The authors review various modifications of the transport problem and consider instrumental methods for its solution. A comparative analysis of various methods of solution is carried out. Problems with various restrictions on power transmission are considered. Examples of application of mathematical methods for solving transport problems in real practice for solving problems in various fields of professional activity are given.

**Key words:** Optimality criterion, transport problem, linear programming, mathematical energy problems, transport problem with power transit, throughput, initial reference plan, transport matrix, specific cost of energy transmission

#### **1.INTRODUCTION**

The source of power for all, without exception, elements of the city's infrastructure is the City's electric networks. Like a web, they cover a certain territory (city, microdistrict or village) entirely with their cable and overhead power lines and current pipelines that intersect at substations or devices for electricity distribution, representing a complex multi-level engineering system. Already at the planning and design stage, urban electric networks are considered as a set of global works to create a workable power supply scheme for residential (intended for population) territories with a predetermined number of industrial and internal infrastructure facilities, and with the prospect of its further development.

The development of the city's power supply scheme should begin with the design of urban electric networks, since all elements of such electric networks are strictly connected and mutually conditioned. It is the design at the early stages of development and approval that allows the best way to plan and implement the location of certain components of the future urban electric network even before construction begins. The design of urban electric networks should take into account the needs of all prospective electricity consumers, regardless of their level of scale and belonging, and on the basis of these calculations, organize electricity supply according to the most optimal scheme. The need to design urban electric networks is explained by economic reasons. Early planning of the city's power supply system makes it possible to calculate the cost of upcoming construction, installation and debugging work that will be required for the supply

In addition, a well-thought-out project helps to take into account various circumstances that can affect the specified operation of electric networks, which, in turn, allows you to provide for the availability of backup or emergency modes of operation of power supply. Also, the design of urban electric networks involves the development of all necessary measures to ensure the safety and comfort of operation of the city's power supply system and minimize its impact on the environment, taking into account the future development of the city's infrastructure.

The power sources (SP) of the city's power supply system are city power stations and step-down substations. A power center (CP) is a distribution device of the generating voltage of an electric station or a secondary voltage distribution device of a step-down substation, to whose buses the distribution networks of a given area are connected. Electric power stations are usually thermal power plants that provide heat and partially electric power to municipal and industrial facilities.

The main groups of electricity consumers in the city's power supply systems are: municipal consumers; industrial enterprises; electrified urban and suburban transport; in some cases, settlements, industrial and agricultural enterprises of suburban zones. Municipal consumers of electricity are residential, administrative, cultural, educational, medical, and similar buildings.

For any energy company, it is necessary to deliver its products, heat and electricity to the consumer without interruption. When transmitting electricity over long distances in different directions, you need to make an optimal scheme of the electrical network. Often, when it is necessary to minimize the cost of electricity transmission, it is necessary to take into account such factors as the capacity of the transmission line, the possibility of transit of the transmitted power, and more. Elena I. Mironova et al., International Journal of Advanced Trends in Computer Science and Engineering, 9(4), July - August 2020, 4474 - 4482

## 2.METHODOLOGY

Taking into account the above, we need to do thorough work to find the optimal location of power lines, find ways to transport finished products to consumers with minimal costs and in the shortest possible time, determine the type of transmission line taking into account its capacity, and more. Using mathematical methods, it is possible to unify the solution of these problems, for example, using a model of a transport problem. We formulate a condition for the power transmission problem: we need to determine the scheme of the electric network from n power sources to m consumers along power lines, at which the total cost of power transmission will be minimal. At the same time, all the volumes of power sources, the amount of power required by the consumer and the cost of transferring one unit of power (unit cost) are known [1].

The problem is solved in the stages shown in figure 1.

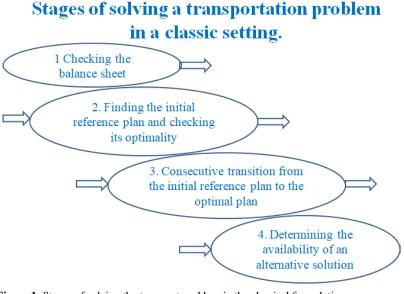


Figure 1: Stages of solving the transport problem in the classical formulation

Let's focus on possible implementations of some stages of the solution. To find the initial reference plan, you can use various methods shown in figure 2.

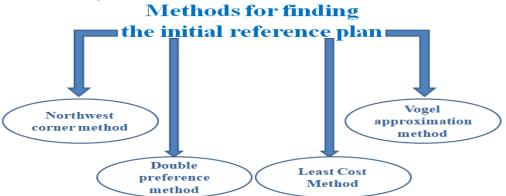


Figure 2: Methods for finding the initial reference plan for a transport problem

## **3.DISCUSSION AND RESULTS**

When implementing the Northwest corner method, the table is filled in without taking into account the unit cost. From the first source to the first consumer is sent or the necessary amount of power (assuming that the needs of the first user is less than the available supply from the first source), or the amount that can provide the first source (assuming that the needs of the first user more available stock from the first source). Then, in the first case, we send the remainder available from the first source to the second consumer, and in the second case, we take the missing volume of products for the first consumer from the second source and continue using this algorithm until we meet all the needs of all consumers. Considered example. The projected power supply system has five nodes with power sources A1, A2, A3, A4, A5, which have respectively 25, 50, 75, 25, 75 units of power. In this case, the four nodes consumers B1, B2, B3, B4 need 50, 45, 50 and 105 units of power, respectively. The mutual arrangement of nodes is such that the construction of possible lines of the electric network from any source to all consumers. The cost of transferring one unit of power from each source to all nodes to consumers is set in table 1.

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	Bl	<b>B</b> 2	<i>B</i> <sub>3</sub>	$B_{\mathfrak{A}}$
$A_{l}$	300	200	800	100
$A_2$	200	500	250	300
$A_3$	900	400	600	500
$A_{\mathfrak{A}}$	700	300	1000	300
$A_5$	400	600	700	400

Table 1: Source data for the transport task

The amount of reserves at the sources is equal to 25+50+75+25+75=250 power units, the volume of needs is equal to 50+45+50+105=250 units of capacity.

The volume of reserves at sources is equal to the volume of needs, the balance condition is met and this task is closed

Find the original reference plan using the Northwest corner method.

We will send the entire power reserve from the first source to the first consumer, but they will need another 25 units of power, which we take from the second source. At the same time, the needs of the first consumer will be met, the

The initial reference plan has the following format X =0

> 0 0

The cost of the electric network is equal to the sum of the products of unit costs on the values of transmitted power from sources to consumers. In this case they are equal:

 $25 \cdot 300 + 25 \cdot 200 + 25 \cdot 500 + 20 \cdot 400 +$ 

+50.600+5.500+25.300+75.400=103000m onetary units.

It should be noted that from the point of view of theoretical electrical engineering, an electrical network is an electrical circuit and for this network, in particular, the first law of Kirchhoff is applicable. For each *i*-th power source, the sum of the power flowing through the lines to all consumer nodes is equal to the power of this source.

For each *i*-th consumer, the sum of the power flowing through the lines from all sources is equal to the power of this consumer. These power balances in each of the nodes are constraints when solving a transport problem. The total number of restrictions is equal to the number of source and consumer nodes (n+m).

It is known from theoretical electrical engineering that for any electrical network, the number of independent equations compiled according to the first Kirchhoff law is one less than the number of nodes and is (n+m-1). Therefore, the number of independent constraints is (n+m-1)1). The number of basic variables is also equal to (n+m-1). Each basis variable  $x_{ii}$  corresponds to the presence of a line in the diagram between nodes i and j, since the power flowing between nodes *i* and *j* is not equal to 0.

reserves of the first source are exhausted, and the second source will still have 25 units of goods, which we will send to the second consumer. As a result, the reserves of the first and second sources will be zero. We will send the reserves of the third source to the second consumer (20 units of power), the third (50 units of power), and the fourth (5 units of power). As a result, all consumers, except the last one, will be satisfied in their needs. And the missing power to B4 will be directed from the fourth and fifth sources.

25	0	0	0 )	
25	25	0	0	
0	20	50	5.	
0	0	0	25 75	
0	0	0	75)	

When finding the initial reference plan using the lowest cost method, you need to select the cell with the smallest cost for transmitting a unit of power, and send the maximum possible capacity along this line. Then, from the remaining unit costs, choose the smallest, and send the power along this line. And so we continue until all the supplies are exhausted, and all consumers do not get what they want. Let's draw up an initial reference plan for the task already considered.

The lowest unit cost is 100 monetary units when transferring from the first source to the fourth consumer. The maximum possible power that can be sent over this line is 25 units available from the first source. Now we are looking for the lowest unit cost in the remaining part of the table (we exclude the first row, since the reserves of the first source are exhausted). The minimum cost is 200 monetary units between the second source and the first consumer. We will send the 50 units of power required by the first consumer along this line, while the second source will run out of supplies. We will send the inventory of the fourth source to the fourth consumer along the line with the lowest cost (from the remaining table) for transmission. And so we continue until all the power is distributed among consumers [2].

	( 0	0	0	25)	
	50	0	0	0	
The initial reference plan will look like this: $X =$	0	45	30	0	
	0	0	0	25	
The initial reference plan will look like this: $X =$	0	0	20	55)	

In this case, the cost of the electric circuit will be equal  $25 \cdot 100 + 50 \cdot 200 + 45 \cdot 400 +$ 

+30.600 + 25.300 + 20.700 + 55.400 = 92000monetary units.

When using the double preference method, we also search for the lowest value of the unit cost first by rows, then by columns (we mark the cells). The cells marked twice are filled first with values equal to the maximum possible volume of power. The distribution process continues until all power is distributed from sources and consumers are satisfied.

$$X = \begin{pmatrix} 0 & 0 & 0 & 25 \\ 25 & 0 & 25 & 0 \\ 0 & 45 & 0 & 30 \\ 0 & 0 & 0 & 25 \\ 25 & 0 & 25 & 25 \end{pmatrix}$$
 and the costs will be

Another method for finding the reference plan – the Vogel approximation method –involves finding the difference between the two smallest unit costs per transmission in each row and column. This difference is taken modulo. The power transfer starts with the cell with the lowest unit cost, standing in the row or column with the largest difference. Let's analyze the application of the method on the example of solving the above problem. We will add an additional row and column to the transport matrix, where we will write the found difference modules.

The lowest fare in the first column is 200, followed by 300 (these numbers are underlined in the table), the difference modulus is 100, and we write this number in the last row in the first column. In the second column, the lowest rates

Using this method, we will draw up an initial reference plan for the problem that has already been considered. Using this algorithm, the cells marked twice will be those with the cost of 100 and 200 in the first and second rows, respectively, and we will send the initial power to them.

As a result, we will send 25 units of power from the first source to the fourth consumer, and 25 units of goods that are necessary for the first consumer from the second source to the first consumer. And so we continue until the original reference is found. In the problem under consideration it will look like this:

are 250400 3025 the 00 diff 25 n400 4091 v50 write them in the second column of the last row, and continue this way until the last column. Similar differences are calculated for each row (the lowest row rates are shown in bold). Among the obtained absolute values of differences, we will select the largest – 350, which is in this case the third column. We start transferring capacity from the cell with the lowest unit cost in the selected third column. From the second source, we will send 50 units of power to the third consumer with a rate of 250 monetary units per unit of power. The reserves of the second source are exhausted, the needs of the third consumer are satisfied, so we exclude the second row and the third column from consideration. The transport matrix after the first iteration is completed is shown in table 2.

**Table 2:** Transport table after the first iteration.

Users Sources	50	45	50	105	
25	<u>300</u>	<u>200</u>	800	<u>100</u>	100
50	<u>200</u>	500	<u>250</u>	<u>300</u>	50
75	900	400	<u>600</u>	500	100
25	700	<u>300</u>	1000	300	0
75	400	600	700	400	0
	100	100	350	200	

Then we proceed in the same way.

Now the largest modulus of difference 200 has a fourth column. And from the first source to the fourth consumer, we will send the entire available stock -25 units of power,

since this unit cost is the lowest in the entire column. We exclude the first line from consideration, since all the reserves of the first source are used. The results are shown in table 3.

Table 3: Transport table after the second itera	tion.
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Users Sources	50	45	50	105	
25	<u>300</u>	<u>200</u>	800	<u>100</u>	100
50	200	500	250 50	300	_
75	900	400	600	500	100
25	700	<u>300</u>	1000	<u>300</u>	0
75	<u>400</u>	600	700	400	0
	100	100	_	200	

#### Table 4: Transport table after the third iteration.

equal

Users Sources	50	45	50	105	
25	300	200	800	100 25	_
50	200	500	250 50	300	_
75	900	400	600	500	
25	700	300	1000	300	
75	400	600	700	400	
			_		-

														( 0	0	0	25	
													k like $X =$	0	0	50	0	
	W	e con	tinue this	way until	we fi	nd the	origina	l refe	rence plan	, whic	ch wi	11 100	k like $X =$	0	45	0	30	and
														0	0	0	25	
														50	0	0	25)	
the	cost	of	energy	transfer	in	this	case	is	Sol	ution	of	the	proposed	probl	em	by	means	of

Solution of the proposed problem by means of MATHCAD shown in figure 3.

 $25 \cdot 100 + 50 \cdot 250 + 45 \cdot 400 + 30 \cdot 500 + 25 \cdot 300 + 50 \cdot 400 + 25 \cdot 400 = 85500$ monetary unit.

Figure 3: Solving the problem using MATHCAD.

The choice of a rational power supply scheme is often faced with a situation that it is impossible to directly lay a power line from a specific source to one of the consumers. This can be the result of many reasons: the safe distance of a power line to a residential building, natural conditions, and others. Let's try to analyze the solution of the problem of optimal location of power lines with this in mind.

If it is impossible to transfer power between some sources and consumers, we have a model of the problem with ORIGIN = 1

prohibitions. In this case, the solution is to block the transfer by increasing the unit cost (approximately 100 times). In the above example, we will prohibit the transfer from the second source to the third consumer. To do this in this direction we will take the unit cost in the amount of  $250 \cdot 100 = 25000$  monetary unit. Then the solution to the problem will look like this (figure 4).

Figure 4: Solving a problem with bans using MATHCAD.

As a result, there is no transfer of power from the second source to the third consumer, but the cost of the electric circuit, as expected, is greater than in the classical setting. If necessary, the task can take into account a certain amount of power transfer from a specific source to a specific consumer. To do this, enter the desired delivery and do not change it during the solution. In this problem, you need to supply 25 units of power from the first source to the first consumer. The solution to the modified problem is shown in figure 5.

Figure 5: Solution of the problem with fixed energy transfers by means of MATHCAD.

When planning optimal schemes of the electric network in tasks, power engineers often face restrictions on the capacity of power transmission lines. This value depends on the current, voltage, and other characteristics. This is one of the most important factors. Throughput limits the maximum active power that can be transmitted over the line, taking into account all technical limitations. Let's assume that only 30 units of power can be sent from the third source to the second consumer in this problem. When making a decision, proceed as follows:

ORIGIN = 1

- the second column is divided into two conditional consumers with the necessary needs of 30 and 15 power units, i.e. the total power must be 45 units, in accordance with the task condition, and one value must be equal to the capacity;

- and the cell corresponding to the "excess" power needs to be blocked, taking a very large value of the specific cost of energy transfer.

Using MATHCAD tool methods, the solution looks like this (figure 6).

Figure 6: Solving a problem with throughput using MATHCAD.

Often, it is advisable to design electrical network schemes in such a way that power is transmitted through so-called transit nodes. These intermediate nodes can be either power supply nodes or consumer nodes. In such situations, energy is not transferred directly from sources to consumers, but from sources to end users via intermediate (transit) consumers or sources.

Solving a transport problem with power transit with n power sources and m consumers, we give all nodes of the scheme a through numbering 1, 2, 3, ..., (n+m).

The power transfer from node i to node j does not depend on the direction, so the specific cost matrix will be symmetrical relative to the main diagonal, since  $c_{ij}=c_{ji}$ . The unit cost of transmitting transit power through any one is zero, while the elements of the main diagonal are zero in the matrix of unit costs.

It should be noted that the transit variable is included in the solution of the problem with a minus sign, this follows from the laws of electrical engineering.

The objective function and restrictions in this problem are the same as in the classical transport problem.

The algorithm for solving the problem with power transit does not differ from the algorithm for solving the transport problem in the classical formulation. A transport matrix is compiled. note that in this case it will be a square of size (n+m) by (n+m). We should not forget that the transit

power can be transmitted through any node, and when solving all the transit variables are basic.

Let's consider an algorithm for solving a transport problem with power transit using an example.

The projected power supply system has two nodes with power sources A1, A2, which have respectively 110, 40 units of power. At the same time, the two consumer nodes B3 and B4 need 80 and 70 units of power, respectively. The mutual arrangement of nodes is such that the construction of the electric network lines between any nodes is possible. The cost of transmitting one unit of power over lines between nodes is  $c_{12}=3$ ,  $c_{13}=10$ ,  $c_{13}=4$ ,  $c_{23}=6$ ,  $c_{24}=7$ ,  $c_{34}=8$  conventional uni/ power units. Find the optimal scheme of the electrical network. Let's solve the problem taking into account the transit capacity.

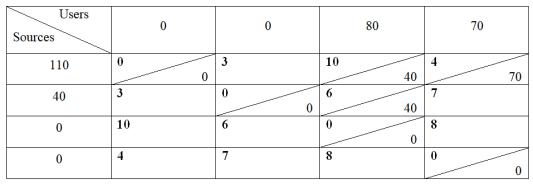
The issue has already adopted end-to-end numbering. Let's make a transport matrix, it will have a dimension of 4 by 4, since there are only 4 nodes in the system.

Note that the first two nodes are sources, then their power consumption is 0, and the third and fourth nodes are consumers, so they can not emit anything, so their power as sources is 0.

All transit variables are basic, so along the main diagonal, all cells will be "filled" with zero dummy delivery.

Let's make an initial reference plan using the doublepreference method. The results of the decision are shown in table 5.

Table 5: Initial reference plan for a transport task with capacity transit



In this case, the cost of the electric circuit will be equal  $10 \cdot 40 + 4 \cdot 70 + 6 \cdot 40 = 920$  monetary units, and the

initial reference plan will look like:  $X = \begin{bmatrix} 40 & 70 \\ 40 & 0 \end{bmatrix}$ . Ir

from the first source are transmitted to the second consumer.

Let's try to optimize the initial plan. Let's perform a redistribution of power transfer, for example, by the method of potentials [2]. We get the transport matrix shown in table 6.

other words, 40 units of power from both sources are transmitted to the first consumer, and all 70 units of power

 Table 6: Solution of the transport problem with capacity transit

Users Sources	0	0	80	70
110	0 0	3 40	10	4 70
40	3	0 - 40	6 80	7
0	10	6	0 0	8
0	4	7	8	0 0

With this scheme, 40 units of power will be transmitted from the first source node to the first consumer, and they are not transmitted directly, but in transit, through the second source node. And another 40 units of power is transmitted to the first consumer by the second source, which generates itself. It turns out that in General, the first source receives 80 units of power from the second consumer, and it generates 40 units itself, and 40 through it transits the first source.

With this power transfer scheme, the costs will be  $3 \cdot 40 + 4 \cdot 70 + 6 \cdot 80 = 880$  monetary unit.

#### 4.SUMMARY

From the considered example, we see that the most "far" from optimal is the reference plan found by the method of the Northwest corner. This was to be expected, according to this method, the cells are filled from top to bottom, from right to left. Power transmission costs are not taken into account. But this method is simple and clear, does not require any additional mental effort.

The case is better with the methods of least cost and double preference. The reference plan obtained using these algorithms is closer to the optimal one, and the cost of the electrical network is lower. These methods are more effective than the Northwest corner method, and are simple enough to implement. Unit costs are taken into account here, so the result is better. But the minimum costs are taken into account in the initial iterations of filling in the table, and in the final ones – energy transfer sometimes occurs at high rates.

This is the problem that the Vogel approximation method eliminates. According to the algorithm, the energy transfer from the beginning to the end is made taking into account the specific costs for power transmission, so the initial reference plan is either optimal (as in the example given), or close to the optimal one. The disadvantage of this method is that it is very labor intensive and time – consuming.

The considered modification of the transport problem - the problem with power transit is a more General problem and has wider possibilities for optimizing the electrical network scheme than the transport problem in the classical formulation. And the cost of energy transfer in this setting is less.

### **5.CONCLUSION**

As a result of the analysis of various problems that arise when designing power lines, we see that the transport problem helps to find simple and short-term solutions to problems. The scope of application of transport tasks is quite wide.

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