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On Environment's Mathematical Model and Multi- legged Walking Robot Stable Evolution

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ABSTRACT
Mathematical environmental modelling, based on our past research on dynamical systems evolution stability, generally in the case of dynamical systems dependent on parameters and approaching the phenomena from reality, is introduced and justified in the first section of this paper. Reality perception is signified by the continuity perception quality for the functions defining a concrete dynamical system from the reality, and additionally by a limited number of possible function discontinuities. Piecewise property continuity for the functions defining the dynamical system which approaches environmental phenomena implies the properties of stable region separations in the dynamical system's free parameters. These stable region free parameters allow the optimization of the evolutions of the dynamical system using compatible criteria for optimization. That is, we are able to realize stability control of the evolution of the dynamical system. We state that every dynamical system depends on parameters from the literature and have separation properties between unstable and stable regions in the free parameter domain. The second part of the paper our theory on dynamical system stability control from the environment is applied for specific multi-legged walking robot models which rely on two dimensional parameters, three dimensional in the case of legs evolution. Our critical position theory on the evolution of the walking robot is recapped and developed on our analysis cases of walking robot models which take their inspiration from the environment.

Key words : Environment's Mathematical Model, Environment's Dynamical System, Stable Region, Multi-Legged Walking Robot, Critical Position, Kinematics Analyze.

1. INTRODUCTION

Any environmental dynamical system can be thought of as a dynamical system in regard to its defining parameters without fixing the values as a physical parameter (particularly mechanical parameters), geometric parameters, potential economic, chemical, biological parameters etc. The matrix defining the linear dynamic system has the matrix components which are assumed to be real values, and the matrices which interfere in the exposing the method are also a real value component. In previous papers we assumed that the mathematical model matrices possess complex values which are accounted for as particular cases of complex values. Such a hypothesis ensures a new analytical method, in the complex domain, on dynamical system stability. Out theory on dynamical system control stability is used here for specific walking robot models dependent on parameters. The critical position of the evolution of the walking robot is reminded and analyzed in some instances of walking robot legs in kinematic analysis [1-11, 13, 22-33].

A crucial idea arising from every concrete dynamical system which is dependent on parameters from the literature is separation properties between unstable and stable regions in specific free parameter domains [12. 14-16, 21]. Theses unstable and stable regions are divided into the free parameter domain by a boundary. Separation properties are observed through stable and unstable regions as open sets, except for the boundary points. This separation creates stability control freedom on any stable point neighborhood in the dynamic system's open stable region. We discovered mathematical conditions of the stability regions existence for the dynamic systems which take their inspiration from the environment, using various results from the domain of applied mathematics [14, 16, 17-21].

We define the environment mathematical model, analyzing some condition factors of separation between unstable and stable regions in the free parameter domain of the nonlinear or linear dynamical systems, naming the QR algorithm used on the real matrix which defines the linear dynamic system or the "first approximation" of the nonlinear dynamic system, using complex domain operations by the matrix "shift of origin". The matrix eigen values dependent on the matrix components properties interfere in separation property justification. The real matrix components that define the linear dynamical system depend on parameters which are assumed to be continuously piecewise in the free parameters and analyzed for the environment model's final argumentation.

2. MULTI-LEGGED WALKING ROBOT MODELS

We analyze some walking robot evolution problems by leg evolution synchronization in kinematics hypothesis. Extending this to the dynamical evolution of the walking robot using the above theory is intuitively accepted and is extremely attractive.

First to be studied are the mathematical and physical walking robot model are physical models consisting of a platform and six similar legs joined at the extremities attached to the robot platform and which have a synchronized evolution in the vertical plane (figure.1).

Additionally, in fig. 5 there is another case based and inspired on the initial two dimensional evolution technique used for performing leg evolution, consisting of the three dimensional evolution of each leg of the multi- legged walking robot.

Each two dimensional leg evolution accounts for a leg compounded from two jointed segments, named the knee joint, which describes a circular arc route in leg evolution with the leg base extremity describing an imposed route on the ellipse arc (figure 2). The evolution of the leg base point is assumed to have a constant speed component in a horizontal direction. The hypothesis imposes the robot's concrete leg evolution, except for evolution in the neighborhood of possible critical positions, which is analyzed below.

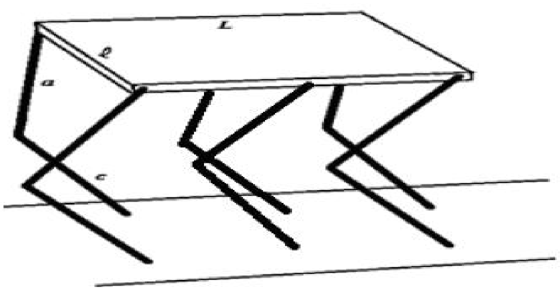

Figure 1: Walking robot physical model.

The physical model and attached mathematical model of the robot leg assures the identifiability of the existence of the critical position on the knee joint's circle arc, an idea introduced by the authors and also detailed in the following.

The walking robot leg physical evolution, in two dimensions, is described in figure 2.

A similar physical model is also used for three dimensional multi-legged walking robot model leg evolution.

The points specifying the extremities of the continuous domain on the circle arc where the knee joint is moving, in two dimensions, have positions where the inferior segment of the leg is usually on the ellipse arc.

The physical model used for three dimensional legs evolution of the multi-legged walking robot emphasizes also possible critical points existence. The critical points for three dimensional legs evolution, if they exist, will specify the extremities of the continuous domain where the knee joint is moving.

The point denoted by O_C is the leg joint extremity attached to the robot platform and the segments denoted by $O_C P$ and PQ have length a, respectively c are the components of the robot leg. The point P is the knee joint of the leg and the point Q is the leg base point.

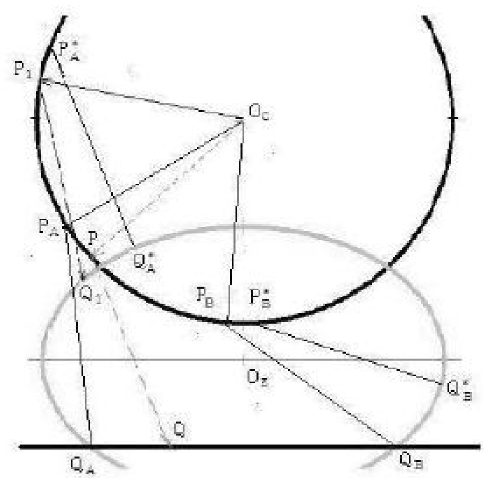

Figure 2: Physical model of walking robot leg.

The knee joint P describes, in robot leg cyclic movement, a circle arc route, and a base point Q of the leg imposes an elliptic route on the superior ellipse arc $Q_B Q_A$.

The circular route is assumed with the radius a and center O_C whilst the elliptic route is assumed with the semi aces a, b and O_E .

The elliptic and respectively circular routes are transverse and involve the imposition of a constant speed component of the leg base point, in a horizontal direction except for a critical position neighborhood.

The robot leg evolution critical point is point P_c on the circular arc and corresponding Q_c point on the elliptic arc where the "knee joint" directional movement is changed.

The orthogonal coordinate system is identified for the physical model from fig.2 by the following coordinate values, defining the figure points: $O_C(a, h)$, $O_E(a, b_1)$, $P(x_P, y_P)$,

$$Q(x_Q, y_Q),\ Q_A(x_A, 0),\ Q_B(x_B, 0).$$

The following conditions are used on the leg mathematical model parameters:

$$a > b > b_1 > 0;\ a + c > h$$

The critical position of the evolution of the leg "knee joint" and in the corresponding leg "base point" position is necessary null speed, a vital property used in concrete robot leg mathematical model description.

Points P^*_A and P^*_B in figure 2 delimit the maximal and can be moved since in this position the segments $P^*_A Q^*_A$ and $P^*_B Q^*_B$ are usually on the ellipse arc and the distance to the ellipse arc in the neighborhood is a minimum. This separation property between the existence regions (implicit stability) and inexistence (instability) are emphasized by analysis of the knee joint P kinematics evolution from the environment.

The robot leg mathematical model corresponding to physical model from fig. 2 is described by the relations:

$$\begin{aligned}&(x-a)^2 + (y_P - h)^2 - a^2 = 0.;\\&(x-a)^2/a^2 + (y_Q - b_1)^2/b^2 = 1.\end{aligned} \qquad (1)$$

A definition of a covering domain for the variables from (1).

$$x \in [0,\ 2a],\ y_P \in [h-a,\ h+a),\ y_Q \in [0,\ b+b_1]. \qquad (2)$$

Explicit functions deduced from (1) have the form:

$$y_P = h \mp \qquad {}^2)^{1/2}\ ; \qquad (3)$$

$$y_Q = \pm\ b/a\ (2ax - x^2)^{1/2} + b_1 \qquad (3')$$

Let $P(xp,yp)$ be the circle arc point and $Q(x_Q,y_Q)$ the ellipse arc point which corresponds to one position of the robot leg evolution. The condition

$(x_P - x_Q)^2 + (y_P - y_Q)^2 \ -c^2 = 0$ is imposed.

The segment $O_C Q$ in the triangle $O_C PQ$ can be expressed as function of the angle $(O_C PQ)$ denoted θ:

$$(O_C Q)^2 = (O_C P)^2 + (PQ)^2 - 2(O_C P)(PQ)\cos\theta \qquad (4)$$

The following analytical relation is formulated, where the variable x_Q is assumed an independent variable and is denoted by x:

$$\begin{aligned}&(a-x)^2 + [h - b_1 \mp \qquad ax - x^2)^{1/2}]^2 =\\&= a^2 + c^2 - 2ac\ \cos\theta\end{aligned} \qquad (5)$$

The relation (5) is used to evaluate the angle $(O_C PQ)$ as function of variable x, for $x \in [0,\ 2a]$.

The points $Q_A(x_A, 0)$ and $Q_B(x_B, 0)$ have the abscises $x_A = a - a\ (1-(b_1/b)^2)^{1/2}$, $x_B = a + a\ (1-(b_1/b)^2)^{1/2}$, with x_A, x_B being strictly between 0 and $2a$.

Below is the linear evolution of the variable x between 0 and $2a$. where the constant speed ω and initial condition x_0 are considered:

$$x(t) = \omega\ t + x_0 \qquad (6)$$

A robot leg evolution cycle can begin from the point Q_B, moving on the superior ellipse arc up to point Q_A, using an evolution law on horizontal axis defined by (6), excepting a critical points neighborhood.

The variables covering domain is reminded below.

$$x \in [0, 2a],\ y_P \in [h-a, h+a],\ y_Q \in [0, b+b_1] \qquad (7)$$

The selected explicit functions for the physical models from figures 3 and 4, are:

$$\begin{aligned}&y_P = h - (2ax - x^2)^{1/2}\\&y_Q = b/a\ (2ax - x^2)^{1/2} - b_1\end{aligned} \qquad (8)$$

In the first explicit case of a physical model of walking robot leg the below relation is assured, where $x \in [x_{QA}, x_{QB}]$:

$$\begin{aligned}&(a-x)^2 + [(b/a)(2ax - x^2)^{1/2} -\\&- (b_1 + h)]^2 = a^2 + c^2 - 2ac\ \cos\theta\end{aligned} \qquad (9)$$

As the knee joint speed is natural of zero value in critical positions where the sense of motion is changed, it is necessary to also use Cauchy conditions of neighborhood continuity of critical points to ensure continuous displacement and speed values as functions on time.

We next describe specific walking robot leg cases using physical models from figures 3 and 4. The walking robot physical model is defined with assistance from the corresponding robot leg definition.

Figure 3 shows the physical robot leg model, from which we see the pivot point O_P of the leg, the mobile joint P of the leg that describes a circle arc in the leg's evolution on cycle and the robot leg point Q base positioned in our case on an ellipse arc of center O_Q and semi axes length a and b . The orthogonal system of coordinate axes is similar as for figure 2.

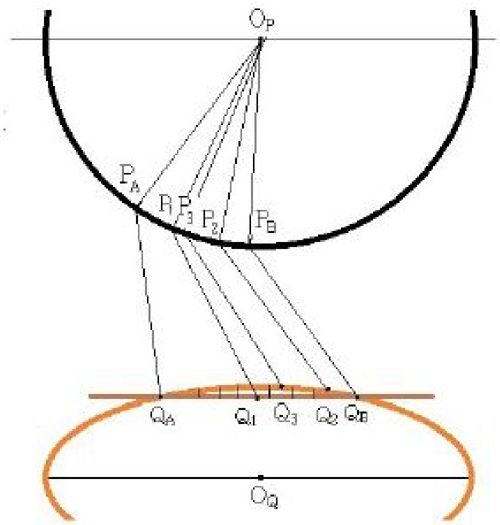

Figure.3: One physical model for walking robot leg

Let O_P (a, h), O_Q (a, □b_1), Q_A (x_A , 0), Q_B (x_B , 0) be the points from figs. 3 and 4. A leg base point from fig.3 evolution cycle is defined by the successive evolution QA , Q1, QB , on the horizontal axis followed by the succession Q2 , Q3 , QA on the superior ellipse arc. The critical points are unidentified on the figure.3 physical model, but they exist on the physical model from figure.4.

These two models differ through the value of parameter h , in the first instance the value h_1 and in the second the value $h_2 > h_1$ implies the corresponding modification of the parameter b_1.

The conditions $a > b > b_1 > 0$; $a + c > h$ are consumed, where the inferior component length of the robot le is c . The physical model from figure.3 is assumed $c > a$.

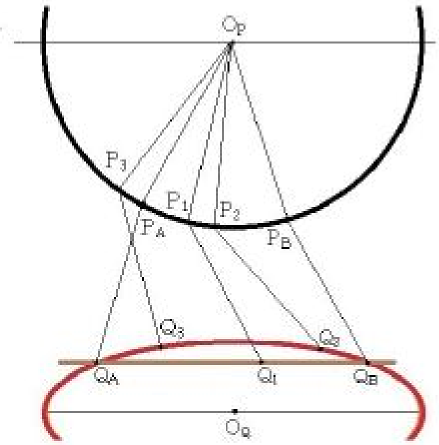

Figure 4: Other physical model of walking robot leg.

Our definition of the critical position in robot leg evolution, is the position where point P_C on the circular arc and corresponding Q_c point on the elliptic arc change properties in altering the movement direction of the "knee joint".

In the critical position of the leg "knee joint" evolution and in the corresponding leg "base point" evolution we remark upon the necessary null speed of leg "knee joint" and we reiterate that this is an important property used to assure concrete evolution of the robot leg.

The critical points P_c and Q_c define the maximal domain extremities on the circular arc where point P can be moved and segment P_cQ_c is usually on the ellipse arc. The sense of movement changes on the circle arc in the continuous domain extremities of evolution of the "knee joint". This critical point can be generalized into walking robot movement where the ellipse arc is substituted by another continuous curve or in three dimensional legs evolution.

The identification analytical method of critical points on the circle arc and implicit critical points on the ellipse arc, for a concrete physical model from figure 4, is answered by the solving of the equation depending on x_Q imposed by the condition of segment P_cQ_c orthogonal position.

Figure 5 describes a three dimensional leg evolution physical model for multi-legged walking robots, [15]. The critical point is necessary to assure the zero value of the leg base point speed.

We state that the walking robot stability (existence) position is also maintained in a neighborhood for one such position.

The leg is compounded from superior component BtQt as defined by the extremities points Bt , Qt jointed in pivot point Bt attached to the robot body and inferior component QtPt with "knee joint" Qt and base point Pt . For the length of Component lengths BtQt and QtPt assume a constant value.

The Pt base point moves between point P_I and P_F in the vertical plane, orthogonal on axis O_y using uniform accelerated displacement on the horizontal direction up to the median point P_M, on the ellipse arc, and symmetric displacement assured up to the final point P_F

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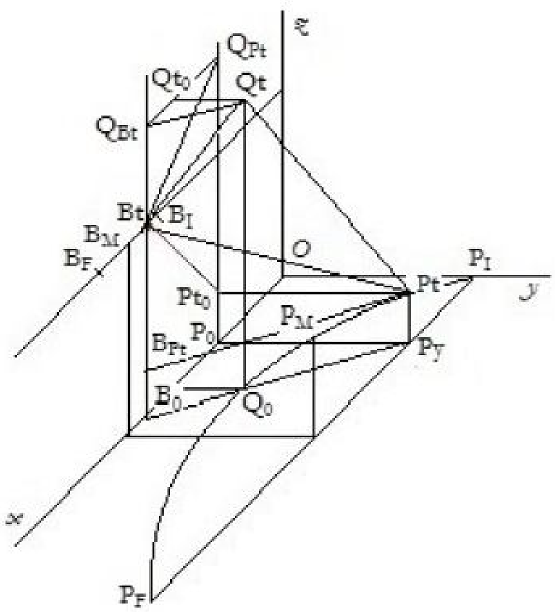

Figure 5: Physical model of three dimensional walking robot leg.

The joint point Bt attached to the robot body moves simultaneously with a linear route parallel to the axis *Ox* , using uniform displacement between initial point B_I up to median point B_M and symmetric displacement is assured between median point B_M and final point B_F.

The "knee joint" point Qt trajectory, uniquely identified in the vertical plane, is defined by the points Bt, Qt, Pt, in each time *t*, with the definition of the distance between the length of segment BtPt dependent on time *t* , is researched for possible critical point identification, which is similar in two dimensional cases as described above. The following formulas, the geometric data from figure 5 and physical data, are used.

The three dimensional trajectory of the point Qt is orthogonally projected on the plane *xOz* using the projected point Qt_0 from this plane. The point Qt critical position of, in its three dimensional evolution, is identified by a critical position on the plane projected trajectory of the point Qt_0 where the directional movement needs to be altered. The point Qt_0 is orthogonal projection of the point Qt on the side $Q_{Bt}Q_{Pt}$, which is parallel with the axis *Ox* , where the triangles $QtQ_{Bt}Q_{Pt}$, $Q_0B_0P_0$ have the sides respectively parallel. The uniform accelerated displacement of the point donated P, on the horizontal line, from the P_i up to middle of the P_IP_F, is described by the relation

$$x_P(t) = a_P t^2 / 2 \qquad (10)$$

The parameter a_P is constant acceleration in horizontal coordinate displacement of the point denoted P.

Point Bt uniform displacement on parallel line with axis *Ox* , from the point B_I up to middle of the segment $B_I B_F$, is described by the relation:

$$x_{Bt}(t) = v_B t \qquad (11)$$

The parameter v_B is a constant speed in point Bt displacement on the parallel line with axis *Ox* .

The parameter *L* is introduced for correlating data from the three dimensional mathematical walking model.

The six leg walking robot is considered, so that it is close

robot leg. The following lengths are defined $\overline{BtQt}=2L$, $\overline{QtPt}=3L$, $\overline{BtB_0}=1.5L$, $\overline{PtPt_0}=1.5L$.

to one evolutionary cycle, such the following relation arises $P_I P_F = 6\, B_I B_F$. This relation is justified by the hypothesis on the separation and successive displacement of each leg in cycling evolution.

The required time for simultaneously arrival in the middle of the routes B_IB_F, P_IP_F, by point Bt respectively Pt , is denoted by t_M. The following relations are true:

$$v_B t_M = d/12,\ \ v_B = d/t_M/12; \qquad (12)$$
$$a_P t_M^2/2 = d/2,\ a_P = d/t_M^2$$

Point Bt, Pt coordinates are identified in function on time as below:

$$x_{Bt} = x_{B_I} + v_B\ t,\ y_{Bt} = 0.,\ z_{Bt} = 1.5L$$
$$x_{Pt} = a_P\ t^2/2,\ y_{Pt} = 1.5L, \qquad (13)$$
$$z_{Pt} = (b/a)(a^2 - (x_{Pt} - d/2)^2)^{1/2} - b_1$$

In the above relations $0 \le t < t_M$, *a*, *b* are ellipse semi axes and $0 < b_1 < b$. Coordinate *z* is negatively imposed for the ellipse centers from the fig. 5 so that $z_{Pt} \in [0, b - b_1)$. The length of the segment BtPt is calculated from:

$$\overline{BtPt} = ((x_{Pt} - x_{Bt})^2 + (y_{Pt} - y_{Bt})^2 + (z_{Pt} - z_{Bt})^2)^{1/2} \quad (14)$$

The vertical triangle angles BtQtPt are calculated using triangle side lengths, by the relations of type:

$$\cos(\square)=\frac{(\overline{BtQt})^2 + (\overline{BtPt})^2 - (\overline{QtPt})^2}{2\overline{BtQt} \times \overline{BtPt}} \quad (15)$$

The values of the angles of the vertical triangle BtQtPt from the physical model described on fig.5 allow evaluation of all necessary sides or angles from the physical model. The critical point where the direction of movement changes to projected point Qt_0 is searched for in the parameter time in the interval $[0, t_M)$.

The three dimensional legs evolution mathematical model for multi-legged walking robot, as explained above, allows identification of the possible existence of the critical position for knee joint point, calling for a specialized computer program, this is planned for the next paper.

3. CONCLUSION

In the environmental mathematical characterization and application to the walking robots dynamical system stability control, the walking robot is considered as a particular case of a dynamical (kinematical) system which is dependent upon parameters, is conducted.

The mathematical conditions on the separation between unstable and stable regions, in the dynamical system free parameter domain, discovered by us, are emphasized, calling for algebraic operations in the complex domain which allow new results in the stability theory. These separation conditions represent the conditions which describe the environmental dynamical system.

The separation properties described in the paper, also encountered in numerous other dynamical systems from the literature, which are only contemplated without mathematical justification, is a crucially important property since it creates possibilities for stability control of a dynamical system from the environment by optimizing the system evolution using compatible criteria in the stability regions of the free parameters domain. The research for dynamic systems can be performed, with conservations towards the fundamental idea, and for kinematics of the walking robots. The stable region separation property is described in our cases of walking robot leg with the possibility to be extended in alternative cases.

The critical position of the walking robot is defined in the paper and a mathematical method of its identification is performed. The study has not exhausted the environment mathematical characterization issue and the dynamical systems stability control problem, or on kinematical stability control. However, this has exposed an interesting environmental domain.

ACKNOWLEDGMENT

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