



Intuitionistic fuzzy Logic System and its Application to Global Carbon Dioxide Emissions Prediction

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ABSTRACT

In this study, a Takagi-Sugeno-Kang based intuitionistic fuzzy logic system is proposed for the prediction of global carbon dioxide emission for the first time. The intuitionistic fuzzy logic system is an integration of artificial neural network learning and intuitionistic fuzzy logic reasoning. The gradient descent back propagation is applied in the optimization of the parameters of the proposed model. The model is evaluated based on some performance metrics. Results of evaluation revealed that the intuitionistic fuzzy logic system outperforms other existing models in the literature in terms of prediction accuracy.

Key words: Carbon dioxide emissions, gradient descent backpropagation, hesitation index, intuitionistic fuzzy set.

1. INTRODUCTION

Climate pollution from carbon dioxide (CO₂) has been an important and challenging task globally. The forecasting of CO₂ emission has attracted researches in many fields in recent times. According to [1], CO₂ emission has affected countries in diverse ways including health, agriculture, economics, climate and tourism. Naturally, there should be a balance between the CO₂ emitted from animals and other sources and the CO₂ utilized by plants during photosynthesis, this balance has been distorted by human activities. Reference [2] pointed out that this imbalance is due to greenhouse effect (global warming, melting of polar ice sheet, rise in sea level and coastal inundation, and damage to agriculture and natural ecosystem). Many human activities have also resulted in an increasing emission of global greenhouse gas (GHG), largely by burning fossil fuels to generate electricity, heat and cool buildings, and power vehicles—as well as by clearing forests [3]. According to [3], carbon dioxide, methane, nitrous oxide, and fluorinated gases are the major greenhouse gases that people have added to the atmosphere. Appropriate methods have to be formulated in order to predict the amount of emission of these gases into the atmosphere. In this study, the prediction of CO₂ emission is considered because according to

[4], CO₂ constitute a significant percentage of atmospheric air pollution.

Many methods have been adopted in the literature for the prediction of CO₂ emission and many have studied the relationship of CO₂ with other economic indicators. For instance, [1] proposed an artificial neural network approach for the estimation of CO₂ emission. The author used four input variables namely global oil, natural gas, coal and primary energy consumption to predict CO₂ emission. Reference [2] adopted the autoregressive integrated moving average (ARIMA) models to forecast yearly CO₂ prediction in Bangladesh. Different parametric models of ARIMA were constructed and different metrics were adopted to evaluate each ARIMA model. Reference [5] proposed a swarm intelligence methodology for the forecast of global CO₂ emission. Reference [6] employed bee algorithm and artificial neural network to forecast world CO₂ emission. Reference [7] presented a comparison evaluation of neural network learning algorithms for the CO₂ emission prediction in Malaysia for the period 1980-2009. The comparison was made between Levenberg-Marquardt and gradient descent backpropagation algorithms for learning the parameters of neural network. Results of findings revealed that Levenberg-Marquardt algorithms showed better performance compared to gradient descent in CO₂ emission prediction. Reference [8] proposed methods for forecasting CO₂ emission based on machine learning methods. The authors exploited the strengths of random forest and support vector machines in their analysis. Results revealed that support vector-based method produced better forecasting results. Reference [9] predicted CO₂ emission in all provinces in China using K-means cluster based logistic model. The K-means cluster analysis method was able to split the CO₂ emissions into five types while the logistic model forecasted the CO₂ emissions. Analysis of results revealed that the CO₂ emission of China was continuously increasing. Reference [10] forecasted medium- and long-term CO₂ emissions for provincial power grid using life cycle assessment and gray methods. The proposed method in [10] was found to exhibit good results in terms of prediction accuracy. Reference [11] predicted global CO₂ emission using two artificial neural network models namely neural network auto-regressive with exogenous input model and the evolutionary product unit neural network model (EPUNN). The authors in [11] adopted the same input and output variables as those reported in [5]. Reference [11] concluded

that the evolutionary approach provided more stable result in the test data than the multilayer neural network. Reference [12] forecasted CO₂ emission using neural network. The authors adopted solid fuel, oil, natural gas, electricity consumption, gross domestic product and resident population obtained from the national and provincial Italian statistics as their input indicators. The authors in [12] concluded that oil and natural gas contributed immensely to CO₂ emission. Reference [13] proposed a hybrid model involving fuzzy linear regression and back propagation network for global CO₂ concentration prediction. The authors concluded that the forecasting accuracy of the hybrid approach was better than other models in the literature. Reference [14] applied genetic algorithm to forecast global CO₂ emission. The global energy consumption dataset from 1980 to 2010 was adopted for the analysis. The authors pointed out that genetic algorithm model exhibited good performance as the predicted values were in good agreement with the observed data. Reference [15] considered two input indicators namely energy consumption and economic growth to predict CO₂ emission in G20 countries using adaptive neuro fuzzy inference system (ANFIS). The authors in [15] revealed that ANFIS provided efficient CO₂ emission prediction based on the two input indicators. However, ANFIS is a traditional type-1 fuzzy system and may not handle some indifference in the set definition as it is only defined by membership function with implicit assertion that non-membership function values are complementary to membership function values. This may not always be the case in real world applications. The global CO₂ is highly uncertain [13] and applying type-1 fuzzy logic defined by only the membership functions may not be very suitable.

This work seeks to forecast global CO₂ emission by utilizing the intuitionistic fuzzy logic system (IFLS) which incorporates separate specifications for the membership and non-membership functions and enables hesitation. According to [16], intuitionistic fuzzy set (IFS) provides an efficient means of expressing a fuzzy set where available information is insufficient to define an imprecise concept using the traditional fuzzy sets. In the same vein, [17] pointed out that using IFS provides a more natural form of decision making where more than two answers are involved compared to traditional FLS. This work adopts the same dataset as presented in [11]. The global CO₂ is collected from 1980 to 2010 and includes inputs such as global oil, natural gas, coal and primary energy consumption. The contributions of this work are as follows: 1) the use of IFLS with membership and non-membership function to predict global CO₂ emission for the first time. 2) The IFLS enables hesitation which is often neglected when analyzing global CO₂ emission prediction. It is believed that with the independently defined membership and non-membership functions of IFS, the IFLS becomes a more powerful modelling tool compared to the traditional type-1 FLS and the prediction is more accurate and closer to human reasoning than the type-1 FLS.

To the best knowledge of the authors, this is the first work that employs IFLS for the prediction of global CO₂ emission. The rest of the paper is organized as follows: In Section 2, a brief description of IFS is given. Section 3 describes the parameter update rule using gradient descent backpropagation learning algorithm. In Section 4, the evaluation of the proposed model is carried out and the conclusion is drawn in Section 5.

2. INTUITIONISTIC FUZZY SET

Fuzzy set, (FS), introduced by [18] is an extension of the binary set that has value 0 or 1. In real life problems (data) where some forms of uncertainty and fuzziness are encountered, binary set may not be appropriate in such cases. The use of FS is an alternative means of addressing the problems of uncertainty in many real-world problems and can provide better solutions. For FS however, the membership function for each element has values in a closed interval [0,1] and these values may not express the concept of “neither this nor that” otherwise known as neutrality or lack of knowledge. In other words, traditional FS does not incorporate hesitation degree. According to [19], the sum of membership and non-membership degree of element may be less than one, implying an extra degree of neutrality (hesitation). In such cases, the traditional FS may not suffice. Intuitionistic fuzzy set (IFS) [20] is the extended version of the traditional fuzzy set [18] and constitutes a fuzzy set with membership and non-membership functions. An interesting part about IFS is that it enables hesitation such that the addition of the membership and non-membership functions of IFS is not always complementary (an assertion implicit in traditional fuzzy set). Shown in Figure 1 is the plot of IFS. For instance, input value 4 will be $IFS_{A^*}(4) = [0.77, 0.20]$.

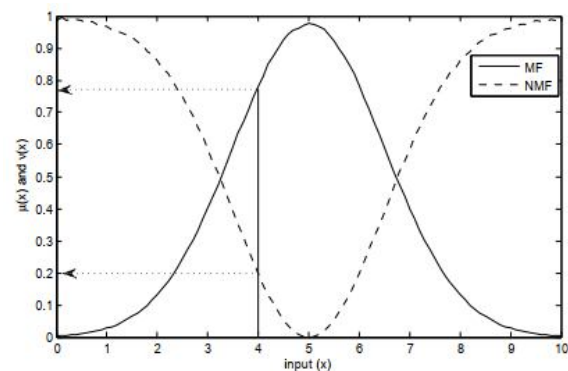


Figure 1: Intuitionistic Fuzzy Set [21]

Definition 1: Given a finite universal set, X , a subset A^* with element, x in A^* , is specified as $A^* = \{ (x, \mu_{A^*}(x), \nu_{A^*}(x)) | x \in X \}$ with some hesitation index (π) such that $\mu_{A^*}(x) + \nu_{A^*}(x) \leq 1$ [20].

where the function:

$\mu_{A^*} : X \rightarrow [0,1]$ such that for every $x \in X$, $\mu_{A^*}(x) \in [0,1]$ and this represents the membership function degrees of IFS.

and the function:

$\nu_{A^*} : X \rightarrow [0,1]$ such that for every $x \in X$, $\nu_{A^*}(x) \in [0,1]$ and this represents the non-membership function degrees of IFS.

Obviously, for IFS, $0 \leq \mu_{A^*}(x) + \nu_{A^*}(x) \leq 1$. For every element, there is the π such that $\pi_{A^*}(x) = 1 - (\mu_{A^*}(x) + \nu_{A^*}(x))$ which implies that $0 \leq \pi_{A^*}(x) \leq 1$. This measures the degree of hesitancy of the element x to the IFS A^* .

According to [20], when the membership and non-membership functions add up to 1, the traditional fuzzy set is recovered. In this case, the non-membership function is a complement to the membership function which is a special case of IFS.

i.e. for traditional FS,

$$Y = \{(x, \mu_{A^*}(x), 1 - \mu_{A^*}(x)) | x \in X\},$$

then $\pi_{A^*}(x) = 1 - (\mu_{A^*}(x) + 1 - \mu_{A^*}(x))$. In essence, FS is a special case of IFS when $\pi_{A^*}(x) = 0$.

Recently, the intuitionistic fuzzy inference has been extensively researched and applied in many problem domains such as multicriteria decision making, system identification and prediction, clustering, time series forecasting with promising results. For instance, [16] proposed an intuitionistic fuzzy inference system for predicting ozone (air pollutant) time series in Pardubice micro-region. The same authors in [22] also proposed a Takagi-Sugeno-type intuitionistic fuzzy inference system (IFIS) for regression problems. In their paper, different optimization algorithms namely gradient descent, Kaczmarz algorithm, Kalman filter and Moore-Penrose pseudo-inverse were adopted for the tuning of the model free parameters. Results reveal that Kalman filter and Moore-Penrose pseudo-inverse were more suitable for the optimization of the consequent parameters of the IFIS. Reference [23] also applied IFS with incomplete certain information on weights for multicriteria decision making. The authors pointed out that using IFS provides appropriate fuzzy decision-making procedure and satisfies a situation with incomplete certain information. Reference [24] applied IFS to model uncertainty in some regression problems. Also, [25] applied IFLS to solve a gas compression system (GCS) time series dataset with accuracy that matches that of traditional interval type-2 FLS. Reference [26] proposed an IFS for time series analysis in plant monitoring and diagnosis. Intuitionistic FS has also been applied for bankruptcy forecasting [27]. A system that adopts IFS in the rule base is known as IFLS. The IFLS consist of the intuitionistic - fuzzifier, rule base, inference engine and defuzzifier.

During training of the model, the external inputs are passed into the IFLS to obtain membership and non-membership function values of the inputs. The input-output relationships are expressed as intuitionistic fuzzy IF...THEN rules represented as:

$$R_k : \text{IF } x_i \text{ is } A_{ik}^* \text{ and } \dots \text{ and } x_n \text{ is } A_{nk}^* \text{ THEN } y_k = \sum_{i=1}^n w_{ik} x_i + b_k \tag{1}$$

The generic rule can be formulated for membership and non-membership functions respectively as follows:

$$R_k^\mu : \text{IF } x_i \text{ is } A_{ik}^{*\mu} \text{ and } \dots \text{ and } x_n \text{ is } A_{nk}^{*\mu} \text{ THEN } y_k = \sum_{i=1}^n w_{ik}^\mu x_i + b_k^\mu \tag{2}$$

$$R_k^\nu : \text{IF } x_i \text{ is } A_{ik}^{*\nu} \text{ and } \dots \text{ and } x_n \text{ is } A_{nk}^{*\nu} \text{ THEN } y_k = \sum_{i=1}^n w_{ik}^\nu x_i + b_k^\nu \tag{3}$$

where A^{*} 's are IFS, x 's are inputs, y_k 's are outputs of each rule, w is the weight and b is the bias (the weight and bias are consequent parameters).

Using a t -norm, in this case, product t -norm, the inference engine combines these rules and produces a mapping from the type-1 intuitionistic fuzzy input sets to a type-1 intuitionistic fuzzy output set. For a TSK IFLS, the output of each rule is computed directly because of the functional dependencies of output variable on input variables and requires no defuzzification [22].

According to [27], the final output of a TSK-type IFLS is defined as follows:

$$y = (1 - \beta) \sum_{k=1}^M f^{u_k} y_k^\mu + \beta \sum_{k=1}^M f^{v_k} y_k^\nu \tag{4}$$

where: $f^{u_k} = \frac{f_k^\mu}{\sum_{k=1}^M f_k^\mu}$ (5)

and $f^{v_k} = \frac{f_k^\nu}{\sum_{k=1}^M f_k^\nu}$ (6)

and f^{u_k} and f^{v_k} are normalized firing signals for membership and non-membership functions respectively and β is the user defined parameter that determines the magnitude of the non-membership function. It is obvious that if β is 1, the output is formed from only the non-membership function and if it is 0, then only the membership function contributes to the final output.

3. PARAMETER UPDATE RULE

In this study, the popular gradient descent (GD) back propagation algorithm is used to tune the parameters of IFLS. The GD searches through the solution space to find a function that has the lowest possible cost. The cost function for a single output is defined as:

$$E = \frac{1}{2} (y^a - y)^2 \tag{7}$$

where y^a is the actual output and y is the model prediction.

The generic parameter update rule using GD is as follows:

$$\theta_{ik}(t+1) = \theta_{ik}(t) - \gamma \frac{\delta E}{\delta \theta_{ik}} \tag{8}$$

where γ is the learning rate (step size) that must be carefully chosen to enhance stability of the learning model as a large value may lead to instability, and small values may lead to a slow learning process. The parameter θ is the generic parameter to be tuned. The learning rate and IF-indices used in this work are not adjusted. The consequent parameters include the weights (W) and biases (b) with the update rule:

$$w_{ik}(t+1) = w_{ik}(t) - \gamma \frac{\delta E}{\delta w_{ik}} \tag{9}$$

$$\text{and } b_k(t+1) = b_k(t) - \gamma \frac{\delta E}{\delta b_k} \tag{10}$$

respectively. The derivative in (7) is computed as in (9) and (10) for the weights,

$$\frac{\delta E}{\delta w_{ik}} = \frac{\delta E}{\delta y} \frac{\delta y}{\delta y_k} \frac{\delta y_k}{\delta w_{ik}} = \sum_{k=1}^M \frac{\delta E}{\delta y} \left[\frac{\delta y}{\delta y_k^{\mu}} \frac{y_k^{\mu}}{\delta w_{ik}^{\mu}} + \frac{\delta y}{\delta y_k^{\nu}} \frac{y_k^{\nu}}{\delta w_{ik}^{\nu}} \right] \tag{11}$$

$$= (y(t) - y^a(t)) * \left[(1 - \beta) \left(\frac{f_k^{\mu}}{\sum_{k=1}^M f_k} \right) + \beta \left(\frac{f_k^{\nu}}{\sum_{k=1}^M f_k} \right) \right] x_i \tag{12}$$

while the derivative in (8) is computed as (11) and (12) respectively for the biases.

$$\frac{\delta E}{\delta b_k} = \frac{\delta E}{\delta y} \frac{\delta y}{\delta y_k} \frac{\delta y_k}{\delta b_k} = \sum_{k=1}^M \frac{\delta E}{\delta y} \left[\frac{\delta y}{\delta y_k^{\mu}} \frac{y_k^{\mu}}{\delta b_k^{\mu}} + \frac{\delta y}{\delta y_k^{\nu}} \frac{y_k^{\nu}}{\delta b_k^{\nu}} \right] \tag{13}$$

$$= (y(t) - y^a(t)) * \left[(1 - \beta) \left(\frac{f_k^{\mu}}{\sum_{k=1}^M f_k} \right) + \beta \left(\frac{f_k^{\nu}}{\sum_{k=1}^M f_k} \right) \right] \tag{14}$$

The Gaussian function are used in the definition of the membership and non-membership functions of the IFLS. Mathematically, the Gaussian membership function is defined as follows:

$$\mu_{ik}(x_i) = \exp \left(\frac{(x_i + c_{ik})^2}{2\sigma_{ik}^2} \right) \tag{15}$$

which is modified as (14) and (15) to reflect membership and non-membership functions of IFS respectively.

$$\mu_{ik}(x_i) = \exp \left(\frac{(x_i + c_{ik})^2}{2\sigma_{ik}^2} \right) (1 - \pi_c(x_i)) \tag{16}$$

$$v_{ik}(x_i) = (1 - \pi_{var}(x_i)) - \mu_{ik}(x_i) \tag{17}$$

where π_c and π_{var} are intuitionistic fuzzy index of center and variance respectively.

The antecedent parameters are the centre (c) and standard deviation (σ) which are updated in the same manner as the weight and bias.

$$c_{ik}(t+1) = c_{ik}(t) - \gamma \frac{\delta E}{\delta c_{ik}} \tag{18}$$

$$\text{and } \sigma_{ik}(t+1) = \sigma_{ik}(t) - \gamma \frac{\delta E}{\delta \sigma_{ik}} \tag{19}$$

where the derivative $\frac{\delta E}{\delta c_{ik}}$ in (16) is calculated as follows:

$$\frac{\delta E}{\delta c_{ik}} = \sum_k \frac{\delta E}{\delta y} * \left[\frac{\delta y}{\delta f_k^{\mu}} \frac{\delta f_k^{\mu}}{\delta \mu_{ik}} \frac{\delta \mu_{ik}}{\delta c_{ik}} + \frac{\delta y}{\delta f_k^{\nu}} \frac{\delta f_k^{\nu}}{\delta \nu_{ik}} \frac{\delta \nu_{ik}}{\delta c_{ik}} \right] \tag{20}$$

and the derivative in (17) is computed as follows:

$$\frac{\delta E}{\delta \sigma_{ik}} = \sum_k \frac{\delta E}{\delta y} \left[\frac{\delta y}{\delta f_k^{\mu}} \frac{\delta f_k^{\mu}}{\delta \mu_{ik}} \frac{\delta \mu_{ik}}{\delta \sigma_{ik}} + \frac{\delta y}{\delta f_k^{\nu}} \frac{\delta f_k^{\nu}}{\delta \nu_{ik}} \frac{\delta \nu_{ik}}{\delta \sigma_{ik}} \right] \tag{21}$$

Due to space constraint, the individual derivatives are omitted here.

4. MODEL EVALUATION

In order to evaluate the proposed model, three performance criteria are adopted. These include the root mean squared error (RMSE) and mean absolute error (MAE) which are defined as follows:

$$RMSE = \sqrt{\frac{1}{N} \sum_{i=1}^N (y^a - y)^2} \tag{22}$$

$$MAE = \frac{1}{N} \sum_{i=1}^N |y^a - y| \tag{23}$$

where y^a is the actual measurement and y is the predicted output of IFLS. The dataset for the analysis is obtained from [11] and depicted in Table 1. The data consists of oil consumption, natural gas (NG) consumption, coal consumption and primary energy (PE) consumption. The data values are normalized to lie within a small range of [0,1]. The data is split into training set (1980 - 2003) and testing set (2004 - 2010).

Table 1: Actual values of global energy consumption and CO₂ emission per year

Year	Oil Consumption (Mtoe)	NG Consumption (Mtoe)	Coal Consumption (Mtoe)	PE Consumption (Mtoe)	CO ₂ Emission (Mt)
1980	2972.2	1296.9	1806.4	6624	19322.4
1981	2863	1309.5	1820.6	6577.5	19073.2
1982	2770.7	1312.5	1846.9	6548.4	18900.7
1983	2748.3	1329	1897.7	6638.2	19072.1
1984	2810.1	1440	1983.2	6960.2	19861
1985	2804.7	1488.3	2056	7137.5	20246.7
1986	2894.1	1503.6	2089.2	7307.5	20688.3
1987	2946.8	1579.6	2169	7555.7	21344.5
1988	3038.8	1654.9	2231.7	7833.5	22052.2
1989	3093	1729.2	2251.2	8001.7	22470.2
1990	3148.6	1769.5	2220.3	8108.7	22613.2
1991	3148.2	1807.5	2196.4	8156	22606.5
1992	3184.8	1817.9	2174.6	8187.6	22656.7
1993	3158	1853.9	2187.6	8257.5	22710.6
1994	3218.7	1865.4	2201.9	8357.6	22980.3
1995	3271.3	1927	2256.2	8577.9	23501.7
1996	3344.9	2020.5	2292.2	8809.5	24089.8
1997	3432.2	2016.8	2301.8	8911.6	24387.1
1998	3455.4	2050.3	2300.2	8986.6	24530.5
1999	3526	2098.4	2316	9151.4	24922.7
2000	3571.6	2176.2	2399.7	9382.4	25576.9
2001	3597.2	2216.6	2412.4	9465.6	25800.8
2002	3632.3	2275.6	2476.7	9651.8	26301.3
2003	3707.4	2353.1	2677.3	9997.8	27508.7
2004	3858.7	2431.8	2858.4	10482	28875.2
2005	3908.5	2511.2	3012.9	10800.9	29826.1
2006	3945.3	2565.6	3164.5	11087.8	30667.6
2007	4007.3	2661.3	3305.6	11398.4	31641.2
2008	3996.5	2731.4	3341.7	11535.8	31915.9
2009	3908.7	2661.4	3305.6	11363.2	31338.8
2010	4028.1	2858.1	3555.8	12002.4	33158.4

Mtoe = Million tonne oil equivalent

Mt = Million tonne

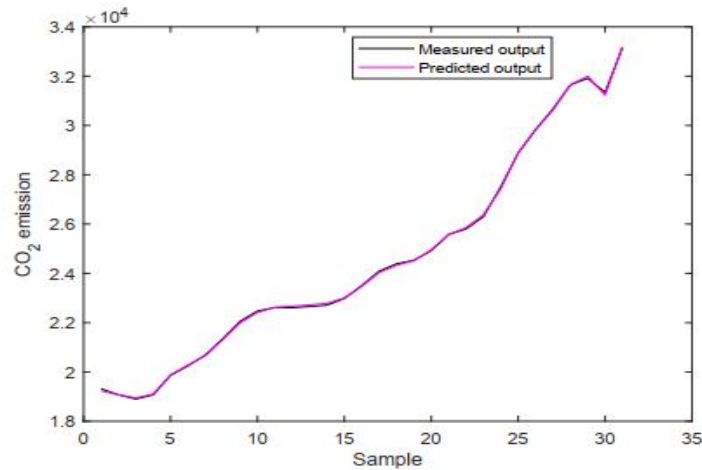


Figure 2: Actual and predicted output of global CO2 emission using IFLS

Table 2: Comparison of IFLS prediction with other models in the literature.

Actual CO ₂ emission	PSO prediction [5]	PSO error [5]	MLP prediction [11]	MLP error [11]	EPUNN prediction [11]	EPUNN error [11]	Proposed model prediction	Proposed model error
19322.4	20953	1630.6	20940	1617.6	19324.5	2.1	19229	93.4
19073.2	20084	1010.8	20031	957.8	19061.5	11.7	19075	1.8
18900.7	19354	453.3	19268	367.3	18858.4	42.3	18952	51.3
19072.1	19177	104.9	19083	10.9	19018.6	53.5	19106	33.9
19861	19665	196	19593	268	19837.9	23.1	19880	19
20246.7	19623	623.7	19548	698.7	20210.7	36	20282	35.3
20688.3	20331	357.3	20290	398.3	20694.6	6.3	20657	31.3
21344.5	20750	594.5	20728	616.5	21360.7	16.2	21307	37.5
22052.2	21484	568.2	21498	554.2	22089.4	37.2	21988	64.2
22470.2	21918	552.2	21954	516.2	22515.1	44.9	22413	57.2
22613.2	22364	249.2	22422	191.2	22668.7	55.5	22619	5.8
22606.5	22360	246.5	22419	187.5	22656.8	50.3	22672	65.5
22656.7	22655	1.7	22728	71.3	22710.4	53.7	22723	66.3
22710.6	22439	271.6	22502	208.6	22753.9	43.3	22785	74.4
22980.3	22927	53.3	23016	35.7	23030.4	50.1	23005	24.7
23501.7	23351	150.7	23462	39.7	23545.6	43.9	23468	33.7
24089.8	23946	143.8	24090	0.2	24116.5	26.7	24031	58.8
24387.1	24654	266.9	24837	449.9	24412.3	25.2	24322	65.1
24530.5	24842	311.5	25037	506.5	24545.5	15	24511	19.5
24922.7	25417	494.3	25644	721.3	24917.9	4.8	24954	31.3
25576.9	25789	212.1	26038	461.1	25553.1	23.8	25577	0.1
25800.8	25998	197.2	26260	459.2	25762	38.8	25856	55.2
26301.3	26285	16.3	26564	262.7	26244.1	57.2	26378	76.7
27508.7	26900	608.7	27216	292.7	27412.2	96.5	27426	82.7

28875.2	28145	730.2	28539	336.2	29038.9	163.7	28896	20.8
29826.1	28556	1270.1	28976	850.1	29941.6	115.5	29828	1.9
30667.6	28860	1807.6	29300	1367.6	30729.5	61.9	30617	50.6
31641.2	29373	2268.2	29847	1794.2	31630.7	10.5	31649	7.8
31915.9	29284	2631.9	29751	2164.9	31877.7	38.2	32001	85.1
31338.8	28557	2781.8	28978	2360.8	31335.1	3.7	31221	117.8
33158.4	29545	3613.4	30031	3127.4	32997.8	160.6	33198	39.6
Total error		24419		21894		1412.2		1408.3

Table 3: Comparison of IFLS prediction with Kavooosi *et al.* [14]

Year	2004	2005	2006	2007	2008	2009	2010	Average
Actual data	28875.2	29826.1	30667.6	31641.2	31915.9	31338.8	33158.4	--
GA-CO ₂ exponential	28033.8	29481.9	30217.8	30859.0	31630.7	31728.3	31980.5	--
Relative error (%)	-2.914	-1.154	-1.467	-2.472	-0.893	1.243	-3.552	1.956
GA-CO ₂ linear	27586.2	29624.2	30530.8	31324.7	32233.9	32171.0	31317.7	
Relative error (%)	-4.464	-0.677	-0.446	-1.000	0.996	2.655	-5.551	2.256
Proposed model	28896	29828	30617	31649	32001	31221	33198	
Relative error (%)	0.072	0.006	-0.165	0.025	0.267	-0.376	0.119	0.147

Figure 2 shows the actual and predicted values of global CO₂ emissions using IFLS. Tables 2 and 3 show the prediction accuracy of IFLS for comparison against other models in the literature. As shown in the table 2 and Figure 2, IFLS predict CO₂ emission as closely as possible to the actual values. This demonstrates an acceptable performance as depicted in the smallest absolute error. Closely following the prediction of IFLS is the EPUNN model, an evolutionary approach. Table 3 shows the comparison of IFLS prediction with the test set in [14] utilizing genetic algorithm. The relative error of IFLS is the lowest compared to the error in the linear and exponential genetic algorithm models in [14].

Table 4: Comparison of IFLs with other models in terms of Performance metrics

Model	Train set	Test set	Train MAE	Test MAE
Abdelfatah <i>et al.</i> [5]	537.86	2121.4	-	-
Sheta et al. – MLP [11]	33.279	673.97	33.279	555.33
Sheta et al – EPUNN [11]	41.52	100.95	35.75	79.17
Proposed model	25.97	83.86	21.09	38.22

Table 4 shows the performance of IFLS with other models in terms of RMSE and MAE metrics. As shown in the table 4, IFLS exhibits superior performance compared to other models in the literature. This is an indication that IFLS can be considered as a suitable candidate for global CO₂ emission prediction.

5. CONCLUSION

The prediction of CO₂ has been a hot topic in recent years as a result of global warming. It is therefore important to develop accurate models for predicting CO₂ emission for effective management and control of air pollution. This study has presented the prediction of global CO₂ emission problem using IFLS. As discussed above, IFLS is constructed using both membership and non-membership functions such that the sum of the two terms are not complementary. This allows IFLS some flexibility and the capacity to provide accurate CO₂ estimate better than other models in the literature. As demonstrated through experimental analysis, the outputs of

IFLS are as close as possible to the actual global CO₂ emission values.

In the future, we intend to utilize a higher order intuitionistic fuzzy logic system namely; interval type-2 intuitionistic fuzzy logic system for the prediction of global carbon dioxide and to adapt the parameters with different learning algorithms such as particle swarm optimization, extended Kalman filter and Simulated annealing.

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