ISSN 2278-3091
Volume 9, No.3, May - June 2020
International Journal of Advanced Trends in Computer Science and Engineering
Available Online at http://www.warse.org/IJATCSE/static/pdf/file/ijatcse33932020.pdf
https://doi.org/10.30534/ijatcse/2020/33932020

Construction of Membership Functions in Fuzzy Modeling Tasks using the Analytic Hierarchy Process

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ABSTRACT

The paper presents an overview of existing approaches to the problem of fuzzy set membership functions and suggests a method of constructing the membership functions with many arguments for the linguistic variables terms in fuzzy modeling and control problems, which combines statistical data analysis and expert evaluation based on the analytic hierarchy process. The method involves fuzzy clustering of the linguistic variables universal set and the formation of the set of its terms and tabular membership functions. The paper presents an approach to the formation of a set of analytical functions types for each term and their expert evaluation based on the advantages for the conditions of the modeling. To calculate the parameters of the analytic term membership functions it is suggested to use the coordinate centers of the respective clusters and optimization methods for the paired criteria for the accuracy of approximation, as shown in the example of the cone-shape membership function. The criterion for selecting an analytic type of membership function is formalized based on the analytic hierarchy process, using the criteria of approximation accuracy and expert evaluation of the advantages for the modeling object conditions. We suggest to set the priority of selection criteria separately for each modeling object, taking into account the quality of statistics, the level of experts' expertise, the requirements for the accuracy of modeling. In general, the developed method can be applied to construct the membership function of many arguments in the development of information systems based on fuzzy logic.

Key words: analytic hierarchy process, fuzzy clustering, fuzzy modeling, information system, membership function.

1. INTRODUCTION

Fuzzy logic is widely used in the construction of intelligent information systems of various applications. It is applied in tasks of modeling, data analysis [1], control and decision-making support [2]. Fundamentals of the application of fuzzy logic in practical problems are laid in the works of L. Zadeh [3]. One of the most responsible steps in fuzzy modeling, which determines the quality of the problem solution and the accuracy of the results obtained, is the construction of membership functions for fuzzy sets or linguistic variables terms.

1.1 Problem analysis

Defining the membership function is a stage that allows to start using a fuzzy set. As a rule, the membership function is constructed on the basis of statistical information or with the participation of an expert (expert group). In the first case, the membership function must have a frequency interpretation, in the second case, the degree of membership is approximately equal to the intensity of the manifestation of some property. Expert methods of constructing the membership function are divided into direct and indirect.

In direct methods, the degree of membership is assigned directly in a table, graph or formula. Direct group methods are a variety of direct methods. A group of experts is presented with a specific object and everyone has to answer: does this object belong to a given set? The value of the object's belonging function to the fuzzy set is defined as the number of positive responses divided by the total number of experts [4], [5]. After selecting the membership function type, experts are involved in determining its parameters [6]. The values of the membership function parameters can be calculated based on statistics, for example, using genetic algorithms [7]. Direct methods of assigning the membership function are used for measurable values or when thresholds values can be distinguished. We must note that when using direct methods, there is often a "slipping" of experts' evaluations to thresholds values. In the absence of elementary measurable properties, indirect methods should be used.

In indirect methods, expert evaluations are processed using certain algorithms with the aim to reduce the subjectivity of expert judgment [6]. Among the indirect methods of assigning membership functions are the statistical methods and pairwise comparisons method. Statistical methods for constructing membership functions are based on the experimental data sets and include fuzzy clustering methods and the potential method [8], [9]. The pairwise comparisons method relies on the use of evaluation matrices which contain expert evaluations of the relative membership of elements in a set or the degree of belonging to the properties formalized by a fuzzy set. To find the values of the membership function, it is necessary to form the eigenvector of the matrix of evaluations. When using expert groups, the methods of conducting and processing the results of the expert survey are used [10].

These approaches are mainly used to construct membership function of a single variable, the types of which are systematized in the works of A.P. Rothstein, A.V. Leonenko, S.D. Shtovba and other scientists [5], [11]-[13]. The classes of the membership function of a single variable are as follows:

- piecewise linear function (where the most popular is the triangular function);
- Z-shaped and S-shaped functions;
- U-shaped features.

However, the development of fuzzy logic also requires considering the construction of membership functions for many arguments. Membership functions of many arguments are used to define fuzzy relations and control problems [14], [15]. One of the options for classifying the membership functions of many arguments is based on the generalization of the membership functions of one variable [16]. We must note that this approach does not systematically cover the whole problem of formalizing the type of membership function of many arguments and requires continued research in this field, based on the experience of constructing the membership function of a single variable.

1.2 Aim of the article

One of the main tasks in fuzzy modeling is the formation of sets of linguistic variables terms and the construction of their membership functions. The aim of this paper is to formulate an approach to constructing term membership functions in general as a function of many arguments, using a combination of statistical processing methods and expert-based evaluation methods based on the analytic hierarchy process.

2. THE MAIN SECTION

Given the complexity of presenting and analyzing the membership functions of many arguments, we suggest to use statistics of the modelling object to construct them, and then perform the following steps:

- fuzzy clustering of statistics of the membership function arguments (universal set of linguistic variables);
- determination of the set of membership function types;
- calculation the values of the membership function parameters;
- selection of the membership function type, taking into account the accuracy of approximation and expert evaluation.

In general, the task of clustering is to divide a given set of experiments into groups of "similar" elements. We suggest that the degree of similarity is a distance between the vectors of data. The use of fuzzy clustering allows each vector of experimental data to belong to several clusters with a certain degree of membership. These clusters can form the basis for the formation of a linguistic variable terms set. To determine the number of clusters, an expert in this subject domain must make a decision based on the semantic load of a particular linguistic variable. In the absence of reliable knowledge, it is possible to apply mountain clustering to determine the number of clusters and the preliminary evaluation of their centers. We suggest to use the c-means method as it is the most common fuzzy clustering method. A large number of modifications is developed for this method, which differ in the form of clusters that are released during the operation of the method.

For each cluster in the domain of the membership function argument resulting from the use of the fuzzy c-means clustering method, the name is defined and a table membership function is formed, thereby specifying the set of terms $\{t_1, t_2, ..., t_K\}$ of linguistic variable β. The name of the term is determined by the expert, based on the specifics of the subject domain.

The table-defined term membership function is formed by comparing the values of experimental matrix and the transposed row of the membership matrix for this cluster. Thus, the table-defined term membership function is represented as a matrix:

$$\mu_k^\beta(\overline{x}) = \left[\begin{array}{cccc|c} x_{11}^\beta & x_{12}^\beta & \dots & x_{1m}^\beta & y_{k1}^\beta \\ x_{21}^\beta & x_{22}^\beta & \dots & x_{2m}^\beta & y_{k2}^\beta \\ & & \dots & & \\ x_{L1}^\beta & x_{L2}^\beta & \dots & x_{Lm}^\beta & y_{kL}^\beta \end{array}\right], \quad (1)$$

where x_{ij}^β is the value of the j-th argument in the i-th row of data, y_{ki}^β is the value of the k-th term membership function.

To facilitate the computation of table-defined function during fuzzy modeling and control we suggest to perform the approximation with the corresponding analytically-defined function. For this purpose, the expert selects the variants of the analytic type of the approximating function for the table-defined function (1). The expert uses knowledge of the physical characteristics of the process, the semantic load of the term. For a small number of function arguments $\mu_k^\beta(\bar{x})$ it is possible to construct its graph and its projection on coordinate planes. The expert can use the variants of the analytic assignment of the membership functions given in [16] or define own types. In general case, a set of possible types of membership functions of many arguments is formed:

$$F_{\mu_k^\beta(\bar{x})} = \{ f^1_{\mu_k^\beta(\bar{x})}(\bar{x},\bar{a}_1),\dots, f^N_{\mu_k^\beta(\bar{x})}(\bar{x},\bar{a}_N)\}, \qquad (2)$$

where $f^i_{\mu_k^\beta(\bar{x})}(\bar{x},\bar{a}_i)$ is the type of the membership function of many arguments; $\bar{a}_i$ is a parameter vector of the membership function type; N is a number of membership functions types allocated to a given table-defined membership function $\mu_k^\beta(\bar{x})$.

Each type of multiple membership functions $F_{\mu_k^\beta(\bar{x})}$ should be evaluated using expert evaluation methods. To this end, we suggest to use a pairwise comparisons method with the rating scale provided by Saati for the analytic hierarchy process. Based on this method, pairs of possible types of membership functions are selected from the set $F_{\mu_k^\beta(\bar{x})}$ and for each of them the expert determines the preference of one option over the other according to the scale given in Table 1 [17].

Table 1: Pairs of comparisons

Relative importance (points)	Definition
1	equal importance of the options
3	the first option is a little more important than the second
5	essential advantage of the first option over the second
7	significant advantage of the first option over the second
9	absolute advantage of the first option over the second
2,4,6,8	intermediate estimates between adjacent statements

Within this method, using the scale given in Table 1, a matrix of the results of pairwise comparisons is formed:

$$C_{\mu_k^\beta(\bar{x})} = \begin{bmatrix} c^k_{11} & c^k_{12} & \dots & c^k_{1N} \\ c^k_{21} & c^k_{22} & \dots & c^k_{2N} \\ & & \dots & \\ c^k_{N1} & c^k_{N2} & \dots & c^k_{NN} \end{bmatrix}. \qquad (3)$$

We must note that according to the requirements of the pairwise comparisons method, the product of elements of the matrix (3) symmetrical about the main diagonal must be equal to 1. That is, the estimation of the advantage of one option over another when switching the options becomes inverted. In addition, the values of the elements on the main diagonal of the matrix (3) should be equal to 1. If it is possible to involve several experts from the processes of modeling object, the pairwise comparison method should be applied to the estimates of all experts. For each variant of the function type from the set $F_{\mu_k^\beta(\bar{x})}$ the parameter values should be calculated. The calculation of of the approximation error of a table-defined membership function should be based on one of the paired criteria, since they are less sensitive to errors in the experimental data. We will use the approximation error criterion in the following general form:

$$G_k^{i\beta}(\bar{a}_i) = \sum_{n=1}^{L} (f^i_{\mu_k^\beta(\bar{x})}(\bar{x}_n,\bar{a}_i) - y^\beta_{kn})^2 \to \min \cdot \qquad (4)$$

To get the unknown values of a parameter $\bar{a}_i$ the minimum of criterion (4) must be calculated. Depending on the type of function $f^i_{\mu_k^\beta(\bar{x})}(\bar{x},\bar{a}_i)$ the least-squares method, the Monte Carlo method, or other analytical or numerical optimization methods can be used. Let $\bar{a}_i^*$ be a vector of parameter values that provide the minimum of criterion (4), and $G_k^{i\beta}(\bar{a}_i^*)$ is a corresponding minimum criterion value.

Most analytically represented types of membership functions of many arguments use the parameters of the center of geometric shapes that describe the set of allowable values in the plane of arguments [16]. To simplify the calculations when finding the minimum of criterion (4), we suggest to use the coordinates of the center of the cluster as the coordinates of the center of this geometric shape on the basis of which the table-defined function of the corresponding linguistic variable term is formed. This greatly simplifies the problem of extremum search and optimization procedures, such as the least-squares method.

In addition, to simplify the calculation procedures when applying optimization methods (such as least squares method), we suggest to replace the variables in the function $f^i_{\mu_k^\beta(\bar{x})}(\bar{x},\bar{a}_i)$ in order to linearize the dependencies in criterion (4). Consider the example of calculating the parameters of a cone-shape function by the least squares method, an example of graph is shown in Figure 1 [16].

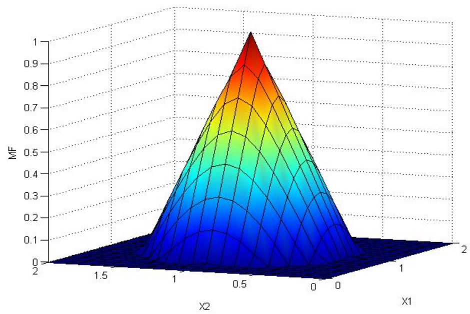

Figure 1: An example of a cone-shape membership function

We assume that the analytic type of the membership function corresponds to the general type of cone-shape function represented by the formula [16]:

$$\mu(\bar{x}) = \begin{cases} 1-\sqrt{\sum_{j=1}^{m}\frac{(x_j - x_j^o)^2}{(h_j)^2}}, if \sqrt{\sum_{j=1}^{m}\frac{(x_j - x_j^o)^2}{(h_j)^2}} < 1, \\ 0,\ else \end{cases} \quad (5)$$

where x_j is the value of the j-th component of the vector of variables; x_j^o is the j-th value of the cone base center; h_j are non-zero numerical parameters which define scaling by vector coordinates $\bar{x}$; *n* is the number of variables in a vector $\bar{x}$.

The coordinates of the cone base center x_j^o will be considered known as the values must correspond to the coordinates of the cluster center. To search for values of parameters h_j we will use only those values of the table-defined membership function y_{kn}^{β}, which are within $0< y_{kn}^{\beta} \le 1$.

Based on this assumption and given that $\mu(\bar{x}_n) = y_{kn}^{\beta}$, on the basis of (5) next transformations are performed:

$$1-\sqrt{\sum_{j=1}^{m}\frac{(x_{nj} - x_j^o)^2}{(h_j)^2}} = y_{kn}^{\beta},$$

$$\sqrt{\sum_{j=1}^{m}\frac{(x_{nj} - x_j^o)^2}{(h_j)^2}} = 1 - y_{kn}^{\beta},$$

$$\sum_{j=1}^{m}\frac{(x_{nj} - x_j^o)^2}{(h_j)^2} = (1 - y_{kn}^{\beta})^2.$$

We make the substitution of variables:

$$\begin{cases} \tilde{h}_j = (1/h_j)^2, \\ \tilde{y}_{kn}^{\beta} = (1 - y_{kn}^{\beta})^2, \\ \tilde{x}_{nj} = (x_{nj} - x_j^o)^2. \end{cases}$$

As a result, criterion (4) with unknown function $f^{i}_{\mu_k^{\beta}(\bar{x})}(\bar{x}, \bar{a}_i)$ will be obtained in linear form:

$$G = \sum_{n=1}^{L}(\sum_{j=1}^{m}\tilde{h}_j \cdot \tilde{x}_{nj} - \tilde{y}_{kn}^{\beta})^2 \to \min.$$

Based on this criterion, the unknown values of parameters $\tilde{h}_j$ can be found using the standard least squares method to find the linear function coefficients. After finding the values of parameters $\tilde{h}_j$ it is needed to perform a reverse replacement to find the parameters $\tilde{h}_j$ of cone-shape function.

The calculation of the parameters of other types of analytically defined membership functions of many variables can be considered similarly.

After calculating the parameters of all types of functions from the set $F_{\mu_k^{\beta}(\bar{x})}$, it is needed to select the function that most closely matches the process modeled on the basis of fuzzy logic and the data obtained from the experiment. In order to select the analytic membership function of many arguments, we suggest to use the analytic hierarchy process of Saati [17].

Forming the membership function of many arguments, the following criteria are used to select the corresponding function:

- estimation of the approximation error by the analytic function of the table-defined membership function;
- expert evaluation of the correspondence of the type of analytically-defined membership function to the processes formalized by a linguistic variable.

To estimate the approximation error by the analytic membership function of a table-defined membership function of a linguistic variable term, it is necessary to use criterion (4), which defines the parameter values of the analytical membership function.

According to the requirements of the analytic hierarchy process, it is necessary to scale the values of the approximation error:

$$\hat{G}_k^{i\beta}(\bar{a}_i^*) = \frac{\sqrt{G_k^{i\beta}(\bar{a}_i^*)}}{\sum_{i=1}^{N}\sqrt{G_k^{i\beta}(\bar{a}_i^*)}}, \qquad (6)$$

where $\hat{G}_k^{i\beta}(\bar{a}_i^*)$ is a scaled optimal estimate of the approximation error.

Similarly, it is necessary to scale the value of the expert evaluation in correspondence with the type of analytically-defined membership function to the simulated processes obtained by the pairwise comparisons method.

According to the analytic hierarchy process, to obtain a vector of scaled expert evaluations $\bar{c}_k^{\beta}$ =($c_k^{1\beta}, \ldots, c_k^{N\beta}$) of conformity to the simulation conditions of each type $f^i_{\mu_k^\beta(\bar{x})}(\bar{x}, \bar{a}_i)$ of analytically defined membership function it is necessary to find the eigenvector of the matrix (3) at its maximum eigenvalue by solving the system of equations:

$$\begin{cases} C_{\mu_k^\beta(\bar{x})} \cdot \bar{c}_k^{\beta} = \lambda_{\max} \cdot \bar{c}_k^{\beta} \\ c_k^{1\beta} + \ldots + c_k^{N\beta} = 1 \end{cases}, \qquad (7)$$

where $\lambda_{\max}$ is the maximum eigenvalue of the matrix $C_{\mu_k^\beta(\bar{x})}$.

The usage of formula (7) in information technology requires the implementation of numerical methods to calculate eigenvalues and eigenvectors of matrices. Therefore, to reduce the complexity of calculations, it is possible to use less accurate than (7), but easier to implement, formula for obtaining vector values of scaled expert evaluations:

$$c_k^{i\beta} = \frac{\sum_{j=1}^{N} c_{ij}^k}{\sum_{i=1}^{N}\sum_{j=1}^{N} c_{ij}^k}, \qquad (8)$$

where c_{ij}^k is the element of the matrix $C_{\mu_k^\beta(\bar{x})}$.

Using the analytic hierarchy process, it is necessary to evaluate the significance of each criterion for generating a priority for each analytically defined membership function type. To this end, the pairwise comparisons method should be applied to evaluate the above two criteria. As the comparison concerns the evaluation of the priority of the magnitude of the table-defined function approximation error and the correspondence of the analytical function type to the character of the processes of the modeling object, we suggest to establish this priority in each specific practical problem separately, since there may be significant differences between the objects of modeling experiment and data collection, expert qualification level, precision requirements for fuzzy modeling, etc. Let Z^1 denote the criterion for approximation error evaluated by the analytic function of the table-defined membership function, and Z^2 denote the criterion for expert evaluation of the correspondence of the analytically-defined membership function type to those processes that are formalized by a linguistic variable.

Using the pairwise comparisons method with the rating scale presented in Table 1, the expert should form an appropriate matrix of estimates:

$$Z_k^{\beta} = \begin{bmatrix} 1 & c^{k\beta}_{Z^2Z^1} \\ c^{k\beta}_{Z^1Z^2} & 1 \end{bmatrix}, \qquad (9)$$

where $c^{k\beta}_{Z^1Z^2}$ is an evaluation of the priority of the criterion Z^1 over the criterion Z^2 when constructing the term membership function t_k of linguistic variable *β*, moreover $c^{k\beta}_{Z^1Z^2} \cdot c^{k\beta}_{Z^2Z^1} = 1$.

Using the matrix (9) and according to the analytic hierarchy process weight coefficients γ_{Z^1} and γ_{Z^2} of the criteria Z^1 and Z^2 respectively are calculated as:

$$\begin{cases} \gamma_{Z^1} + \gamma_{Z^2} = 1, \\ 0 \le \gamma_{Z^1}, \\ 0 \le \gamma_{Z^2}. \end{cases}$$

In order to calculate the coefficients γ_{Z^1} and γ_{Z^2} we can use approaches similar to (7) or (8), depending on the requirements of the specific task and features of the implementation of information technology.

The following generalized (integral) criterion should be used to determine the analytic type of the membership function of many arguments:

$$I^*(f^i_{\mu_k^\beta(\bar{x})}) = \gamma_{Z^2} \cdot c_k^{i\beta} - \gamma_{Z^1} \cdot \hat{G}_k^{i\beta}(\bar{a}_i^*) \to \max. \qquad (10)$$

Each type of function $f^i_{\mu_k^\beta(\bar{x})}(\bar{x}, \bar{a}_i)$ of the set $F_{\mu_k^\beta(\bar{x})}$ is assigned according to the criterion value (10), the optimal criterion value of functions $F_{\mu_k^\beta(\bar{x})}$ is calculated using the method of searching a finite set. Such function $f^{i}_{\mu^{\beta}_{k}(\bar{x})}(\bar{x},\bar{a}_i)$ is selected as the term membership function for which the value of criterion (10) is maximum. We can use a vector as the function parameter values $\bar{a}_i^*$. However, at high values of approximation error, an expert can sort out many types of functions $F_{\mu^{\beta}_{k}(\bar{x})}$ and start the method from the beginning, or abandon the construction of the analytic membership function in favor of interpolation methods or the use of repeated fuzzy clustering.

3. CONCLUSION

A method for constructing membership functions for linguistic variables terms has been developed based on the processing of modeling object statistics and expert evaluations. Using the analytic hierarchy process allows to take into account the accuracy criteria for the approximation of the tabular data function obtained by fuzzy clustering, and the expert evaluation of the advantages of the membership function types for particular simulation conditions. This approach can be used in general case to construct the membership functions of many arguments in the development of information systems based on fuzzy logic.

Further studies should consider the particularities of applying the approaches proposed when working with the evaluations of a group of experts.

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