

Modelling Heterogeneous Data Transmission Systems Based on Queueing System Networks



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ABSTRACT

The range of problems associated with the analysis of computer systems based on switches and hubs is considered. It is assumed that the corresponding computing system serves small-scale production, which has several standard subsystems. The formulation of the system design problem with minimising the message delay time is discussed. A mathematical model of the switch is built, its analysis is carried out.

Use of a device that provides analysis and routing of traffic arriving at its ports, as well as high-speed parallel exchange between several different pairs of ports, as a central network module, leads to a better distribution of the intensities of incoming messages in the inbound communication channels. Representation of the mathematical model of the system in the form of a Queueing System network greatly simplifies the finding of mean of information message flow rates between local systems of business units, reduces the complexity of experimental research and the necessary experimental measurements.

Key words: heterogeneous system, data transmission, traffic routing, queueing system, mathematical model, Markov model.

1. INTRODUCTION

Many units that are part of a modern industrial enterprise make high requirements for the speed and reliability of data transfer over a computer network. Computing systems can be created to perform various tasks, and the specificity of the data transmitted and processed in them determines the special requirements for the computing system architecture. When creating a data transmission system, the main attention should be paid to control of the data flow in them [1, 2, 3]. Depending on the functional purpose of the business unit, the nature of the data flow of the computing system changes. At the same time, the computer networking should be performed in such a way as to ensure efficient and adaptive production

process control [4, 5], collection and processing of large amounts of information, its storage and presentation in a form convenient for operating personnel. In addition, the capabilities of the data transmission system should provide continuous monitoring of the state of the control objects and timely response to all their changes. At the same time, the computing system should be easily customizable, easy to maintain, and expandable in the future [6, 7].

When designing a computer system, one of the main problems is the problem of choosing the general structure of the system and the bandwidth of the system channels, since the timely delivery of messages and the cost of both cable lines and network equipment directly depend on this [8, 9].

Considering the computer system as an object of design, it is necessary to highlight several formal features and properties that characterize the data transfer system in terms of a system. These features and properties include:

- presence of many interconnected and interacting elements;
- multifunctional and hierarchical structure;
- existence of a common goal for the functioning of the system, and general purpose, which determines the unity of complexity and organization;

stochastic nature in the interaction of subsystems and elements in the structure of the system.

2. CHARACTERISTICS OF A TYPICAL COMPUTING SYSTEM FOR SMALL-SCALE PRODUCTION

In the designed computer system, most of the messages sequentially pass through several queueing systems (network devices). In this case, a computer system model can be represented as a network of queueing systems, which is characterized by the relationships between individual QS (Queueing System) and the properties of the systems themselves.

In the study of the mathematical model, it can be assumed that the outbound message flows from the workplaces are distributed according to the Poisson law, i.e. they possess ordinarity, no aftereffect and stationarity. In real computer networks, the transmitter protocol stack sends the message in batches, breaking it into frames of a certain length. On the receiver side, the reverse operation is performed - analysis and collection of packets into a single message.

However, when creating a mathematical model, we can assume that the designed data transmission system is a message-switched network, i.e. division of the transmitted message into packets does not occur. The next assumption is that network devices (workstations, servers, switches, hubs) have size-unlimited transfer memory buffers. This simplification is quite justified, given the considerable size of buffers of modern network devices, as well as the principles of data transfer protocol operation, which consist in the fact that when the device buffer is full, the sender and receiver exchange service information packets that block the transmission of the main data flow until the buffer memory is freed.

Considering a computer system as a network of queueing systems, it is necessary to determine what is a queue block and what is a service device for each QS within mathematical model. In this approach to creating a mathematical model, we assume that:

- the queue blocks in all QSs are the memory buffers of the corresponding network devices;
- Request service devices in all QSs are data channels with certain bandwidths.

The assumption that network devices have size-unlimited memory buffers allows us to consider the queueing system as QS with a length-unlimited queue.

Since in the method of accessing the CSMA/CD data transfer media, which is the basis for the Ethernet technology, all computers on the network are equivalent in terms of priority for message transmission, and there is no possibility of setting priority for any type of message, queueing in QS can be considered as non-priority type queueing. The principle of operation of the Ethernet network does not allow the use of duplicate or parallel alternative routes for data transfer, which makes it possible to consider all QSs included in the model as single-channel ones.

The experimental determination of the message flow rates through each transmission channel presents considerable difficulties. Representation of a computer system model as a network of queueing systems greatly simplifies finding the mean number of messages passing through communication channels per unit of time. To do this, it is necessary to experimentally determine the intensity for only one data transmission channel and have expert estimates of the relative

distributions (probabilities) of the number of messages transmitted over the data transmission channels. The probability of timely delivery of messages in a computer system will be discussed separately.

3. MATHEMATICAL MODELS OF SWITCHING DEVICES

Despite the differences in the principles of constructing switches, the construction of a mathematical model of the switch is carried out regardless of its operating features. Since the main purpose of the switch is to divide the network into logically independent subnetworks, then, when creating a mathematical model, each port to which the network segment is connected can be considered as a single-channel device for servicing requests from the queueing system. The law by which the servicing will be provided will be determined by the law of the probabilistic distribution of the lengths of messages transmitted in the system.

The source of requests in this case is a network device (hub, router, switch) directly connected to the port. If the LAN of the unit is connected to the port, then the message (request) flow to the corresponding port is formed as the sum of the flows from each workplace of the network segment in question. A message from the source of requests arrives directly to the service device (port microprocessor unit) or, if it is busy, it enters the queue (port microprocessor unit buffer). Given the principles of operation of modern data transfer protocols, we can assume that the port buffer has an unlimited RAM size. When constructing a mathematical model, this allows us to assume that the waiting queue for servicing a request has an unlimited length. After servicing in the microprocessor unit of the switch port, the message arrives at the input of the corresponding microprocessor unit of the port, depending on the destination address. The message is forwarded to the corresponding microprocessor unit of the port via the data bus inside the switch. Therefore, the overall request flow arriving at the input of the service device is summed up from the request flow of the external segment of the network and the request flow arriving via the internal bus from other service units of the switch model.

As a result, each switch port in use can be modelled by a single-channel queueing system with an unlimited queue. Laws of service and arrival of requests are determined by the specifics of operation of automated workplaces that form transmitted messages.

Since the switch is used to combine at least two local segments of the system, the switch model will represent the relationship of several QSs of the same type, i.e. it will form a network of queueing systems. Because of this, to calculate the intensities of messages passing through the ports of the switch, the mathematical apparatus for calculation of QS networks with the exponent rules of the distribution of

probability densities can be used. Figure 1 shows an example of a triple-ported switch model.

Since bridges and routers perform the same functions as switches (but only on a smaller scale - 2 ports), their mathematical models can also be represented as a QS relationship.

The application of the QS theory mathematical apparatus will be shown by the example of modelling the computer

system shown in Figure 2. An example of a local area network of a unit included in a computer network of an enterprise is shown in Figure 3.

This computer system includes most of the modern network devices used for the synthesis of computer networks. The transmission graph of the model of the computer system under consideration is shown in Figure 4 (0 - virtual request source).

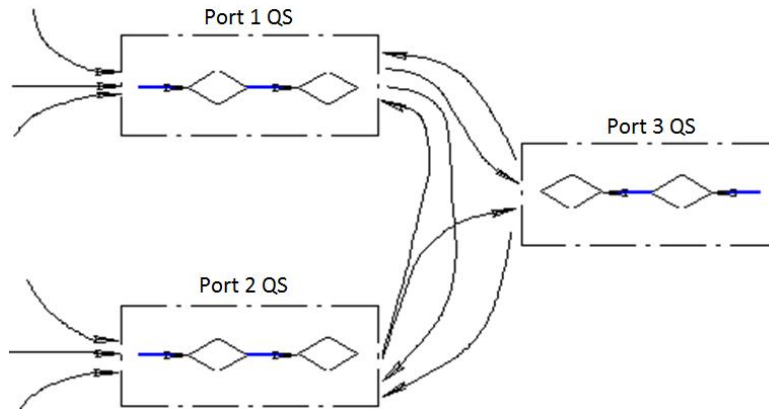


Figure 1: Triple-ported switch model

In the model of a computer system, two types of constituent subsystems can be distinguished. The structures of

the models of these subnetworks are as follows.

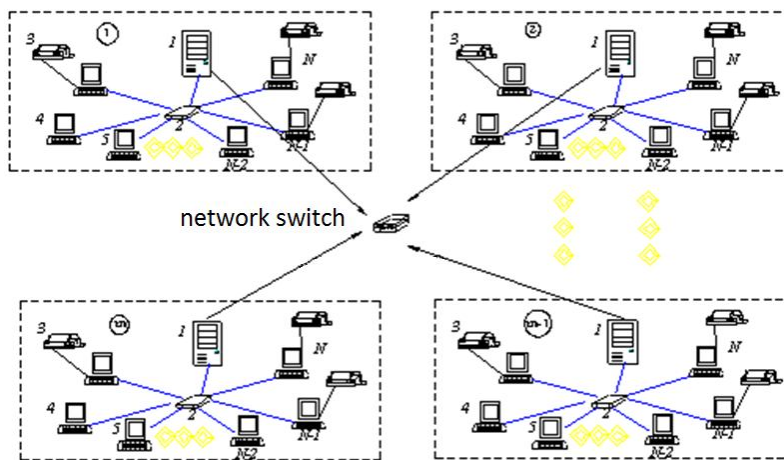


Figure 2: The structural diagram of the computer system of a standard enterprise

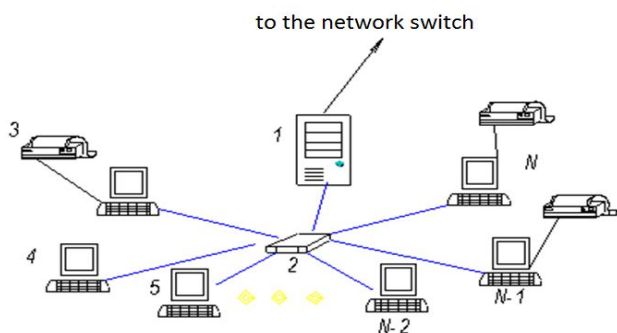


Figure 3: The structure of the computer system of the unit: 1 – server router; 2 - hub; 3..N - operator workplaces

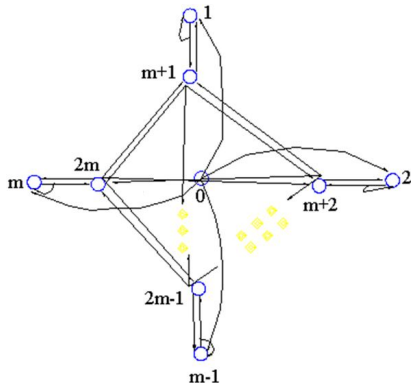


Figure 4: Transmission graph of a computer system: each of the arcs has its own probabilities of arrival of messages from QS_i to QS_j

1. The request source is a LAN hub. Service time is the time of transmitting the message from the workstation of the local network to the server router of this network, and more precisely, to the port of the server router related to the local segment. In the diagram shown in Figure 4, these QSs are shown with the numbers 1..m. The number of such QSs in the local network coincides with the number of hubs (or servers) in the general data transmission system.

2. Sources of requests are the ports of the server router related to the switch port and all but one of the switch ports. Service time is the time of transmitting the message from the port of the server router related to the port of the switch of this network or from all switch ports but the one under consideration. Based on the logic of the switch operation, its model is represented as the relationship of QS - ports of the switch. As a result, the number of QSs in the switch model is equal to the number of active ports in the switch. These QSs are shown under numbers (m+1)..2m in Figure 4.

The complexity of calculating QS networks lies in the fact that the simplest request flow coming into a service system will generally possess an aftereffect at its output. In this case, we cannot use the Markov QS analysis apparatus. However, based on mathematical studies of the correlation properties of request flows, we can assume that if the service durability is distributed according to the exponential law on all devices in the network, then the request flows outgoing from QS will be Poisson. Knowing the transition probabilities and one of the request flow rate based on the graph shown in Figure 4, a system of linear algebraic equations is composed. The unknowns in this system are the message flow rates arriving at each QS in the network.

After calculating the intensities of the requests arriving at the inputs of the QS network, the characteristics (mean request sojourn time in the system, mean waiting time in QS) of each QS are calculated. If necessary, the total mean request sojourn time in the network is determined based on the found parameters of each QS.

The model of the computer system contains $2m$ QS and one source of requests. Requests arrive from the source of requests directly to the j -th system with probability P_{0j} . Requests arrive from the i -th system to the j -th system with probability P_{ij} . To determine the characteristics of the QS network, first, it is necessary to determine the request flow rates in each system. The mean number of requests leaving the system is equal to the mean number of incoming requests,

and therefore, for the steady state, $\lambda_j = \sum_{i=0}^{2m} \lambda_i P_{ij}$, where $j = 1, 2, \dots, 2m$, is true. In the matrix form $\lambda = \lambda T$ or $\lambda = T' \lambda$, T' is the transposed matrix with respect to T . We introduce the dynamic matrix $D: D = T - I$, where I is the identity matrix, D' is the transposed matrix with respect to D , $D' \lambda = 0$. This relation forms a system of linear homogeneous equations with respect to unknowns λ_i (the intensities of incoming requests). The solution of the system is λ_i as a function of experimentally determined λ_k .

4. ANALYSIS OF THE MARKOV MODEL

Let us derive the main characteristics of QS with Poisson distribution laws. By E_n we denote the state of the system when there are n requests in it. In the time interval $[t, t+dt]$, transitions can occur with the corresponding probabilities, which are determined by the transition matrix:

	E_0	E_1	E_2	E_3
E_0	$1 - \lambda dt$	λdt	0	0
E_1	μdt	$1 - (\lambda + \mu) dt$	λdt	0
E_2	0	μdt	$1 - (\lambda + \mu) dt$	λdt
E_3	0	0	μdt	$1 - (\lambda + \mu) dt$

Since the interval dt is small, the approximate equalities are used: $e^{\lambda dt} \approx 1 - \lambda dt$, $e^{\mu dt} \approx 1 - \mu dt$.

The probability of the transition $E_0 - E_0$ is determined by the probability of the absence of request arrivals for the interval dt and is equal to $1 - \lambda dt$. The probability of the transition $E_n - E_{n+1}$ is determined by the probability of the arrival of one request and the probability of not servicing the request being serviced and is equal to $(1 - \mu dt) \cdot \lambda dt \approx \lambda dt$. The probability of transition $E_i - E_{i-1}$ is determined by the probability of servicing one request during the time dt and the probability of non-arrival of the next request, and is equal to $(1 - \lambda dt) \cdot \mu dt \approx \mu dt$. The transition probability $E_i - E_i$ is determined by the probability of a compound event $(1 - \lambda dt) \cdot (1 - \mu dt) \approx 1 - (\lambda + \mu) dt$. The state equations are obtained from the transition matrix in the form $P(t+dt) = P(t)J$, or

$$P_0(t+dt) = (1 - \lambda dt)P_0(t) + \mu dt P_1(t); \quad (1)$$

$$P_n(t+dt) = \lambda dt P_{n-1}(t) + (1 - (\lambda + \mu) dt) P_n(t) + \mu dt P_{n+1}(t). \quad (2)$$

After standard conversions, we obtain the main parameters of the queueing system that interests us.

$$P_0 = 1 - \frac{\lambda}{\mu} = 1 - \psi, \psi = \frac{\lambda}{\mu}, P_n = (1 - \psi)\psi^n.$$

The mean number of nodes in the system is defined as $\bar{n} = \frac{\psi}{1 - \psi}$.

The mean number of requests in the queue is $\bar{v} = \frac{\psi^2}{1 - \psi}$.

For one request, the mean sojourn time in the system is equal to the ratio of the mean number of requests in the system to the mean number of nodes passing through the system per unit time. Since the queue does not grow indefinitely, the mean number of incoming requests is equal to the mean number of leaving requests $\lambda = (1 - P_0)\mu$. The mean sojourn time in the system is

$$\bar{u} = \frac{1}{\mu} \frac{1}{1 - \psi}.$$

For one request, the mean waiting time in the system is $\bar{\omega} = \frac{1}{\mu} \frac{\psi}{1 - \psi}$.

5. THE PROBABILITY OF TIMELY SERVICE

To determine the probability of timely delivery of a message in a computer system, we artificially introduce another characteristic of the message - the “ageing” time. We assume that the “ageing” probability density function is distributed according to the exponential law with mean intensity ν .

Two basic random processes occur in a data transmission system: a waiting process characterized by a random time t_w and a servicing process characterized by a random time t_c . Due to additivity, a random delivery time t_v will be defined as

$$t_v = t_w + t_c. \quad (3)$$

For the corresponding model, we apply the Laplace transform method. Due to the multiplicativity of the Laplace transform, the probability of timely message delivery for such a model will be defined as

$$Q = w(\nu) = w(\nu)h(\nu) \quad (4)$$

where $w(\nu)$ is the Laplace transform of the waiting time distribution function; $h(\nu)$ is the Laplace transform of the service time distribution function.

For the probability of timely delivery, we finally obtain:

$$Q = \frac{\mu - \lambda}{\mu - \lambda + \nu}. \quad (5)$$

Given the value of the mean amount of data transmitted over the network, as well as the restrictions on the time of message transmission over communication channels, formula (5) allows us to determine the required bandwidth of data channels in a real computer system.

6. SYNTHESIS OF THE DATA TRANSFER NETWORK STRUCTURE

Using modern methods of integrating computer systems allows us to synthesize the following version of the enterprise computer network structures shown in Figure 5. With this method, data exchange between local networks is carried out through a common network device - a switch.

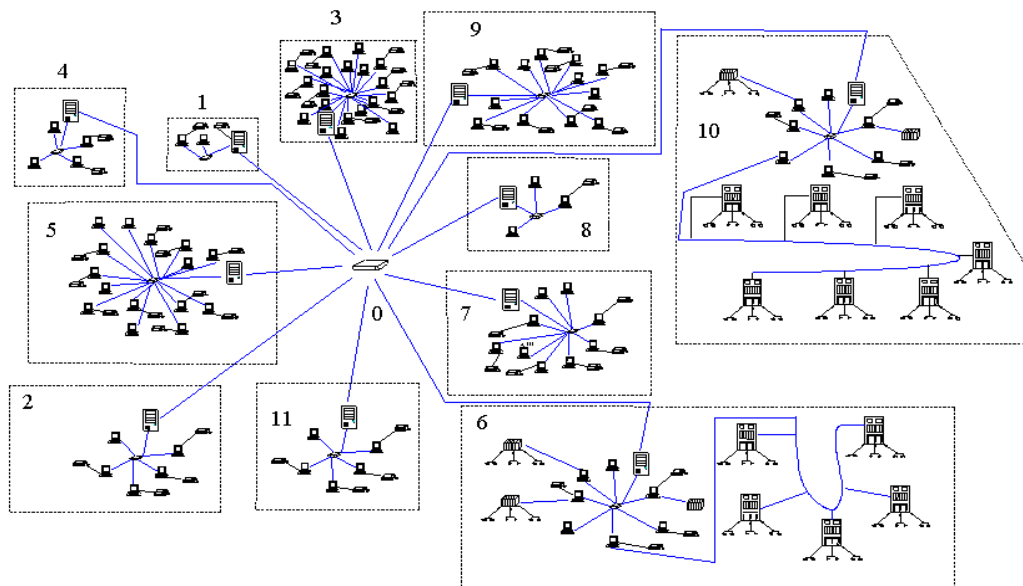


Figure 5: Version of the structure with a central switch

The mathematical models of the computer systems under consideration are networks of queueing systems. Figure 6 shows the transition graph of the proposed version of the computer system. The numbers indicate the queueing systems through which requests (messages) pass during their transmission over the network.

The analysis of the results shows that with a change of λ in the range $[0.. \lambda_{trans}]$, the probability of timely delivery has an approximately constant value, and in the range $[\lambda_{trans}.. \lambda_{crit}]$, there is a sharp decrease in the probability of timely delivery.

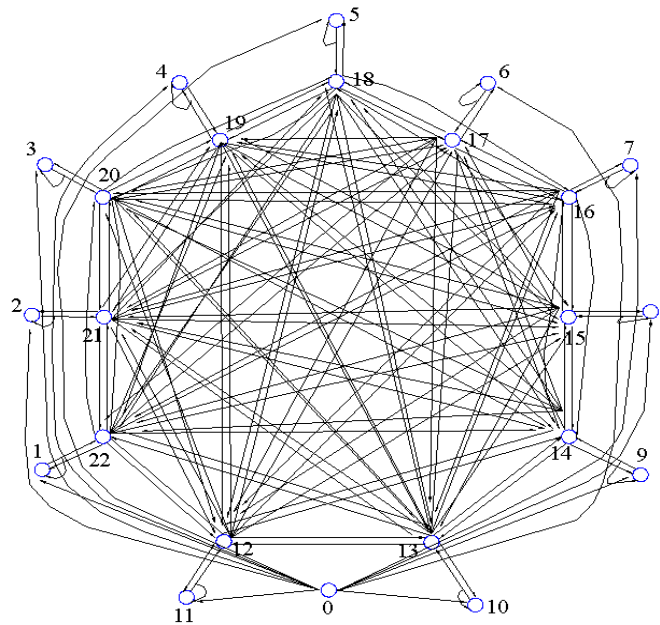


Figure 6: QS transition graph

The analysis of dependencies allows us to draw the following conclusions. When flow rates of the circulating messages are low, the probability of timely delivery and the sojourn time in the system are approximately the same for all variations of the schemes and have acceptable values (for probability value of more than 0.9) for given network parameters. With low traffic of incoming data packets, the probability of timely delivery in the system using a hub as the central switching node has the greatest (although with a minimum difference), compared with other structures of the systems, value. However, an increase in the external flow rates of transmitted messages in the networks of the enterprise units leads to a sharp increase in traffic through communication channels incoming to the ports of the hub. These weaknesses are largely corrected in the proposed version of the system structure (Figure 5). Use of a device that provides analysis and routing of traffic arriving at its ports, as well as high-speed parallel exchange between several different pairs of ports, as a central network module leads to a better distribution of the intensities of incoming messages in the incoming communication channels.

The values λ_{trans} and λ_{crit} are determined by the service intensity of the considered queueing system. For λ_{crit} , there is an overload of data transmission channels and the queue (waiting time) of messages increases unlimitedly, the probability of timely delivery for $\lambda \rightarrow \lambda_{crit}$ tends to zero. All dependencies have a plateau when λ is changing.

7. CONCLUSION

Representation of the mathematical model of the system in the form of a QS network greatly simplifies the finding of mean of information message flow rates between local systems of enterprise units, reduces the complexity of experimental research and the necessary experimental measurements.

Use of a device that provides analysis and routing of traffic arriving at its ports, as well as high-speed parallel exchange between several different pairs of ports, as a central network module, leads to a better distribution of the intensities of incoming messages in the inbound communication channels. Representation of the mathematical model of the system in the form of a Queuing System network greatly simplifies the finding of mean of information message flow rates between local systems of business units, reduces the complexity of experimental research and the necessary experimental measurements.

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