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Performance Optimization Method in OFDM Based on Majorization and Minimization Technique

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ABSTRACT

Phase Noise is considered a remarkable problem which causes significant degradation in detecting packet- based Orthogonal Frequency Division Multiplexing (OFDM) signals. Therefore, its estimation is essential to reduce the interference of channels in signals. Basically, this noisy signal might be entangled with OFDM signal according to many reasons. One of these important reasons is related to oscillator itself which generates the carrier signals and causes inter channel interference (ICI). The second main reason is multipath fading channel which causes a delay of OFDM signal and results of inter symbol interference (ISI). Another type of noise is known as Additive White Gaussian Noise (AWGN) or thermal noise whose effect is negligible comparing to phase noise. In this paper, we demonstrate and simulate a practical algorithm to mitigate phase noise which induces ICI .This algorithm is termed as Tight Quadratic Majorization (TQM) and is derived for phase noise estimation. Basically, TQM principles are based on time-domain OFDM symbols Majorization and Minimization (MM) technique. Therefore, we explain in details the behavior of the signal whose phase noise **(**PHN**)** is modeled as Wiener Process and channel is a Circularly Symmetric Complex Gaussian ($\mathcal{CN}$).While, the channel impulse response is considered as static channel with slow-fading in its energy. Moreover, we clarify the idea of MM technique with its conditions. Then, we illustrate the derivation of TQM by the assumption of applying projection matrix whose maximum eigenvalue is one. Mainly, this assumption is utilized in order to simplify TQM derivation. Finally, we evaluate the OFDM modulation signal in time-domain at receiver-side. This signal is affected by large Wiener Process PHN and estimated by TQM algorithm. MATLAB simulation results reveal that TQM implementation has outperform to approximate PHN in OFDM systems with high number of subcarriers (Nc).

Key words : Inter Channel Interference, Majorization-Minimization (MM), Orthogonal Frequency Division Multiplexing(OFDM) systems, Wiener Process PHN.

1. INTRODUCTION

Nowadays, both broadcasting,[digital video broadcast (DVB)] and wireless local area networks (WLAN), adapt multi-carrier transmission to combat the hostility of wireless channel and providing high data rate communications [1]. OFDM (Orthogonal Frequency Division Multiplexing) is a special form of multi-carrier transmission where all subcarriers are orthogonal to each other [2]. Normally, dealing with ever unpredictable wireless channel at high data rate communications is not an easy task. Thereby, OFDM suffers from some limitations. One of these limitations is frequency synchronization errors (highly sensitive to analog radio frequency RF impairments) which refers to contradiction in carrier frequency, [also known as frequency offset (CFO)] between transmitter and receiver side. This problem is caused by separately used local oscillators (LO) and being relative motion between both of them in transmitter and receiver that gives the permission to Doppler spread to arise [2,3]. Synchronization needs to be done in three factors: symbol, carrier frequency, and sampling frequency synchronization. The crucial side effect of frequency synchronization error is phase noise. Generally, Phase noise is a random process which manifests itself in two ways- a common phase error (CPE), which is an identical phase rotation in all subcarriers while inter-carrier interference (ICI) is a result of losing the orthogonality among subcarriers [3]. In other words, phase noise is the phase (θ) difference between the phase of the carrier signal and the phase of LO. CPE can be removed easily since it is the same rotation in all carriers [4].On the other hand, the major effect of PHN revolves around ICI since any change in one phase of subcarrier will transfer to the next subcarrier that will cause a severe damage to the OFDM signal (OFDM transmission block corruption) [4].Basically, it is not a big manner to deal with single carrier since modulation method can track the fast variation of phase noise adaptively in a decision directed manner [5] while in multiple carriers OFDM transmits data symbols over low rate subcarriers, this technique makes OFDM very difficult to track and compensate phase noise. Therefore, applying estimation method for PHN is crucial. Phase noise is measured by two-metrics the bit-error-rate (BER) and signal-to-noise ratio (SNR) [4,6]. As aforesaid the first cause of phase noise is LO. In reality, there are two kinds of oscillator- open-loop (e.g., free running oscillators) and closed-loop (e.g., phase locked loop PLL oscillators) [1].It is well known experimentally and theoretically that PHN is modeled as Wiener Process in free running oscillators [1]. This paper is targeted Wiener Process PHN and measured its effect by SNR on OFDM signal when

TQM algorithm is executed [4] since ICI PHN is dominant over CPE PHN and it has high impact on OFDM signal.

2. OFDM SIGNAL PHASE NOISE-WIENER PROCESS

Combating the hostility of wireless channel and providing high data rates are the main reasons to appear Multi-Carrier transmission. The idea of Multi-Carrier is considered the core of OFDM signal where all subcarriers are orthogonal to each other. Therefore, OFDM can provide a high user data rate transmission capability at a reasonable complexity and precision. An OFDM signal consists of numbers N of sinusoids waves that are modulated by data symbols a_m which have a duration time T with spacing channel 1/T [6].By applying Fast Fourier Transform (FFT) algorithm at transmitter and receiver side Eq(1) , OFDM can be performed efficiently. Therefore, we need to know the difference of OFDM signal behavior in both single carrier and multiple carrier. In single carrier at symbol a_m duration time T, is considered in [6]:

$$s(t) = \sum_{m=0}^{N-1} a_m e^{j2\pi(\frac{m}{T})t} \quad (1)$$

The data symbols a_m is modulated at frequency m/T during the period of time T. T is represented $T_{symbol} = 1/B \ \mu s$ symbol duration in single carrier scenario where B is the total bandwidth. N is the total number of symbols and by measuring the total symbol rate R in OFDM single carrier transmission is equal to N/T. While in multiple-carrier transmission scenario, the data symbols s_k is transmitted at $T_{symbol} = 1/B\,/Nc\ ms$ where bandwidth B is divided by the number of subcarriers Nc. Thereby, low symbol rate is transmitted through a longer time period which leads to prevent Inter Symbol Interferences (ISI), e.g., [9]

$$\frac{1}{\frac{B}{Nc}} ms \gg \frac{1}{B} \mu s$$

Equation (2) describes OFDM signal in multiple-carrier transmission scenario [4]:

$$x_n = \sum_{k=0}^{Nc-1} s_k e^{(j2\pi nk/Nc)}, n = 0, 1, \dots, Nc - 1 \quad (2)$$

Where s_k is the data symbol modulated during the frequency $n * k/Nc$.In both aforementioned situation OFDM signal is performed in the absence of carrier phase impairments [6].However, when PHN affects OFDM signal, signal behavior is expressed in Eq(3) will be [6]:

$$s(t) = (\sum_{m=0}^{N-1} a_m e^{j2\pi(\frac{m}{T})t}) e^{\theta(t)} \quad (3)$$

Where $\theta(t)$ is illustrated a time varying phase caused by either a Carrier Frequency Offset (CFO) between the receiver and transmitter carrier, or PHN of carriers themselves. In the first case, $\theta(t)$ is represented the variation between the Local Oscillators (LO) in transmitter and receiver side. Therefore, the following equation can express it [6]:

$2\pi\Delta F + \theta_0$, where ΔF is the ***CFO*** (4)

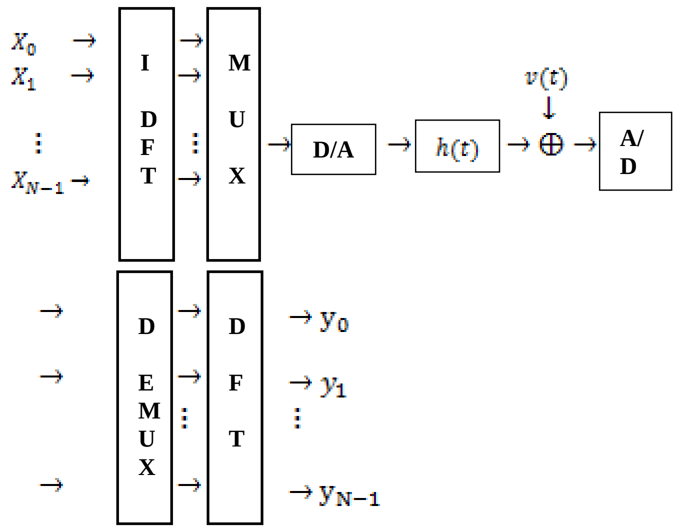

Fig.1: OFDM Model with channel $h(t)$ and (AWGN) $v(t)$

In the second case, $\theta(t)$ is modeled as Wiener Process (Brownian motion) [4,6]:

$$\theta t - \theta t - 1 \sim \mathcal{N}(0, \sqrt{2\pi\Delta f3dB}/fs) \quad (5)$$

Where $\mathcal{N}(0, \sqrt{2\pi\Delta f3dB}/fs)$ represents Gaussian normal distribution which has zero mean (μ) $\varepsilon[\theta(t)]=0$, $\varepsilon[\theta(t+t_0) - \theta(t_0)]^2 = 4\pi\beta[Hz]|t|$ which represents the variance (σ^2) , fs is the sampling rate of θt and $\beta[Hz]$ denotes the one sided 3dB linewidth of the Lorentzian power density spectrum of the free-running carrier generator.

2.1 OFDM Signal Evaluation

The design of the OFDM block is expressed in Figure 1[7,8] and taken into account the behavior of signal through slow varying channel (impulse response channel).Where, x_k are the transmitted symbols,$h(t)$ is the channel impulse response,$v(t)$ is the complex Gaussian channel noise and y_k are the received symbols. The transmitted symbols are taken from multi amplitude signal constellation. The D/A and A/D converters contain ideal low-pass filters with bandwidth $1/Ts$, where Ts is the sampling interval period of the system. A cyclic prefix (CP) of time length T_G is used to eliminate inter symbol interference (ISI) and preserve the orthogonality of the sine waves. Now, we are going to express channel impulse response based on two models:

1) Channel evaluation is treated as a time limited pulse train [7]:

$$g(t) = \sum_m \alpha_m \, \delta(t - \tau_m T_s), \qquad (6)$$

Where $g(t)$ is a channel attenuation vector space (channel impulse response), m is multiple paths which assigned the possibility that the signal could take, τ_m is an amplitude complex valued (different values are mapped to different paths) and $\tau_m T_s$ is distributed over Cyclic Prefix (CP) $(0 \leq \tau_m T_s \leq T_G)$ which represents a power delay during time t.

2) Channel evaluation for L taps with lth user [9]:

$$h_u(\tau, t) = \sum_{l=0}^{L} h_{u,} l(t) \, \delta_c(\tau - \tau_l) \qquad (7)$$

Where $h_{u,} l(t)$ of complex gain of lth multipath component for the uth user at time t. The channel is assumed to be constant (static) for the duration of one OFDM symbol, and the path gain coefficients for each path contribution are supposed to be uncorrelated. Rewrite Eq(7) as illustrated in Eq(8):

$$h_u, s(t) = \sum_{l=0}^{L} h_u \, l[s] \delta c(t - \tau_l), \qquad (8)$$

where $h_u, l[s] = h_{u,} l(t)$ and $sTs \leq t \leq (s+1)Ts$, Ts is the total OFDM symbols, sTs is a previous total symbols while $(s+1)Ts$ is a post total symbols. Precisely, both aforementioned scenarios are represented the probability of total numbers of paths that specific OFDM signal is taken through transmission process and are known as Rayleigh fading channel which is a statistical model depends on probability theory. In this paper, the channel is assumed static Rayleigh fading channel h with slow-varying[4] whose response does not change with the transmission of OFDM blocks and is denoted by Eq(9):

$$h = [h_0, h_1, \ldots, h_{L-1}]^T \; N_c \gg L \qquad (9)$$

Let Eq(10) represents OFDM transmission symbols *x* [4]:

$$x = [\mathcal{X}_0, \mathcal{X}_1, \ldots, \mathcal{X}_{Nc-1}]^T \qquad (10)$$

v is a zero-mean circularly symmetric complex Gaussian channel noise vector with distribution $CN(0, 2\sigma^2 I)$; denotes thermal noise [4] is expressed in Eq(11).

$$v = [v_0, v_1, \ldots, v_{Nc-1}]^T \qquad (11)$$

As a result, Eq(11) describes the OFDM received channel *y* vector space[4]:

$$y = [\mathcal{Y}_0, \mathcal{Y}_1, \ldots, \mathcal{Y}_{Nc-1}]^T \qquad (12)$$

With cyclic extension (CP) appending and removal, we can obtain the OFDM transmission model [4] as in Eq(13):

$$\mathcal{Y} = \mathcal{X} \circledast \begin{bmatrix} h \\ 0 \end{bmatrix} + v \qquad (13)$$

the subscription $\circledast$ is a convolution process between data symbols $\mathcal{X}$ and channel impulse response *h*.

According to Eq(3) ,phase noise is expressed by Eq(14) [4]:

$$e^{j\theta} := [e^{j\theta 0}, e^{j\theta 1}, \ldots, e^{j\theta Nc-1}]^T \qquad (14)$$

therefore the aforementioned OFDM model is affected by phase noise and expressed by Eq(15):

$$\mathcal{Y} = e^{j\theta} \odot (\mathcal{X} \circledast \begin{bmatrix} h \\ 0 \end{bmatrix}) + v, \qquad (15)$$

where $\odot$ is a Hadamard product.

3. TRANSMISSION FOR SINUSOIDAL SIGNAL

Regarding to Eq(2) ,sine waves are transformed to unitary discrete Fourier transform whose vectors are orthonormal (orthogonal to each other and their lengths are equal to one) in order to reduce the computational complexity.Let F be the Nc × Nc DFT matrix (square matrix).Therefore, we can rewrite Eq(2)[4] as in Eq(16):

$x = F^H s$, where $s = [s_0, s_1, \ldots, s_{Nc-1}]^T$, represents symbols vector space (16)

For consistency of the channel impulse response with (DFT) matrix, we build Semi-Unitary-Matrix $F^{\sim}$ $Nc \times L$,where number of its columns equal to L tap channels. Consequently, we obtain: $(F^{\sim} h)$ .Moreover, for easy computation, we rearrange symbols vector space to diagonal matrix, therefore: $S = Diag(s)$.Finally, in this paper the last pattern of OFDM block transmission based on Eq(15) will be described in Eq(17) [4] :

$$y = \sqrt{Nc} diag(e^{j\theta}) F^H S F^{\sim} h + v \qquad (17)$$

4. PHASE NOISE COMPENSATION

Phase Noise estimation and compensation can reduce ICI between data subcarriers and directly facilitate information decoding at receiver side. Therefore, we can see a posteriori channel estimator in paper [10] has exploited the statistical properties of phase noise. The issue with this method, the authors apply Taylor expansion to approximate the nonlinear optimization objective function, which proves only a small phase noise. While in [5] ,the authors utilize the advantage of previous method with simple modification by applying estimation technique in two stages pilot type for time-domain and comb OFDM type at receiver side. The critical issue with their method is could not deal with phase noise constraint for each iterative sub-problem. A novel method is proposed in [3] for disentangling phase noise and channel estimation .However, their method is computationally unstable with a singularity issue which provides inaccurate solution. Based on [3], semi-definite-programming (SDP) is provided a solution for the problem of phase noise estimation separately from channel estimation. Nevertheless, the problem with their method is number of subcarriers Nc which spreads in OFDM are not too high and phase noise emerges in a small level. These drawbacks with previous optimization methods lead to formulate the objective problem of phase noise [4] as we explain in flowing section.

4.1 Problem Formulation

The model which describes the optimization problem is called proximal gradient algorithm. This model is divided into two terms, first term is to formulate the objective problem and get the local error while second term is to describe the convex function in order to be very close to the solution. Therefore we can express it in model (M)[11]:

$F(x) = f(x) + g(x): x \in \mathbb{E}$ (18), where $\mathbb{E}:$ is an Euclidean distance ,$f(x)$: is an objective function and $g(x)$ is a convex function.

$LSE\, min_x \{\| Ax - b \|^2 + \lambda R(x)\}$ (19), is a regularized least squares (RLS),where

$f(x) = \| Ax - b \|^2$ (20), is a Least Squares Error (LSE) and $g(x) = \lambda R(x)$, where λ represents a regularization parameter providing the tradeoff between fidelity to measurement and noise sensitivity while $R(x)$ is a convex regularizer stabilize the solution.Thereby, the objective function is derived based on Eq(17) [4] will be:

$$\left\| y - \sqrt{Ncdiag\,(e^{j\theta})F^H SF^{\sim} h} \right\|^2 \quad (22),$$
$$minimize\ h, \theta$$

By solving Eq(22) for *h* and θ, we can get the least-squares channel estimate, which is described in Eq(23) and Eq(24) respectively;

$$\hat{h} = \frac{1}{\sqrt{Nc}} \left(F^{\sim H} S^H S F^{\sim}\right)^{-1} F^{\sim H} S^H F diag\,(e^{j\theta})^H y \quad (23),$$

$$\epsilon(\theta) = y^H Diag\,(e^{j\theta}) F^H (I_{Nc} - B) F Diag\,(e^{j\theta})^H y \quad (24),$$

where $B = SF^{\sim}\left(F^{\sim H} S^H F^{\sim} S\right)^{-1} F^{\sim H} S^H$

B is formulated as diagonal matrix of symbols *S* multiplies with semi-unitary matrix $F^{\sim}$ in order to make a projection matrix. In other words, it is the type of transformation for OFDM data symbols block from transmitter side to receiver side. From Eq(24), phase noise estimation problem can simplify as:

$V = F^H (I_{Nc} - B) F$ and $u = e^{-j\theta}$,then

$$\theta^{\wedge} = u^H Diag(y)^H V Diag(y) u \quad (25),$$

Thus, u is minimized according to the unimodular property of *PHN* which demonstrates the magnitude of *PHN* for each subcarrier has to be equal to *1*.Then, the joint phase noise and channel estimation problem depends on the constrained of Eq(26).In other words, values of θ has to be normalized to unity,i.e., $|e^{j\theta n}| = 1$,where n=1,…,Nc-1 (26).

5. MAJORIZATION AND MINIMIZATION (MM) TECHNIQUE

The idea of MM technique is derived from gradient-based algorithm method which is considered a very old method going back as early 1847 with the initial work of Cauchy[11]. The main idea of MM depends on finding an approximation to the objective function F of Model M that satisfies:

$$M(x, x) = F(x) for\ every\ x \in \mathbf{E} \quad (27)$$

Where **E** *is an Euclidean distance*

$$M(x, y) \geq F(x) for\ every\ x, y \in \mathbf{E} \quad (28)$$

From this explanation we understand that $x \to M(x, y)$ lies above $F(x)$ and is tangent at $x = y$.Therefore, MM technique is determined to solve:

$$x_k \in argmin\, M(x, x_{k-1}), x \in \mathbf{E} \quad (29)$$

Where $argmin$ stands for argument of the minimum, and is defined analogously.
This means:

$$M(x, x_{k-1}) \leq M(x, x_{k-1}) for\ every\ x \in \mathbf{E} \quad (30)$$

Thus, from Eq(27) **,**Eq(28) and Eq(30) ; we understand:

$$F(x_k) \leq M(x, x_{k-1}) \leq M(x_{k-1}, x_{k-1}) = F(x_{k-1}) \quad (31)$$
$$for\ every\ k \geq 1$$

The above sequence will produce a descent scheme to minimize the problem in M.

5.1 Tight Quadratic Majorization (TQM)

By applying MM technique, TQM method for noise estimation is derived [4]. In such case of optimization method to recover signal from PHN, MM technique is capable to mitigate high level of PHN by replacing the objective function in (25) with majorized function (surrogate function) based on Eq(18**)** **,**Eq(19),Eq(27),Eq(28) and Eq(30).Therefore, PHN estimation problem can be separated from channel estimation problem which belongs to Eq(20) and Eq(22).That leads to minimize the problem. Moreover, *g(x)* is formulated as quadratic function at u_0 after fulfill MM technique's criterion corresponding to $f(x)$.Thereby, by applying the Eq(32) [11] and Eq(33)[12, lemma1]:

$$f(x) \leq f(y) + 2 < A(x - y), Ay - b > + < D(x - y), x - y >$$

(32), where, $D \geq A^H A$

$$f(x) = x^H M x + 2Re(x^H (L - M) x_0) + x_0^H (M - L) x_0$$

(33), where, *L* and *M* are Hermitian matrix and equal at $x_0 \in C^1$.Therefore, $M - L \geq 0$.

C^1 is a complex unimodular, its radias is equal to unit 1. Therefore, the steps to derive TQM algorithm is summarized as follows [4]:

1) Following the model in Eq(32) with formulating matrix A as unitary orthogonal projection matrix where its diagonal values either *1* or *0*, $P = A(A^H A)^{-1} A^H$.This assumption is taken in order to simplify the objective function in Eq(25).

2) For simplicity, let us rewrite Eq(25) as: $\boldsymbol{V^{\sim} = Diag(y)^H V Diag(y)}$. Hence, the objective problem will be as Eq(34):

$$\boldsymbol{f(x) = \theta^{\sim} = u^H V^{\sim} u} \qquad (34)$$

3) Following the model in Eq(32) by substituting *f(y)* and *g(x)* according to Eq(33) [11,lema 1],we will have:

$$\boldsymbol{f(y) = 2Re\{u_0^H (V^{\sim} - \lambda I_{Nc}) u\}} \qquad (35)$$

$\boldsymbol{g(x) = 2\lambda \parallel u \parallel^2 - u_0^H V^{\sim} u_0}$.Based on Eq(32),we will get:

$$\boldsymbol{u^H V^{\sim} u} \leq \boldsymbol{2Re\{u_0^H (V^{\sim} - \lambda I_{Nc}) u\} + 2\lambda \parallel u \parallel^2 - u_0^H V^{\sim} u_0} \qquad (36)$$

The model of inequality in Eq(36) is obviously satisfy the condition of majorized problem $g(x)$ in Eq(28).

4) Following the step of number 1 where a projection matrix A is assumed whose greatest $eigenvalue\ \lambda_{max}$ is equal 1 and formulated with the objective function $V^{\sim}$.Therefore,

$\lambda_{max} V^{\sim} \leq \lambda I_{Nc}$, where $\lambda I_{Nc} = \parallel y \parallel_{\infty}^2$.

This constraint above satisfies the condition of MM technique based on Eq(28).

5) After substitute the feasible point u_0 with *u(t)* ($u_0$is any feasible point PHN starts with), $\parallel y \parallel_{\infty}^2$ instead of λI_{Nc} , replacing the terms $(V^{\sim} - \lambda I_{Nc})$ with $(\parallel y \parallel_{\infty}^2 \mathrm{I}_{Nc} - V^{\sim})$ by taking the negative sign outside since $\lambda_{max} V^{\sim} \leq \lambda I_{Nc}$ and derive the majorized function which situated at the second term of Eq(36), since $2\lambda \parallel u \parallel^2 - u_0^H V^{\sim} u_0$ is just constant ,their derivation is equal to *0*. Therefore, we will get:

$$\boldsymbol{-2Re\{(u(t))^H (\parallel y \parallel_{\infty}^2 \mathrm{I}_{Nc} - V^{\sim}) u\}}$$

6) Subject to $|u_n| = 1, where\ n = 0, \dots, Nc - 1$, $V^{\sim}\ is\ a\ magnitude\ of\ |y|^2\ and\ u = e^{-j\theta}$, therefore, after the substitution the closed form solution will be:

7) $$\boldsymbol{u(t+1) = \exp[jarg\left((\parallel y \parallel_{\infty}^2 \mathrm{I}_{Nc} - V^{\sim}) u(t)\right)]}$$

8) After substitute the value of $V^{\sim}$ and distribute the term $u(t)$ on both sides, we will get Eq(37):

$$u(t+1) = \exp[jarg\left((\parallel y \parallel_{\infty}^2 1 - |y|^2) \odot u(t) + Diag(y)^H F^H BF Diag(y) u(t)\right)]$$

Eq(37).

Which is called Loos Quadratic Majorization (LQM).By applying other assumption, LQM is shrunk into Tight Quadrature Method (TQM) [4] by supposing λ is equal *1* and cancelling the term of magnitude. Thus, TQM will include the angle of OFDM transformation signal ***y*** .After times of iterations the input PHN which is entangled with signal ***y*** and modeled as Wiener process will converge with the PHN estimation by TQM at receiver side. Consequently, TQM algorithm is extracted in Eq(38):

9) $$\boldsymbol{u(t+1) = \exp[jarg(Diag(y)^H F^H BF Diag(y) u(t)]} \qquad (38).$$

Algorithm for Phase Noise Estimation with TQM

1: Compute signal (y)

y=$\sqrt{Nc}\, diag\ (e^{j\theta}) F^H\ Diag\,(S) F^{\sim} h + V$.

2: Compute B =$S\ F^{\sim}\ (F^{\sim H} S^H S F^{\sim})^{-1}\ F^{\sim H} S^H$, set step t=0; initialize $u(t)$= $e^{j\theta t}$

3: repeat

4: u (t+1) =$exp^{[jarg(diag(y)^H F^H BF diag(y) u(t))]}$

5: t $\leftarrow t + 1$

6:$\theta = -\arg(u)$

7: $mse = mean\,(\theta - \theta)^2$

8: until convergence in iteration 9th.

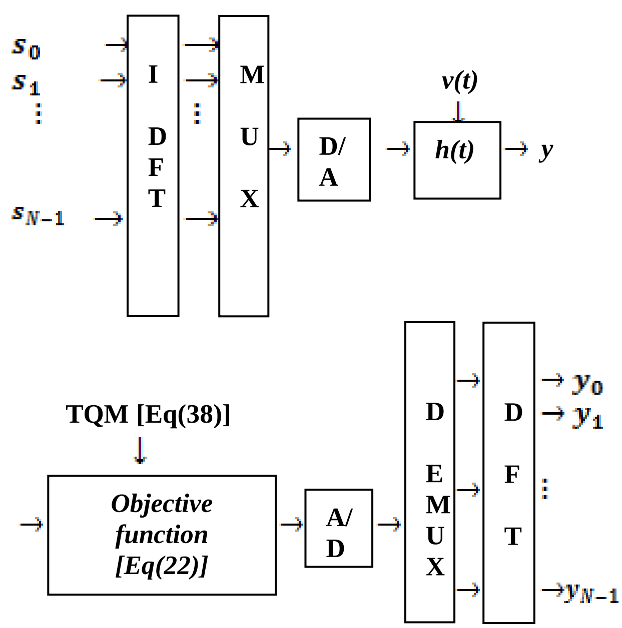

Fig.2 : OFDM Model with TQM and LSE objective function Eq(22)

Remark1: After the TQM algorithm is converged in ninth iterations, the phase nois estimation $\theta = -\arg(\hat{u})$ is entangled with reciprocal ambiguity phase rotation (θc). θc is behaved like CFO does not change with each subcarrier. Basically, θc is produced by objective problem Eq(22) for channel estimation and phase noise estimation, respectively.

Therefore, it could be eliminated by applying the rotation $\hat{u} = \hat{u}/\hat{u}_0$ or $\theta - \theta c 1_{Nc}$.

Remark2: mse is a mean squared error between the observed theta (θ) and estimated theta ($\hat{\theta}$).

6. SIMULATION RESULTS AND PERFORMANCE EVALUATION

In this part, we put our efforts to simulate signal y and TQM algorithm. Firstly, we utilize the Eq**(**16) to simulate matrix $F_{N\times N}$ and $F^{\sim}_{Nc\times L}$.Secondly, We plot the graphs which illustrate Wiener process phase noise $e^{j\theta}$ according to Eq(5), symbols S (data) belongs to Eq(16**)** as well in $F_{N\times N}$ and Rayleigh fading channel h according to Eq(9) .These parameters ($e^{j\theta}$, h) have the ability to affect the signal y and change its behavior. Then, parameter V (thermal noise) is formulated as Circularly Symmetric Complex Gaussian according to Eq(12).Therefore, in this section we discuss the values we apply for each parameter , starting from OFDM signal *y* regarding to Eq(17)till applying TQM algorithm for PHN estimation according to Eq(38). All simulations were run in MATLAB on a PC with a Intel ® core ™ i5-2410 M CPU @ 2.30 GHZ 2.20 GHZ and 10.0 GB RAM.

1) $F_{Nc} \times F_{Nc}$ is modeled as unitary matrix whose columns are orthonormal with subcarriers N_c=1024;as in Figure 3.

2) $F^{\sim}_{Nc\times L}$ is modeled as semi unitary matrix which columns *L*=10; as in Figure 4.

3) Static and slow-fading channel h is independently distributed with each tap *L* and exponentially degreasing power of rate=0.7,e.g., $h = e^{(-(0:L-0.7))}$ as in Figure 6.

4) S is defined as Circularly Symmetric Complex Gaussian with zero-mean μ and variance $\sigma^2 = 2I \sim CN(0,2I)$.

5) ***V*** is formulated as Circularly Symmetric Complex Gaussian with zero-mean μ and variance $\sigma^2 = 0.01 \sim CN(0,0.01)$ with amplitude according to SNRdB=15:5:55.

Remark3: *SNR* dB is computed as: $SNRlinear = 10^{snrdB/10}$ and $power\ of\ noise(p) = Es/SNRlinear$, where *Es* is signal's energy while p utilized as the amplitude of thermal noise (*V*)

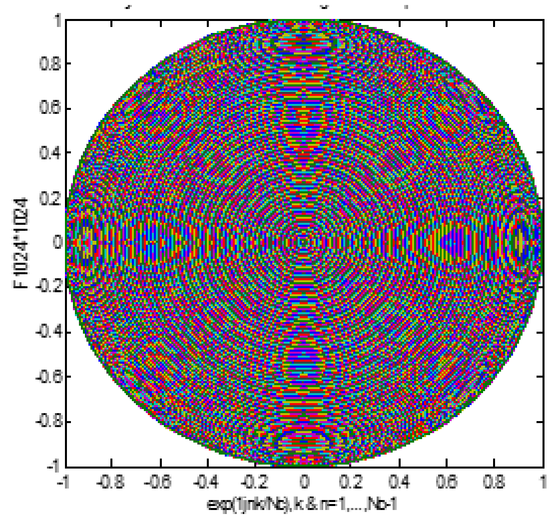

Fig.3: Unitary Matrix (Fourier Series)FNc*Nc

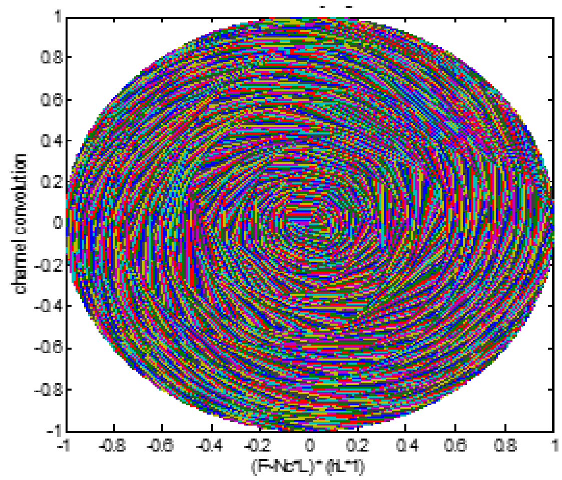

Fig.4: Semi Unitary Matrix F^~ Nc*L

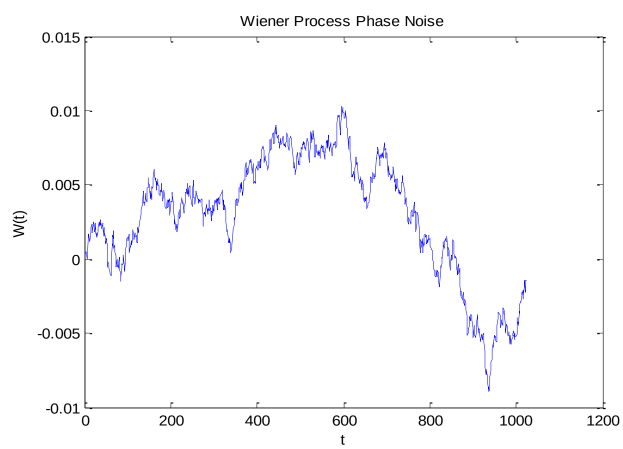

Fig.5: Phase Noise Modulation as Wiener Process

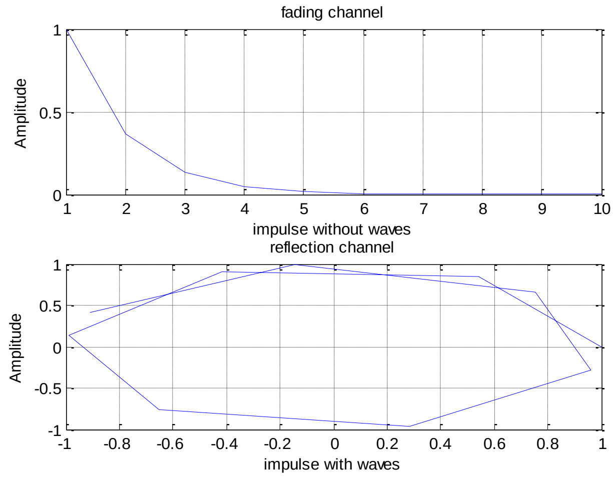

Fig.6: channel Impulse Response h10*1

6.1 Case Studies and Results Discussion

After many times of implementation TQM algorithm to estimate phase noise in signal *y* and taking the average for all executions (applying Monte Carlo simulation) with Nc=1024 and SNR range 15:5:55 and Phase Noise Δf=500 Hz, we observed the objective function in Eq(22) has severe decline from first iteration and stabilize at the same line till 10^{th} iteration, see Figure 7. Therefore, we obtained approximately the same results in [4] according to the convergence of TQM algorithm in 9^{th} iterations and the difference between our benchmark and the results in [4] according to Figure 7 is about 0.0770%. That it means our result of benchmark is very close to [4] and TQM algorithm is capable to estimate high PHN from OFDM signal ***y*** in high performance with high number of subcarriers.

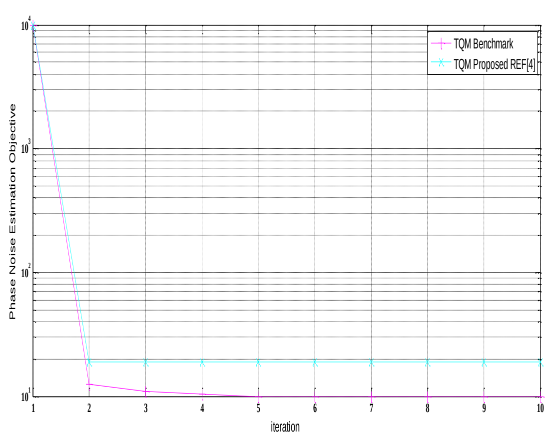

Fig.7: Objective problem according to TQM Convergence. Nc=1024.Δf 3dB=500Hz.

7. CONCLUSION

In this correspondence, we clarify in details the drawbacks of phase noise in terms of OFDM. Add to that, we put our effort to shed light on Majorization Minimization (MM) technique which is an optimization gradient model based on surrogate function with iterative schemes. Also, we get the benchmark of applying TQM algorithm for estimation phase noise which demonstrates by graphs. Literally, we explain time-domain OFDM signal's behavior **y** in existence of Wiener Process PHN before and after applying TQM. Consequently, we illustrate the numerical results based on our benchmark which interpret the degradation in MSE according to SNR from one side and the severe decline for objective problem based on 9^{th} iterations when TQM is applied.

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