

A Chaotic Flower Pollination Algorithm with Sinai Map for Solving Pressure Vessel Design Optimization Problem



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ABSTRACT

In this paper, a new manner of incorporating chaos search to nature-based Evolutionary Algorithms (EA)s is presented. The proposed approach makes benefit from generated chaotic random numbers during search. Also, it inherits the goodness of the generated sequences of two dimensional chaotic map so-called Sinai map. The two dimensional map is employed in a novel way to update the locations of population's members. The proposed approach is validated by tackling the well-known Pressure Vessel Design Optimization Problem.

Key words: Chaos; Constrained Optimization; Evolutionary Algorithms; Flower Pollination Algorithm; Pressure Vessel Problem.

1. INTRODUCTION

Engineering is the most field of science to apply optimization theories and methods. Nowadays, every branch need optimization models to assign optimal values or undertake optimal actions. In design optimization is the class of prior optimization models. Non-optimal design may costs a lot especially if the case is one time done. Design models in construction and civil engineering as well as communication engineering and more engineering branches are well established models and tackled by many scholars globally. Mechanical design is still not yet explored to the limit of other branches. Most of the mechanical design problems found in literature are nonlinear constrained problems which involve non differentiable functions in some cases which requires special treatment and modifications.

Engineering design optimization problems are the most challenging type of problems in literature and basically reflect the goodness of new introduced optimization algorithms [4]. This type of nonlinear engineering problems is tackled by many researchers. Many proposals refer to classical (traditional) methods while recently many other scholars are trying to tackle the aforementioned type by employing nature-inspired methods so called Evolutionary

Algorithms (EA)s. This category of problems includes Constraints which are very critical in attaining a solution in engineering design problems, since they are normally are very hard to satisfy [3].

For more than four decades scholars proposed many nature-inspired search algorithms capturing optimization schemes and methods from natural surroundings. Some of these algorithms are bio-inspired methods that mimic animals, ants, birds, and many others. The simulation of the bio-inspired methods extends to food hunting, mating, and co-evolution relationships. Dozens of proposed algorithms are introduced to solve many optimization models and the super power of computing encourages scholars to extend the bio-inspired methods to tackle new limits and real-life problems. Some of these algorithms mimic a social phenomenon as brain storming in humans or playing music like harmony search. Some of them are chemical-reaction based or inspired from physical rules and phenomena. Others are inspired from theories like big bang theory. Also, these algorithms adopted many versions to enable solving constrained optimization as well as multi-objective optimization proposals which requires special modifications in order to be valid to deal with multiple objectives.

All of these algorithms are different in design and mechanisms to reach the solution but they are common in convergence and the aspects of evolution of the good solutions. Simply generating random solutions and processing them along iterations to approximate the solution. Flower Pollination Algorithm (FPA) is relatively a different new nature-based algorithm inspired by the reproduction of flowering plants [8]. Many proposals are introduced to tackle constrained problems which are a big challenge in designing an algorithm. It is well known that unconstrained optimization is much easier than constrained problems due to limiting the search and needs to guide the stochastic search.

Chaos is a field of mathematics that imposes irregularity and defined rules in the same time in nonlinear systems. In recent years, a great interest of researchers in the relations between chaos and different types of EAs are investigated in

many studies. In This paper, an extended version of FPA is introduced. This proposed method is based on combining chaos to the original standalone algorithm to enhance its exploration and exploitation efficiencies. The proposed approach is adopted to solve aforementioned problem to stand over its capability. After this brief introduction; section 2 is made for Chaos. Section 3 is assigned the proposed approach; it shows the modifications done in order to incorporate chaotic maps to the original FPA. Section 4 is devoted to the pressure vessel problem; finally section 5 is for conclusion.

2. CHAOS

In life, there are many types of phenomena; random, deterministic, irregular, and chaotic. It is really hard to differentiate between these four categories because of their overlapped features. Investigating the difference between irregularity and chaos is really very tough task especially they mostly appear the same. The crucial point makes the chaotic phenomena is discovered as well as defined is chaos is the only category among them that imposes deterministic rules but hidden beyond apparent irregularity which makes it very hard to be discovered. Many of systems behavior can be reconsidered on the chaotic background. Many appearing systems are not as they appear but the randomness and irregularity that they export are not like shown. There is underlying rules governing this behavior but we cannot see. The deepness of the idea of chaos makes it appear like magic or superstitious but that for our lack of knowledge of chaotic systems. Many systems are really contained chaos and analyzing this aspect will lead to better deal and understanding of these systems. Many examples follow under this type of systems like orbit motion of planets, forecasting the population, and many others. The underlying rules of chaos are mathematically represented by functions which have parameters with values; if these values are set at certain levels the systems is controlled and we can benefit from its aspects and irregularity.

Chaos needs time and mathematically interpreted to understand and deal with it. The beauty about chaos is making benefit from both its irregularity appearance and hidden underlying rules. This feature motivates optimizers to incorporate chaos and try to combine it into optimization methods. It is early to say that chaotic dynamics field is mature as a research area; it is believed that still a lot of research gaps to be filled in order to understand chaos and so employ it to optimization.

Recently, generated chaotic sequences have been adopted to substitute random sequences in many applications [1]. There is a highly motivated research stream arouse last few

years is to assess the process of combing chaos to various categories of EAs including Swarm Intelligence techniques and other bio-inspired techniques and to Examine various chaotic maps in the proposed chaotic algorithms to stand over their suitability to various optimization tasks during search.

Mathematically, a map can be defined by a function with the same range and domain. The chaotic maps can be classified into one dimension or two dimension maps [2].

In this paper, two well-known maps are adopted for the proposed approach, one dimension map and the other is two dimensional type.

The **logistic map** is defined by [5]:

$$x_{n+1} = \mu x_n (1 - x_n) \quad (1)$$

Obviously, $x_n \in [0,1]$ under the conditions that the initial $x_0 \in [0,1]$, where n is the iteration number and $\mu = 4$.

The **Sinai map** is defined by the equations [6]:

$$\begin{aligned} X_{n+1} &= X_n + Y_n + a \cos(2\pi Y_n) \bmod(1) \\ Y_{n+1} &= X_n + 2Y_n \bmod \quad (2) \end{aligned}$$

when $a = 1$ it generates chaotic sequences in $(0, 1)$.

3. PROPOSED APPROACH

The number of plants types is almost 250000 different types in the globe approximately. Plants varied a lot over earth in their size, shape, fruits, stems, roots, and flower. Around 80% plants have flowers from the total number of plants. In world of plants the reproduction is done by flowers by carrying pollen from a flower to other cutting distances. The process is called pollination and it is very detailed complicated because of many species and types of flowers allocate a dozens of methods to transfer their pollens in order to reproduce. Most of these types are studied by plant scientists and summarized into clusters and categories. Some flowers used really strange methods but the most common features of this pollination process are studied by Shi and inspired him to propose the algorithm.

The process is associated with the transfer of pollens and so the reproduction is done. The pollinator is carrier of pollens to other flowers in various distances. The pollinators can be insects with their various species, different sizes, genders, and locations. Some pollination is undergone with specific type of insects and cannot be done with others. Animals are sometimes employed for the same task including different types of animals also like insects. The way of carrying pollens varies a lot according to different pollinators or different pollens carried. Some pollinators are carried over the back of the insect or animal, others carried

in mouth, via skin, and many other parts. Birds act also as good pollinators; some of them have co-evolved into a unique flower relationship. Experiments showed that bats and many other night birds are also working as pollinators for special flowers.

Not only insects and birds, but flowers' pollination also employs wind and diffusion. Wind transfer pollens of such plants like grass in a diverse. One of the beauties of this world and also magic is that pollinators pass the flowers of plants employ wind or diffusion and not carry their pollens and transfer to flowers of the first type to carry their pollens across distances. This is done because flowers employ wind needs to maximize reproduction multiples number the other method like in case of grass as mentioned. Of course this process is done for limited Nectar produced and flowers depend on wind for pollination is maintaining this process which called constancy.

Self pollination is done by pollen of same plant as mentioned, while cross pollination when pollen of different plant migrates for fertilization. Another classification is so important; Flower Pollination can be classified into two types: biotic pollination and Abiotic pollination.

3.1 Flower Pollination Algorithm

Based on the above features of flower pollination aforementioned above, the following four rules (steps) of the FPA are the corner stones of the whole issue [7]. It will be illustrated to see the mapping done between the real-life pollination's rules and the design aspects of the algorithm. As mentioned above the following rules (steps) are inspired by Shi in designing FPA as follows:

Rule 1 Cross-pollination

Pollen-carrying pollinators move in a way which obeys Levy flights as represented mathematically

$$x_i^{t+1} = x_i^t + \gamma L(\lambda)(x_i^t - g^*), \tag{3}$$

where x_i^t is the solution vector x_i at iteration t , and g^* is the current best solution. Here γ is a scaling factor, $L(\lambda)$ is the strength parameter, we draw $L > 0$ from a Levy distribution.

$$L \sim \frac{\lambda \Gamma(\lambda) \sin(\pi\lambda/2)}{\pi} \frac{1}{s^{1+\lambda}}, \quad (s \gg s_0 > 0). \tag{4}$$

Here, $\Gamma(\lambda)$ is the standard gamma function.

Rule 2 Local pollination

It is considered as abiotic and self-pollination.

Rule 3 Flower constancy

A reproduction probability that is proportional to the

similarity of two flowers involved.

Both Rule 2 and Rule 3 can be represented as following:

$$x_i^{t+1} = x_i^t + \epsilon(x_j^t - x_k^t), \tag{5}$$

where ϵ is extracted from a uniform distribution in [0,1].

Rule 4 Switching local pollination and global pollination

It can be controlled by a switch probability $p \in [0, 1]$.

Flower Pollination Algorithm (or simply Flower Algorithm)

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Objective min or max  $f(x)$ ,  $x = (x_1, x_2, \dots, x_d)$ 
Initialize a population of  $n$  flowers/pollen gametes with random solutions
Find the best solution  $g_s$  in the initial population
Define a switch probability  $p \in [0, 1]$ 
Define a stopping criterion (either a fixed number of generations/iterations or accuracy)
while ( $t < \text{MaxGeneration}$ )
    for  $i = 1 : n$  (all  $n$  flowers in the population)
        if  $\text{rand} < p$ 
            Draw a ( $d$ -dimensional) step vector  $L$  which obeys a Lévy distribution
            Global pollination via  $x_i^{t+1} = x_i^t + L(g_s - x_i^t)$ 
        else
            Draw  $\epsilon$  from a uniform distribution in  $[0, 1]$ 
            Do local pollination via  $x_i^{t+1} = x_i^t + \epsilon(x_j^t - x_k^t)$ 
        end if
        Evaluate new solutions
        If new solutions are better, update them in the population
    end for
    Find the current best solution  $g_s$ 
end while
Output the best solution found
    
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Figure 1: Pseudo code of the FPA [8].

3.2 Chaotic Flower Pollination Algorithm with Sinai map

The new method so-called Chaotic Flower Pollination Algorithm with Sinai map (CFPS) combines the two dimensional map to FPA. Some modifications are done in order to hybridize them to attain better exploration during search, and so speed up the standalone algorithm. Through the rest of this subsection, the modifications done will be illustrated.

- The initialization is done chaotically by running the logistic map many times to generate the population.
- In the proposed approach substitutes the levy distribution by the Sinai map according to the following manner; a in Eq. (2) will be set at 1, $\gamma \in [0,4]$ in Eq. (3).
- ϵ is drawn chaotically by logistic map in Eq. (5).
- The neighborhood of each member is updated chaotically by using Sinai map.

The modifications above are done to simplify the original algorithm and so reduce complexity and computations by

eliminating levy distribution and easily did its role with Sinai map.

The two dimensional map produces two random numbers, one will be compared to switching probability (p) and the other will be used to update the locations. By amplifying the flight transfer each iteration by setting $\gamma \in [0,4]$ will give better performance than levy flight in the standalone algorithm.

4. PRESSURE VESSEL DESIGN OPTIMIZATION PROBLEM

The problem includes a compressed air storage tank with a working pressure of 3,000 psi and a minimum volume of 750 ft³. The pressure vessel problem is to minimize the total cost of material, forming and welding of a cylindrical vessel as shown in Figure 2. The problem has a nonlinear objective function, a nonlinear and three linear inequality constraints. Since the problem has two discrete variables and two continuous variables, it is a mixed discrete-continuous constrained optimization problem [3] as illustrated in the following optimization model:

Minimize:

$$f(\vec{x}) = 0.6224x_1x_3x_4 + 1.7781x_2x_3^2 + 3.1661x_1^2x_4 + 19.84x_1^2x_3$$

subject to:

$$\begin{aligned} g_1(\vec{x}) &= -x_1 + 0.0193x_3 \leq 0 \\ g_2(\vec{x}) &= -x_2 + 0.00954x_3 \leq 0 \\ g_3(\vec{x}) &= -\pi x_3^2 x_4^2 - \frac{4}{3}\pi x_3^3 + 1,296,000 \leq 0 \\ g_4(\vec{x}) &= x_4 - 240 \leq 0 \end{aligned} \quad (6)$$

with $1 \times 0.0625 \leq x_1$, $x_2 \leq 99 \times 0.0625$, $10 \leq x_3$, and $x_4 \leq 200$

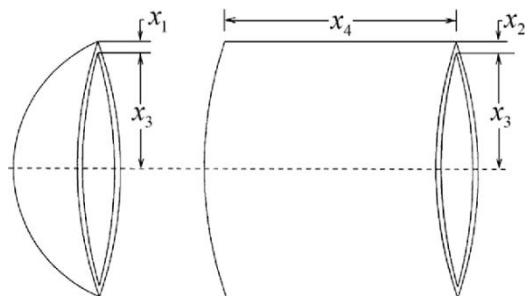


Figure 2: Pressure vessel [3].

The parameters of the proposed approach are set as following; $p = 0.8$, $\gamma \in [0,4]$, $n = 40$, and number of iterations will be set at 100 to solve the given problem. The best solution achieved was $x^* = (0.8125, 0.4375, 42.09845, 176.6366)$ where $f(x^*) = 6,059.71434$.

5. CONCLUSION

The proposed approach successfully incorporated two dimensional chaotic map to FPA which enhance the performance of the original algorithm. The combination of chaos search is done in a new manner to modify many aspects of the original algorithm not only generating set of random numbers as common in literature.

The main research points stemmed out from this paper is amplifying the usage of many dimensions maps in evolutionary algorithms and gets benefit from the geometric aspects of chaos search.

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