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Determination of the lower border of Cramer-Rao for evaluation of the carrier frequency of the radio technical communication channel signal

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ABSTRACT

The problems of signal transmission in modern satellite systems are determined by certain features, both the construction of the system itself, and the problems of signal processing and signal processing. A characteristic feature of the satellite communication channel is the significant uncertainty of the signal received on the frequency (frequency uncertainty of the signal.) Therefore, for the demodulators of satellite modems, the problem of synchronization on the frequency of the carrier oscillation is the most significant. Based on this, when developing algorithms for coherent signal demodulation, the solution to the synchronization problem is crucial. The process of estimating the carrier frequency of a signal received in satellite systems is considered. Quantitative characterization of the minimum boundary variance of satellite carrier frequency estimation is substantiated. Functional dependencies are proposed to determine the Cramer-Rao lower bound, which is proposed as a quantitative characteristic of the minimum boundary variance of the carrier frequency estimation of a radio communication channel. It is established that the Cramer-Rao lower bound, adopted for determining the estimate of the minimum limiting variance of the carrier frequency estimation, is functionally dependent on the energy of the single signal pulse, the interval of information pulses of the complex envelope received, and the interval at which the estimate is made. The dependencies obtained make it possible to set the Cramer-Rao lower bound and the minimum variance of the carrier frequency estimation by its value, provided that other signal parameters are known. Under real conditions, the minimum variance of carrying out an MP estimation of the carrier frequency with the uncertainty of all satellite signal parameters may differ significantly from the minimum variance obtained based on the application of the proposed Cramer-Rao lower bound, which determines the further direction of prospective studies. Prospective research, development and creation of algorithms and techniques aimed at the rules of maximum likelihood of a carrier frequency with a minimum limiting dispersion under the conditions of uncertainty of all signal parameters should be aimed at maximally approximating the indicated dispersion of the estimate of the carrier frequency of the real signal to the lower Cramer-Rao boundary determined for carrier frequency under conditions of certainty of other signal parameters.

Key words: received signal, carrier frequency estimation, minimum limiting variance of carrier frequency estimation, Cramer-Rao lower bound.

1. INTRODUCTION

Problems of signal transmission in modern satellite systems are determined by certain features of both the construction of the system itself and the problems of signal reception and transmission. The existing energy of the satellite communication channel causes an urgent need for coherent signal processing and the use of powerful noise-tolerant coding. The noise-tolerant coding system is an integral part of the satellite modem. The vast majority of systems use high-precision encoding with Viterbo decoding and cascading codes. Widespread are turbo codes and codes with low density of parity checks [1, 2, 3, 4].

Although in recent years, amplitude-phase modulation methods are often used in satellite communication systems, the energy of the satellite channel, as a rule, determines the use of phase modulation [1, 3]. The task of estimating the carrier frequency of the signal, estimating the carrier frequency of the signal is reduced to the problem of estimating the frequency of the maximum in the spectrum of a fragment of a sinusoidal signal against the background of additive Gaussian noise.

1.1 Problem analysis

A characteristic feature of the satellite communication channel is a significant uncertainty of the signal received on the frequency (frequency uncertainty of the signal.) Therefore, for demodulators of satellite modems, the most significant problem is the synchronization of the carrier

frequency. Based on this, in the development of algorithms for coherent signal demodulation, the solution of the synchronization problem is of decisive importance.
The complex signal envelope contains unknown values, namely the frequency, phase, signal delay and transmitted information sequence (ν ,φ ,τ ,d). That is, the task of synchronization is actually reduced to estimating the true parameters of the received signal - ν ,φ ,τ , d , knowledge of the parameters of which is necessary for demodulation of the signal $z(t)$.
The best results can be obtained by a joint assessment of unknown signal parameters. However, in practice, it is not possible to implement such an estimate in a channel with low energy and high frequency uncertainty of the received signal. Therefore, the estimation of the carrier frequency offset of the received signal relative to the nominal value is performed before other synchronization procedures are activated, namely: phase synchronization and clock synchronization [1, 2].
The complexity of the task of estimating the carrier frequency in the satellite channel is determined by the need to process signals in continuous and batch modes.

2. MAIN MATERIAL

2.1 Directions for improving the accuracy of estimating the carrier part of the signal

When solving the problem of estimating the carrier frequency of a modulated signal, we assume that on the observation interval, the duration of K information symbols, a complex envelope of the received signal is given, which is characterized by unknown parameters $\{d , \nu , \varphi , \tau\}$ [1,3]. The task is to estimate one element of the vector $\{d$, ν , φ , $\tau\}$, namely, the parameter ν – the carrier frequency.
To solve the problem of estimating the carrier frequency of the FM signal in conditions of uncertainty of information about the initial phase of the signal (), the value of its delay (τ) and the transmitted information sequence (d), it is advisable to apply the rule of maximum likelihood. It is known that the use of the rule of maximum likelihood to estimate the carrier frequency (MP-estimate) provides asymptotically effective and asymptotically unbiased estimates [5, 6, 7].
In the presence of information about the parameters $\{d$, φ , $\tau\}$ MP-estimation of the carrier frequency can provide the minimum limiting variance, which will be determined by the lower Cramer-Rao boundary [4,5].
Currently, a number of methods are known for estimating the frequency of a sinusoidal signal, which is based on recurrent procedures. Such as Pisarenko method, MUSIK method, auto regression method [6, 7].
However, the variance of estimates that provide these methods, as shown in [8,9], significantly loses the variance of estimates determined by the lower Cramer – Rao boundary.
Thus, the development of a method of MP-estimation of the carrier frequency with a minimum limiting variance, in order to obtain its quantitative values, requires prior development of functional dependences designed to determine the lower Cramer-Rao boundary, which is an urgent scientific problem.

2.2 Analysis of previous works

The question of determining the lower Cramer-Rao codon as which the minimum limiting variance of the carrier frequency estimate according to the rule of maximum similarity will be adopted is devoted to a number of works.
The authors of [10] proposed a weighted Bayesian Cramer Rao-boundary for the joint determination of the synchronization time and carrier frequency shift, which takes into account the previous distribution of estimation parameters and is the exact lower limit for all considered signal-to-noise ratios. The issue of determining the minimum limiting variance of the carrier frequency estimate was not considered in this paper.
In [11, 12, 13], the Cramer-Rao boundary for the variance of the combined Doppler carrier frequency shift and signal delay with an arbitrary signal spectrum is presented. The presented results are proposed to be used for cases when the width of the signal spectrum does not allow the use of other estimation methods within the narrowband model. Direct estimation of the carrier frequency and determination of the minimum limiting variance for it is not considered in the work.
The authors of [14] propose a lower Cramer-Rao boundary for estimating the frequency of a coherent sequence of pulses passively intercepted on a moving antenna. The author proposes to make such an assessment in order to determine the location of the transmitting radar. The direct carrier frequency of these pulses and its evaluation were not considered in the work, although the work is indicative from the point of view of the application of the lower Cramer-Rao limit for the MP of the carrier frequency estimation.
The work [15, 16] is devoted to solving the problem of estimating the signal delay in time, taking into account the change in the shape of the signal pulse during navigation in the premises. The Cramer-Rao lower bound criterion is used to estimate the delay time. The work is quite indicative in terms of signal evaluation, but directly the assessment of the carrier frequency is not considered in it.
In [17], the authors propose an algorithm based on conditional maximization of expectations for joint estimation of the data transmission channel in multiplexing systems with orthogonal frequency distribution channels, phase noise of generators and Doppler carrier frequency offset. For this assessment, the paper proposes a lower hybrid Cramer-Rao boundary. The issues of calculating the lower Cramer-Rao boundary for the carrier frequency and determining the

minimum limiting variance for it are not considered in the paper.
The issue of determining the minimum marginal variance in assessing the quality of the receiving radio system was considered in [18, 19]. The papers directly propose expressions for estimating the minimum noise dispersion (according to the Alan parameter) of both receiving systems and internal synchronization systems, and substantiate the conclusions that such estimation may affect system performance at the level of carrier frequency shift estimation and channel performance estimation. The minimum variance of the frequency estimate (by the Alan parameter) is derived in terms of the spectral noise density at frequencies close to the signal frequency. Determination of the value of the lower Cramer-Rao boundary for the proposed minimum limiting variance was not performed.

2.3 Statement of the research problem

In solving the specific scientific problem of this article, it should be borne in mind that the specified lower Cramer-Rao boundary and the associated minimum variance of the estimate will be used in methods for estimating the carrier frequency of the received phase-modulated signal.
Coherence of signal processing in the demodulator of satellite communication systems causes extremely strict requirements for the accuracy of estimating the phase of the carrier oscillation in the corresponding loop of the demodulator with phase-automatic frequency tuning (PAFT demodulator) [1, 3, 4]. Because of this, the band bandwidth B_C should not normally exceed thousandths of the clock frequency of the received signal - about 10^{-3} 1/T [1, 6, 8]. And for reliable entry of the auto tuning system into synchronization, the value of the variance of the carrier frequency estimate of the FM signal δ_C^2 should not exceed B_C^2. That is, the value δ_C^2 should not be greater than 10^{-5} 1/T^2 [1]. In the following, we assume that for the variance of the FM signal carrier frequency estimation at low signal / noise ratios per bit of information (from 0 to 12 dB) the following requirement must be met $\delta_C^2 T^2 \le 5 * 10^{-6}$ [1, 6, 8]:

$$\delta_C^2 T^2 \le 5 * 10^{-6}. \quad (1)$$

2.4 Determination of the lower limit of the variance of the evaluation of the carrier part of the signal

It is known that the complex envelope of the received signal is given at the interval of observation T_0 by the vector $\alpha = \{\Omega, \varphi, \tau, d\}$ [1, 3]. Where $\Omega = 2\pi\nu$.
Determine the lower bound of the variance of the estimate of one of the elements of the vector $\alpha = \{\Omega, \varphi, \tau, d\}$, namely Ω.

If the signal is characterized by a set of parameters $\beta = \{\beta_0, \beta_{01}, \ldots, \beta_M\}$ and $\bar{\beta}_л(z)$ is an unbiased estimate of some parameter β_k, then the lower limit of the variance of the estimate β_k is determined by the element of the matrix J^{-1}, the inverse of the Fisher information matrix [5, 20].

$$\alpha_k^2 = var\left(\beta_k(z) - \beta_k\right) \ge J^{kk}$$

Where J^{kk} is the element of the matrix J^{-1}.
The elements of the matrix J are defined as follows [21]:

$$J_{ik} = -E_w\left[\frac{\partial^2\left[ln\, p(z \uparrow \beta)\right]}{\partial\beta_i \partial\beta_k}\right]$$

Here E_W means statistical averaging with respect to noise , and the $p(z \uparrow \beta)$ common probability density function of the vector z for a given $\beta = \{\beta_0, \beta_{01}, \ldots, \beta_M\}$.
Given that the noise is uncorrelated Gaussian with zero mean and variance δ^2, we write a common probability density function [20]:

$$p\left(z \uparrow \tilde{\tilde{a}}\right) = \left(\frac{1}{\sigma^2 2\pi}\right)^N exp\left[-\frac{1}{2\sigma^2}\sum_n\left(\left(I_n - i_n\right)^2 + \left(Q_n - q_n\right)^2\right)\right]$$

Where:

$$\tilde{\tilde{a}} = \left\{\tilde{\tilde{\Omega}}, \tilde{\tilde{\varphi}}, \tilde{\tilde{\tau}}, \tilde{\tilde{d}},\right\}$$

$$I_n = R_e\left(z(t_n)\right); \quad Q_n = I_m\left(z(t_n)\right);$$

$\tilde{\tilde{r}}(t_n) = exp\left(j\left(\tilde{\tilde{\Omega}} t_n + \tilde{\tilde{\varphi}}\right)\right)\sum_k \tilde{\tilde{d}}_k h\left(t_n - kt - \tilde{\tilde{\tau}}\right)$ – complex envelope of the reference signal;

$$i_n = R_e\left(\tilde{\tilde{r}}(t_n)\right); \quad q_n = I_m\left(\tilde{\tilde{r}}(t_n)\right);$$

$$t_n = nT_s, n = -\frac{N-1}{2}, \ldots, 0, 1, \ldots, \frac{N-1}{2};$$

T_s – the interval of the samples of the complex envelope of the received signal;

N – - interval of observations, expressed in the number of intervals T_s.

When forming the elements of the Fisher matrix for the parameter $\left\{\tilde{\tilde{\Omega}},\tilde{\tilde{\varphi}},\tilde{\tilde{\tau}}\right\}$ is usually proposed the following approach, which is defined in a number of works [5,6,7,20,21]. It is shown that for large *K (K >> 1)* the components of the matrix can be represented as:

$$F = E_d\left\{\begin{pmatrix} J_{\Omega\Omega} & J_{\Omega\varphi} & J_{\Omega r} \\ J_{\varphi\Omega} & J_{\varphi\varphi} & J_{\varphi r} \\ J_{\tau\Omega} & J_{\tau\varphi} & J_{rr}\end{pmatrix}\right\} \qquad (3)$$

Where $E_d\{*\}$ is the averaging of the elements of the Fisher matrix in the information sequence;

J_{jk} – matrix element for parameters $\left\{\tilde{\tilde{\Omega}},\tilde{\tilde{\varphi}},\tilde{\tilde{\tau}}\right\}$.

It can be shown that up to a constant value that does not depend on the parameters $\left\{\tilde{\tilde{\Omega}},\tilde{\tilde{\varphi}},\tilde{\tilde{\tau}}\right\}$, the logarithm of the likelihood function (2) can be represented as follows [20]:

$$L = \frac{1}{\sigma^2}\sum_n\left(I_n i_n + Q_n q_n - \frac{1}{2}\left(i_n^2 + q_n^2\right)\right)$$

From here:

$$\frac{\partial^2 L}{\partial\tilde{\tilde{a}}_i\partial\tilde{\tilde{a}}_k} = \frac{1}{\sigma^2}\sum_n\left(\begin{array}{l}(I_n - i_n)\dfrac{\partial^2 i_n}{\partial\tilde{\tilde{a}}_i\partial\tilde{\tilde{a}}_k} + (Q_n - q_n)* \\ \left(\dfrac{\partial^2 q_n}{\partial\tilde{\tilde{a}}_i\partial\tilde{\tilde{a}}_k} - \dfrac{\partial i_n}{\partial\tilde{\tilde{a}}_i}\dfrac{\partial i_n}{\partial\tilde{\tilde{a}}_k} - \dfrac{\partial q_n}{\partial\tilde{\tilde{a}}_i}\dfrac{\partial q_n}{\partial\tilde{\tilde{a}}_k}\right)\end{array}\right)$$

Taking into account that $E_w[(I_n - i_n)] = E_w[(Q_n - q_n)] = 0$ we get:

$$J_{ik} = \frac{1}{\sigma^2}\sum_n\left(\frac{\partial i_n}{\partial\tilde{\tilde{a}}_i}\frac{\partial i_n}{\partial\tilde{\tilde{a}}_k} + \frac{\partial q_n}{\partial\tilde{\tilde{a}}_i}\frac{\partial q_n}{\partial\tilde{\tilde{a}}_k}\right) \qquad (4)$$

Where: $\tilde{\tilde{a}}_i, \tilde{\tilde{a}}_k$ – vector element $\tilde{\tilde{a}}$ at i, k = 0,1,2.

Note that, $\tilde{\tilde{r}}(t_n) = i_n + jq_n$ i $\tilde{\tilde{r}}^*(t_n) = i_n - jq_n$..

Let's make a number of transformations $\dfrac{\partial\tilde{\tilde{r}}(t_n)}{\partial\tilde{\tilde{a}}_i} = \dfrac{\partial i_n}{\partial\tilde{\tilde{a}}_i} + j\dfrac{\partial q_n}{\partial\tilde{\tilde{a}}_i}$ and $\dfrac{\partial\tilde{\tilde{r}}^*(t_n)}{\partial\tilde{\tilde{a}}_i} = \dfrac{\partial i_n}{\partial\tilde{\tilde{a}}_k} - j\dfrac{\partial q_n}{\partial\tilde{\tilde{a}}_k}$

From here:

$$\frac{\partial\tilde{\tilde{r}}(t_n)}{\partial\tilde{\tilde{a}}_i}\frac{\partial\tilde{\tilde{r}}^*(t_n)}{\partial\tilde{\tilde{k}}} = \left(\frac{\partial i_n}{\partial\tilde{\tilde{a}}_i}\frac{\partial i_n}{\partial\tilde{\tilde{a}}_k} + \frac{\partial q_n}{\partial\tilde{\tilde{a}}_i}\frac{\partial q_n}{\partial\tilde{\tilde{a}}_k}\right) + \left(\frac{\partial q_n}{\partial\tilde{\tilde{a}}_i}\frac{\partial i_n}{\partial\tilde{\tilde{a}}_k} - \frac{\partial i_n}{\partial\tilde{\tilde{a}}_i}\frac{\partial q_n}{\partial\tilde{\tilde{a}}_k}\right)$$

That is, expression (4) can be represented as

$$J_{jk} = \frac{1}{\sigma^2}\sum_n R_e\left(\frac{d\tilde{\tilde{r}}(t_n)}{\partial\tilde{\tilde{a}}_i} * \frac{d\tilde{\tilde{r}}^*(t_n)}{\partial\tilde{\tilde{a}}_k}\right) \qquad (5)$$

To determine the lower limit of the variance of the parameter estimation Ω , we calculate the component of the matrix (3) taking into account the averaging over the information sequence $-F_{ik} = E_d\{J_{ik}\}$ for i, k = 0,1,2.

Let's define $F_{\Omega\tau}$.

For $\dfrac{d\tilde{\tilde{r}}^*(t_n)}{\partial\tilde{\tilde{\tau}}}$ and $\dfrac{d\tilde{\tilde{r}}(t_n)}{\partial\tilde{\tilde{\Omega}}}$ you can write:

$$\frac{d\tilde{\tilde{r}}^*(t_n)}{\partial\tilde{\tilde{\tau}}} = -exp\left(-j\left(\tilde{\tilde{\Omega}}t_n + \tilde{\tilde{\varphi}}\right)\right)\sum_i\tilde{\tilde{d}}_i^*\frac{\partial h\left(t_n - iT - \tilde{\tilde{\tau}}\right)}{\partial\tilde{\tilde{\tau}}}$$

$$\frac{d\tilde{\tilde{r}}(t_n)}{\partial\tilde{\tilde{\Omega}}} = jt_n exp\left(j\left(\tilde{\tilde{\Omega}}t_n + \tilde{\tilde{\varphi}}\right)\right)\sum_i\tilde{\tilde{d}}_i h\left(t_n - iT - \tilde{\tilde{\tau}}\right)$$

From here

$$\frac{d\tilde{\tilde{r}}(t_n)}{\partial\tilde{\tilde{\Omega}}} * \frac{d\tilde{\tilde{r}}^*(t_n)}{\partial\tilde{\tilde{\tau}}} =$$

$$-jt_n\sum_l\sum_m\tilde{\tilde{d}}_m\tilde{\tilde{d}}_l^*\frac{\partial h\left(t_n - iT - \tilde{\tilde{\tau}}\right)}{\partial\tilde{\tilde{\tau}}}h\left(t_n - iT - \tilde{\tilde{\tau}}\right)$$

We will average the obtained expression in the information sequence.

Usually for $\overset{\approx}{d}_i$ the following model is offered [5,16, 20,22]:

$$E\left\{\overset{\approx}{d}_i \overset{\approx}{d}_k^*\right\} = \begin{cases} D_s, i = k \\ 0, i \neq k \end{cases} \quad (6)$$

Where D_s – some positive value;

$\overset{\approx}{d}_i$ – independent random variables with zero mean and correlation functions.

By condition (6) we obtain:

$$E_d\left\{\frac{\partial \overset{\approx}{r}(t_n)}{\partial \overset{\approx}{\Omega}} * \frac{\partial \overset{\approx}{r}^*(t_n)}{\partial \overset{\approx}{\tau}}\right\} =$$

$$-jt_n D_s \sum_m \frac{\partial h\left(t_n - mT - \overset{\approx}{\tau}\right)}{\partial \overset{\approx}{\tau}} h\left(t_n - mT - \overset{\approx}{\tau}\right)$$

The impulse response of the Nyquist filter is a valid function. So the right part in this expression is an imaginary function, because of this:

$$Re\left(E_d\left\{\frac{\partial \overset{\approx}{r}(t_n)}{\partial \overset{\approx}{\Omega}} * \frac{\partial \overset{\approx}{r}^*(t_n)}{\partial \overset{\approx}{\tau}}\right\}\right) = 0$$

and $F_{\Omega r} = 0$

The same can be shown $F_{\varphi r} = 0$. Analysis of expression (4) shows that the Fisher matrix is symmetric: $F_{ik} = F_{ki}$. That is, the matrix (3) takes the following form:

$$F = \begin{pmatrix} F_{\Omega\Omega} & F_{\Omega\varphi} & 0 \\ F_{\varphi\Omega} & F_{\varphi\varphi} & 0 \\ 0 & 0 & F_{\tau\tau} \end{pmatrix}$$

Hence, the lower limit of the variance of the estimate Ω is determined as follows:

$$var\left(\overset{\approx}{\Omega} - \Omega\right) \geq \frac{1}{F_{\Omega\Omega} - F_{\Omega\varphi}^2 / F_{\varphi\varphi}} \quad (7)$$

Thus, to find the lower limit of the variance of the estimates of the value Ω should be determined $F_{\varphi\varphi}$, $F_{\Omega\Omega}$ i $F_{\Omega\varphi}$.

Let's define $F_{\varphi\varphi}$

For $\frac{d\overset{\approx}{r}^*(t_n)}{\partial \overset{\approx}{\varphi}}$ and $\frac{d\overset{\approx}{r}(t_n)}{\partial \overset{\approx}{\varphi}}$ rightly

$$\frac{d\overset{\approx}{r}(t_n)}{\partial \overset{\approx}{\varphi}} = j\,exp\left(j\left(\overset{\approx}{\Omega} t_n + \overset{\approx}{\varphi}\right)\right)\sum_i \overset{\approx}{d}_i h\left(t_n - iT - \overset{\approx}{\tau}\right)$$

$$\frac{d\overset{\approx}{r}^*(t_n)}{\partial \overset{\approx}{\varphi}} = -j\,exp\left(-j\left(\overset{\approx}{\Omega} t_n + \overset{\approx}{\varphi}\right)\right)\sum_i \overset{\approx}{d}_i h\left(t_n - iT - \overset{\approx}{\tau}\right)$$

From here

$$\frac{d\overset{\approx}{r}(t_n)}{\partial \overset{\approx}{\varphi}} * \frac{d\overset{\approx}{r}^*(t_n)}{\partial \overset{\approx}{\varphi}} = \sum_l \sum_m \overset{\approx}{d}_m \overset{\approx}{d}_l^* h\left(t_n - lT - \overset{\approx}{\tau}\right) h\left(t_n - mT - \overset{\approx}{\tau}\right)$$

Therefore, based on (1) and (6), the expression for this component of the matrix (3) can be represented as:

$$F_{\varphi\phi} = \frac{D_s}{\sigma^2}\sum_n\left(\sum_m h^2\left(t_n - mT - \overset{\approx}{\tau}\right)\right)$$

Similarly

$$\frac{d\overset{\approx}{r}(t_n)}{\partial \overset{\approx}{\Omega}} = jt_n exp\left(j\left(\overset{\approx}{\Omega} t_n + \overset{\approx}{\varphi}\right)\right)\sum_i \overset{\approx}{d}_i h\left(t_n - iT - \overset{\approx}{\tau}\right)$$

$$\frac{d\overset{\approx}{r}^*(t_n)}{\partial \overset{\approx}{\Omega}} = -jt_n exp\left(-j\left(\overset{\approx}{\Omega} t_n + \overset{\approx}{\varphi}\right)\right)\sum_i \overset{\approx}{d}_i h\left(t_n - iT - \overset{\approx}{\tau}\right)$$

From here

$$\frac{d\overset{\approx}{r}(t_n)}{\partial \overset{\approx}{\Omega}} * \frac{d\overset{\approx}{r}^*(t_n)}{\partial \overset{\approx}{\Omega}} =$$

$$t_n^2 \sum_l \sum_m \overset{\approx}{d}_m \overset{\approx}{d}_l^* h\left(t_n - mT - \overset{\approx}{\tau}\right) h\left(t_n - lT - \overset{\approx}{\tau}\right)$$

Based on (5) $F_{\Omega\Omega} = \frac{D_s}{\sigma^2}\sum_n\left(t_n^2 \sum_m h^2\left(t_n - mT - \overset{\approx}{\tau}\right)\right)$ And for $\frac{\partial \overset{\approx}{r}(t_n)}{\partial \overset{\approx}{\Omega}}$ and $\frac{\partial \overset{\approx}{r}^*(t_n)}{\partial \overset{\approx}{\varphi}}$ we will write down

$$\frac{d\overset{\approx}{r}(t_n)}{\partial\overset{\approx}{\Omega}} = jt_n exp\left(j\left(\overset{\approx}{\Omega}t_n + \overset{\approx}{\varphi}\right)\right)\sum_i \overset{\approx}{d}_i h\left(t_n - iT - \overset{\approx}{\tau}\right)$$

$$\frac{d\overset{\approx}{r}{}^*(t_n)}{\partial\overset{\approx}{\varphi}} = -jt_n exp\left(-j\left(\overset{\approx}{\Omega}t_n + \overset{\approx}{\varphi}\right)\right)\sum_i \overset{\approx}{d}{}^*_i h\left(t_n - iT - \overset{\approx}{\tau}\right)$$

So: $F_{\Omega\varphi} = \frac{D_s}{\sigma^2}\sum_n\left(t_n\sum_m h^2\left(t_n - mT - \overset{\approx}{\tau}\right)\right)$

We can show that $N \to \infty (T_S \to 0)$ the expressions for the considered components of the Fisher matrix are rewritten in the form [22]:

$$F_{\varphi\varphi} = \frac{2E_s}{N_0}\int_{-\infty}^{+\infty}\sum_m h^2\left(t - mT - \overset{\approx}{\tau}\right)dt \qquad (8)$$

$$F_{\Omega\varphi} = \frac{2E_s}{N_0}\int_{-\infty}^{+\infty} t\sum_m h^2\left(t - mT - \overset{\approx}{\tau}\right)dt \qquad (9)$$

$$F_{\Omega\Omega} = \frac{2E_s}{N_0}\int_{-\infty}^{+\infty} t^2\sum_m h^2\left(t - mT - \overset{\approx}{\tau}\right)dt \qquad (10)$$

Where E_S is the energy of the elementary signal reference. Changing the order of summation and integration, rewrite expression (8) in the form

$$F_{\varphi\varphi} = \frac{2E_s}{N_0}\sum_m\int_{-\infty}^{+\infty} h^2\left(t - mT - \overset{\approx}{\tau}\right)dt$$

By condition (1) $\int_{-\infty}^{+\infty} h^2\left(t - mT - \overset{\approx}{\tau}\right)dt = 1$. That is $F_{\varphi\varphi}$ for can be written

$$F_{\varphi\varphi} = \frac{2E_S}{N_0}K \qquad (11)$$

Similarly for expression (9) we write:

$$F_{\Omega\varphi} = \frac{2E_s}{N_0}\sum_m\int_{-\infty}^{+\infty} th^2\left(t - mT - \overset{\approx}{\tau}\right)dt .$$

Carrying out the corresponding order of determination $F_{\varphi\varphi}$ of the transformation $F_{\Omega\varphi}$ with respect to and taking into account that the function $h^2(*)$ is even, K is not clear, the value of m varies from $-\frac{K-1}{2}$ to $\frac{K-1}{2}$ for $F_{\Omega\varphi}$ we obtain:

$$F_{\Omega\varphi} = \frac{2E_S}{N_0}K\overset{\approx}{\tau} \qquad (12)$$

In the same way, changing the order of summation and integration, we rewrite expression (10) in the form: $F_{\Omega\Omega} = \frac{2E_s}{N_0}\sum_m\int_{-\infty}^{+\infty} t^2h^2\left(t - mT - \overset{\approx}{\tau}\right)dt$.

Carrying out the appropriate order of definition $F_{\varphi\varphi}$ and $F_{\Omega\varphi}$ transformation relative $F_{\Omega\Omega}$. But using the provisions of Parseval's theorem and the theorem on the differentiation of the Fourier transform to determine J_2 we obtain:

$$F_{\Omega\Omega} \approx \frac{2E_S}{N_0}T^2\frac{K^3}{12} \qquad (13)$$

Thus, substituting (11), (12) and (13) in (7) we obtain the expression of the minimum variance that was sought:

$$var\left(\overset{\approx}{\Omega} - \Omega\right) \ge \frac{N_0}{2E_S}\frac{1}{\left(\frac{K^3}{12} - K\left(\frac{\overset{\approx}{\tau}}{T}\right)^2\right)T^2} \qquad (14)$$

It is obvious that at long observation intervals ($K \rangle\rangle 1$) the normalized Cramer-Rao boundary of the carrier frequency estimation of the phase-modulated signal can be represented as:

$$CRLB(\nu)*T^2 = \frac{1}{2\pi^2}\frac{1}{K^2}\frac{1}{E_S/N_0} \qquad (15)$$

Where $\nu = \frac{\Omega}{2\pi}$.

The Value $CRLB(\nu)*T^2$ (Cramer-Rao lower bound) is called the Cramer-Rao lower bound. The value of which is determined by expression (15) and can be taken to determine the minimum limiting variance of the carrier frequency estimate.

Thus, the lower Cramer-Rao boundary adopted to determine the minimum estimate of the limiting variance of the carrier frequency estimate functionally depends on the energy of a single signal pulse (E_S), the interval of information pulses of the complex envelope of the received signal (T_s) and the interval on which the estimate (N_0).

The proposed expression allows us to determine the lower boundary of Cramer-Rao, provided that other signal parameters are known. That is, the assessment is carried out in ideal conditions.

Typically, the signal, in the conditions of a real satellite channel, part of the information about the parameters of the received signal may be missing. Based on this, the minimum variance of the MP-estimate of the carrier frequency on the real satellite signal may differ slightly from the value of the lower Cramer-Rao boundary obtained under the same conditions [12, 13].

Obviously, techniques and algorithms for estimating the carrier frequency of a real satellite signal based on achieving a minimum estimation variance should ensure that their values relative to these dependencies are as close as possible to the dependence $CRLB(\nu)*T^2$. These provisions identify promising areas for further work initiated in this article.

It should be noted that this approach of determining the lower limit of the estimation variance and, in general, to the estimation of the carrier frequency is to some extent quite effectively used to estimate the carrier frequency of the signal transmitted in packet mode. That is substantiated and presented in [25].

In this work [25] the functional dependences are determined, based on them a rule is formed and an algorithm for estimating the carrier frequency of a signal received by a satellite communication system in packet mode according to the rule of maximum likelihood using sliding fast Fourier transform is proposed. This algorithm makes it possible to estimate the frequency according to the rule of maximum likelihood, taking into account the condition of uncertainty of all parameters of the signal received by the satellite communication system in packet mode at short intervals of observation.

It should be noted that the quality of the carrier frequency estimate is significantly affected by the parameters of the noise environment, which can be formed by various environmental factors, among which are both external and internal noise. Among the list of internal noises of some interest in the process of frequency estimation may take into account the internal noise associated with changes in nonlinear properties of composite materials of the synchronization system under the influence of increasing number of additional tracks of charge carriers due to decay in the material structure of radioisotope inclusions [26]. This can affect the growth of the internal noise of the synchronization system and requires its consideration in the development of advanced systems.

Also an important element of reliability, noise protection and stability of the synchronization system, which directly affects the accuracy of carrier frequency estimation, is the adopted model of the synchronization system. The use of combined synchronized synchronization systems, which have a high order of astatism, can have a good effect on reducing the dynamic errors of the frequency estimation process, as evidenced by the studies presented in [27].

3. CONCLUSION

The paper proposes functional dependences designed to determine the lower Cramer-Rao boundary as a quantitative characteristic of the minimum limiting variance of the carrier frequency estimate of the signal received by the radio communication system.

It is established that the lower Cramer-Rao boundary adopted to determine the minimum estimate of the limiting variance of the carrier frequency estimate functionally depends on the energy of a single signal pulse, the interval of information pulses of the complex envelope of the received signal and the interval on which the estimation is performed.

The dependences obtained in this work make it possible to establish the lower Cramer-Rao boundary and the minimum variance of the carrier frequency estimate by its value, provided that other signal parameters are known.

In real conditions, the minimum variance of the MP-carrier frequency estimation in case of uncertainty of all satellite signal parameters can differ significantly from the minimum variance obtained on the basis of the application of the lower Cramer – Rao boundary.

Prospective research, development and creation of algorithms and techniques aimed at MP-estimation of carrier frequency at minimum limit variance in conditions of uncertainty of all parameters of the received signal should be aimed at maximum approximation of minimum limit variance of carrier signal estimation of real signal to Cramer-Rao lower boundary. determined to estimate the carrier frequency under conditions of certainty of other signal parameters.

REFERENCES

1. Horbatyy I.V. **Systems of remote sensing of the Earth from space: monograph**, SPOLOM, Lviv, 2011. 612 p.
2. Shestakov A.L., Semenov A.S., Ibrieva O.L. **“Estimation of the carrier frequency of a random sequence of pulses by the Proni method”**, Bulletin of SUSU, No. 37 (170), PP. 106–115, 2009.
3. Boyko Yu.M., Svorin N.M **“Analysis of digital methods of modulation / demodulation in sound transmission and information systems”**, News of the Khmelnitsky National University, No. 1, PP. 103-110, 2011.
4. Sana Fathima, Sudershan Reddy Kotla, Sameeha Fahmeen, Quddusa Sultana, Desireddy Krishna Reddy. “**Estimation and Analysis of Instrumental Biases for GPS and NavIC Satellites and Receivers”**, *International Journal of Advanced Trends in Computer Science and Engineering*. Vol.8, PP 391-394, 2019, doi: 10.30534/ijatcse/2019/5981.42019.

5. Levin B. R. **Theoretical Foundations of Statistical Radio Engineering.** Moskov, Radio and Communications, 1989. 656 p.
6. Shelukhin O. I., Lukyantsev N. F. **Digital processing and speech transmission**. Moskov, Radio and communications, 2000, 345 p.
7. Salomatin S. B. **Digital signal processing in electronic systems. Tutorial**. Minsk, BSUIR, 2002, 87 p.
8. Lyons R.G. (2010) **Understanding Digital Signal Processing.** Boston, Prentice Hall, 2010, 992 p.
9. Kootsookos J. A **"Review of the Frequency Estimation and Tracking Problems. CRC for robust and adaptive sustems DSTO"**, Salisbury Site Frequensy and Tracking Project. February, 1999. Access mode: https://www.researchgate.net/publication/37616642.
10. Nasir A.A., Durrani S., Kennedy R.A. **"Particle filters for joint timing and carrier estimation: Improved resampling guidelines and weighted bayesian Cramer–Rao bounds"**. IEEE Trans. Commun, PP.1407–1419, 2012.
11. Gogolev. I.V. **"Cramer-Rao Boundary for Estimation of Doppler Deformation and Signal Delay with an Arbitrary Spectrum Width"**, Bulletin of Higher Educational Institutions of Russia. Radio Electronics, PP.3-6, 2016.
12. Wei H., Ye S., Wan Q. **"Influence of Phase on Cramer-Rao Lower Bounds for Joint Time Delay and Doppler Stretch Estimation"**, Proc. of 9th Intern. Symp. on Signal Processing and Its Applications (ISSPA-2007), 12-15 Feb. 2007. Sharjah, pp. 1-4, 2007.
13. Tong Zhao, Tianyao Huang, **"Cramer-Rao Lower Bounds for the Joint Delay-Doppler Estimation of an Extended Target",** IEEE Global Conference on Signal and Information Processing (GlobalSIP), 14-16 Dec. 2015. Orlando, FL, USA, Orlando, PP. 17-24, 2015.
14. Pourhomayoun M., Fowler M. **"Cramer-Rao lower bound for frequency estimation for coherent pulse train with unknown pulse"**, IEEE Transactions on Aerospace and Electronic Systems, PP. 1304–1312, 2014.
15. Yun S., Kim S., Koh J., Kang J. **"Analysis of Cramer-Rao lower bound for time delay estimation using UWB pulses, Ubiquitous Positioning, Indoor Navigation and Location Based Service (UPINLBS)"**, 3-4 Oct. 2012. Helsinki, Finland, PP. 1–5, 2012.
16. Tavares G., Tavares L., Petrolino A. **"On the true cramer-rao lower bound for data-aided carrier-phase-independent frequency offset and symbol timing estimation"**, IEEE Trans. Commun, pp. 442–447, 2010, 2010.
17. Salim O.H., Nasir A., Mehrpouyan H. **"Channel, phase noise, and frequency offset in OFDM systems: Joint estimation, data detection, and hybrid Cramer-Rao lower bound"**, IEEE Trans. Commun, pp. 3311–3325, 2013.
18. Huang W. C., Li C.P., Li H.J. **"An investigation into the noise variance and the SNR estimators in imperfectly-synchronized OFDM systems"**, IEEE Trans. Wireless Commun, PP. 1159–1167, 2010.
19. Borisov B.D. **"An effective estimation of the instability characteristics of frequency oscillators in the time domain"**, Autometry, pp. 123-126, 2009.
20. Borovkov A.A. **Mathematical statistics**, St. Petersburg, Lan, 2010, 704 p.
21. Meyer H. **Digital Communication Receivers**, New York:John Wiley&Sons, Inc, 1998, 827 p.
22. Lehman E. **Theory of point estimation**, Moscow, Nauka, 1991, 448 p. https://doi.org/10.1007/978-1-4615-8071-3
23. Zalmanzon L.A. **Fourier, Walsh, Haar transforms and their application in management, communications and other fields**, Moscow, Nauka, 1989, 496 p.
24. Dwight G.B. **Tables of integrals and other mathematical formulas**, St. Petersburg, "Lan Publishing House", 2009, 232 p.
25. Oleksandr Turovsky, Oleksandr Drobyk, Anatoliy Makarenko, Oleksiy Khakhlyuk **"Estimates of the carrier frequency of the signal received by the satellite communication system in package mode"**, International Journal of Advanced Trends in Computer Science and Engineering, vol. 9. PP. 3223 – 3228, 2020. https://doi.org/10.30534/ijatcse/2020/115932020
26. Savchenko Vitalii, Vorobiov Oleh, Tkalenko Oksana et all **"Influence of the Composite Materials Nonlinear Properties with Radioisotope Inclusions on Reflected Radiation"**, *International Journal of Advanced Trends in Computer Science and Engineering*, vol. 8. PP. 2716-2720, 2019, 10.30534/ijatcse/2019/05862019
27. Oleksandr Turovsky, Yurii Khlaponin, Muhi-Aldin Hassan Mohamed at al **"Combined system of phase synchronization with increased order of astatism in frequency monitoring mode"**, *CEUR Workshop Proceedings*, vol-2616, Session. 1, pp. 53–62, 2020.