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Exploring applications of Frozen Newton Method in the design of digital filter

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ABSTRACT

Digital filters are popularly used in digital signal processing. The design of these filters can be done using different methods of which the rectangular window method and Frequency sampling methods are commonly used. Filter design using wavelet coefficients also shows better response. In this paper, we explore the use of the Frozen Newton method in improving the response of the filter designed using these methods. This iterative computing is applied considering the Taylor series expansion of the function up to the first order term called the Jacobian.

Key words : Digital FIR filters, Frequency sampling method, Frozen Newton method, Iterative computing, Taylor series expansion, Window method, Wavelet

1. INTRODUCTION

Signal processing has remained in the forefront of developments, providing opportunities for research. FIR (Finite Impulse Response) filters finds its applications in signal processing where a linear phase characteristic is a requirement. Any arbitrary filter characteristics can be approximated by the FIR filters. FIR filters being all-zero filters are also stable. The window method, frequency sampling method and optimal design method are most popularly used FIR filter design methods [1-3]. In the window method, the weighing sequence w(n) of finite duration modifies the Fourier coefficients h(n). Frequency sampling method is used with frequency sampling structures that can be realized with less number of non-zero samples. The optimal filter design method obtains the filter coefficients repeatedly so as to minimize the error. This type of filter design employs different methods.

Wavelet functions are also used widely in signal processing. The signal is broken down into several frequency components and each component is analyzed to obtain the frequency information at the instant of time. Thus the time-frequency representation of the signal can be provided by wavelet transform [4]. Also wavelet coefficients can be used as filter coefficients in the digital filter design [5].

The errors in the design may be minimized by using the Taylor's concept to approximate the function. The Taylor series expansion is applied to study the improvement in the design response. An attempt is made here to use the iterative technique like the frozen newton method to design the digital filter with the better response. This section is followed by the brief discussion of the different methods of designing the FIR filter. Section three describes the frozen newton method and its application to the filter design. Simulation study is presented in section four followed by the conclusion derived by the study.

2. FIR FILTER DESIGN METHODS

2.1 Window Method

The digital filters have the periodic frequency response with the period of 2π. Hence the desired frequency response of the filter $H_{desired}(\omega)$ can be represented as Fourier series representation. The coefficients of this Fourier series h[n] is given by (1) [6].

$$\mathrm{h}[n]=\frac{1}{2\pi}\int_{<2\pi>} H_{desired}(\omega)e^{j\omega n}d\omega \qquad (1)$$

$$\forall \quad n \text{ ranging from } -\infty \text{ to } \infty$$

where the frequency response is obtained by (2)[6-7]

$$H_{desired}(\omega)=\sum_{n=-\infty}^{\infty}\mathrm{h}[n]e^{-j\omega n} \tag{2}$$

Equation (1) represents the impulse response of that of an Infinite Impulse Response (IIR) filter with infinite duration. Rectangular window is used to truncate the impulse response of infinite duration to obtain the Finite Impulse Response (FIR) filter coefficients by varying n of (1) in the range of –N/2 to N/2, with N being the order of the filter. The frequency response of the designed filter is now given by (3)

$$H_{designed}(\omega)=\sum_{n=-N/2}^{N/2}h[n]e^{-j\omega n} \tag{3}$$

Truncating the infinite response leads to the error and overshoot as given by the Gibb's phenomenon [7-9]. The FIR filter is then made causal with the introduction of the sufficient time shift in the impulse response. Now the length of the filter is N.

2.2 Frequency Sampling Method

In Frequency sampling method, the filter is specified by the samples of the desired frequency response, $H_{desired}(\omega)$ spaced uniformly. These N samples represented as H[k], k=0, 1, 2…N-1 can be identified as Discrete Fourier Transform (DFT) of the impulse response of the filter and is given by (4) [6]

$$H[k]=H(\omega)\Big|_{\omega=\frac{2\pi}{N}k} \qquad k=0,..,N-1 \tag{4}$$

and the filter coefficients through inverse DFT of H[k] is given by (5) [6-7]

$$h[n]=\frac{1}{N}\sum_{k=0}^{N-1}H[k]e^{j2\pi nk/N} \qquad n=0,1,...,N-1 \tag{5}$$

As this method does not need the computation of the integrals, the computational complexity here is identical whatever may be the frequency response. Also in this method the error is reduced with the increase in the number of the frequency samples.

2.3 Design using Wavelet Coefficients

Using the Haar transform, a discrete signal can be decomposed into two sub signals, one sub signal being the average and the other, the difference of the two. Each sub signal is of half the length of the discrete signal considered. Haar wavelet is the simplest wavelet called the Daubechies 1 wavelet (db1) [4-5]. A wavelet is represented by the scaling function, φ(t) and wavelet function, η(t). These functions are time scaled and then shifted in time. By adding the time scaled and time shifted signal with some modifications we obtain the scaling function and their difference result in the wavelet function as shown in (6) and (7) respectively [5][10].

$$\varphi(t)=\sum_{k=0}^{2N-1}a_k\varphi(2t-k) \tag{6}$$

$$\eta(t)=\sum_{k=0}^{2N-1}b_k\varphi(2t-k) \tag{7}$$

where a_k and b_k are the scaling coefficients and wavelet coefficients respectively and N is the order of the filter and is even always . These coefficients are considered the filter coefficients for the filter design as there exist the relation between the wavelets and filter. The scaling coefficients show the low pass filter response while the wavelet coefficients shows that of the high pass filter.

These three methods are then subjected to the Frozen Newton method as described in the next section.

3. THE FROZEN NEWTON METHOD

Newton's method of computation is popular in most of the research works. A lot of work is being carried out on various Newton's methods. The comparison of the frozen coefficients method and the Newton's method is studied and is observed that the frozen coefficient method is reliable than the Newton's method [11]. However, this method loses its convergence power and diverges very fast which may be due to the selection of the initial assumption that is away from the solution. Hence the paper [11] suggests using the method of frozen coefficients for first few steps and then the newton's method. The Frozen Newton method, as applied for Taylor series is as given by (8) [12]

$$\begin{aligned}&F(\mu_A)=F(\mu_I)+\Delta\mu\,F'(\mu_I)+\frac{1}{2}\Delta\mu^T F''(\mu_I)\Delta\mu+...\\&F'(\mu_{initial})=Jacobian=J\\&F''(\mu_{initial})=Hessian=H\end{aligned} \tag{8}$$

μ_I is the initial guess and $\Delta\mu$ indicates the strength of iteration.

Taylor series expression helps in representing a function as a sum of infinite terms comprising its derivatives at a point, and was introduced in 1715 by Brook Taylor[13] and is given by the expression given in (9) [14-15]

$$f(x) = f(\mu_I) + (x-\mu_I) f'(\mu_I) + \frac{(x-\mu_I)^2 f''(\mu_I)}{2!} + \dots \approx \sum_{n=0}^{N} \frac{(x-\mu_I)^n}{n!} f^n(\mu_I) + R \quad (9)$$

where $f^n(\mu_I)$ is the n[th] derivative of the function $f(x)$ and R is the remainder term representing the error of approximation. The function is said of have P+1 continuous derivative in an interval [0, 1]. The first two terms of (9) represents the approximation of $f(x)$ around μ_1 [16-17].

The above Taylor series representation of a function as in (9) in terms of its value has been applied towards an iterative process for computing desired parameters with a suitable initial guess, and known as the Frozen Newton method for solving a system of (8). The design of digital FIR filters is well established. Authors in [12] use the frozen newton method to reconstruct the nonlinear system that consists of the second order term, called Hessian. The Taylor series expansion of a point around the initial guess is said to be almost equal to the actual solution, at the first order term, also called the Jacobian.

3.1 Frozen Newton Method Applied to Filters

Our aim in this work is to explore the application of the Frozen Newton method in the design of digital FIR filters. We can identify (8) as the function of initial guess and derivative of function at the initial guess. This equation can be used for iterative design of filters. We have function of the impulse response as given by (10) [7], related through the DFT.

$$H(\omega) = \sum_{n=0}^{N-1} h[n] e^{-j\omega n} \quad where \qquad h[n] = \frac{1}{2\pi} \int_{\langle 2\pi \rangle} H(\omega) e^{j\omega n} d\omega \quad (10)$$

In the design of digital filters, usually we have the desired frequency response, H(ω), and we attempt to design the filters to best represent the desired frequency response. Initial guess is obtained either by window method or frequency sampling method. Since the frozen newton method depends on the derivatives of the function, we need to compute the derivatives of the frequency response. The Discrete Time Fourier Transform (DTFT) has the frequency differentiation property as given by (11)

$$-jnh[n] \leftrightarrow \frac{d}{d\omega} H(\omega) \quad (11)$$

This property translates to a multiplication by n, in the time domain to a differentiation in the frequency domain. Therefore, we now have the application of the Taylor series for digital filter design through the following process. We first compute the impulse response of the desired filter through an established technique, hn-initial. Taking this as the initial guess, we attempt to improve the frequency response through application of the Frozen Newton method together with the frequency differentiation property of the DTFT. We then have the pseudo code as below:

h(n) - taken as initial guess.
Use frequency differentiation property to obtain the derivative of the frequency response.
Apply the Frozen Newton method.

A small perturbation in the initial guess is introduced to obtain an improved frequency response. Of course, since a differentiation in the frequency domain is equivalent to multiplication by n, in the time domain, we now loose the property of linear phase. We attempt to explore possible improvements in the magnitude response of the designed filters, with the initial guess being from three methods: (i) the window method (ii) the frequency sampling method and (iii) design method using the wavelet coefficients.

4. SIMULATION

We now present the implementation of these three different FIR filter design methods for the low pass filter taking the cut-off frequency to be $\omega_c = \pi / 2$. The order of the filter is taken to be 23 for first two methods and 24 for the design using wavelet coefficients.

4.1 Window Method

The filter coefficients are obtained using (1) with n ranging from –N/2 to N/2, as given in (12)

$$h[n] = \frac{\omega_c}{\pi} \text{sync}(\omega_c n) \quad \forall n \text{ ranging from -N/2 to N/2} \quad (12)$$

where, cut-off frequency, $\omega_c = \pi / 2$

Applying this method for filter design, the response obtained is shown in the figure 1. The ripples are observed to be present in the response with a small overshoot at the transition.

Considering this as an initial guess, we then apply the frozen newton method up to the first derivative as in (8). The strength of iteration $\Delta\mu$ is chosen so as to reduce the ripples in the stop band to minimum. The obtained response is shown in figure 2 and is observed that though the amplitude of the response is reduced, the ripples in the stop band are minimized.

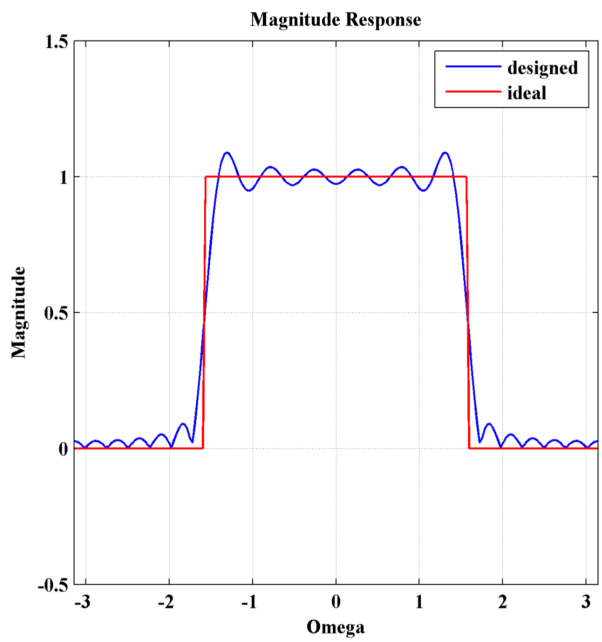

Figure 1: Magnitude response of FIR filter using window method

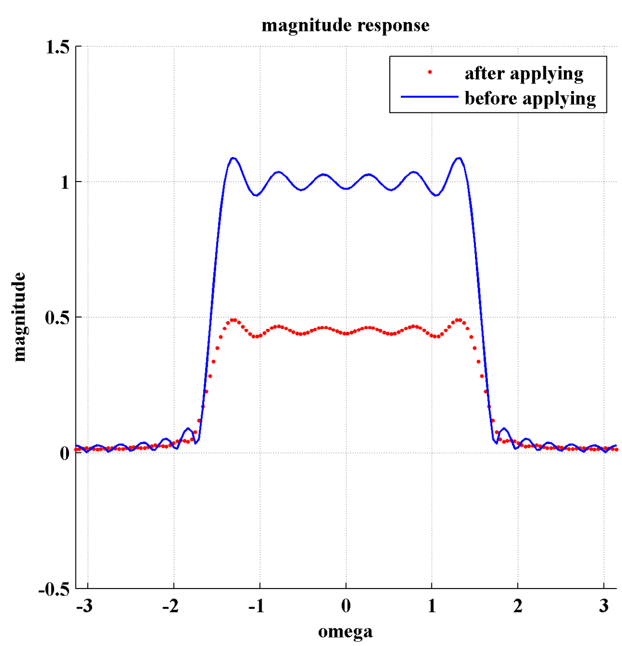

Figure 2: Magnitude response of FIR filter using window method through the application of frozen newton method

4.2 Frequency sampling method

We now implement the filter using Frequency sampling method for an ideal low pass filter. Consider (13) that represent the frequency response of the ideal low pass digital filter

$$H_{desired}(\omega) = \begin{cases} 1 & if\ |\omega| \le \omega_c \\ 0 & if\ \omega_c < |\omega| < \pi \end{cases} \qquad (13)$$

With the cut off frequency $\omega_c = \pi / 2$. The uniformly spaced frequency samples and hence the frequency coefficients are obtained from (4) and (5) respectively. The response obtained is shown in figure 3. When compared with that of the window method, this method is found to be computationally less complex with almost similar response [6]

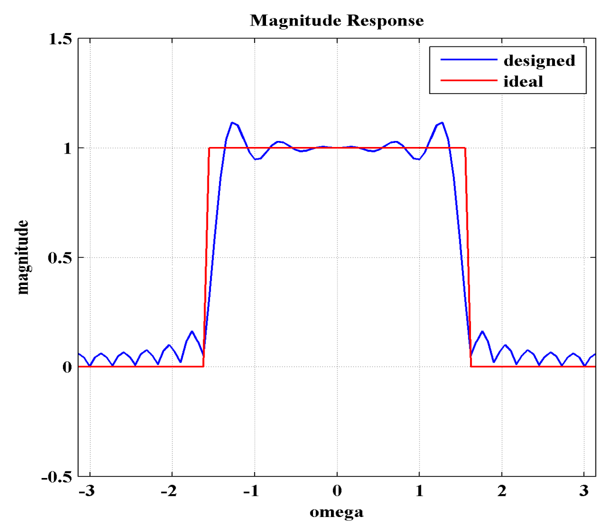

Figure 3: Magnitude response of FIR filter using frequency sampling method

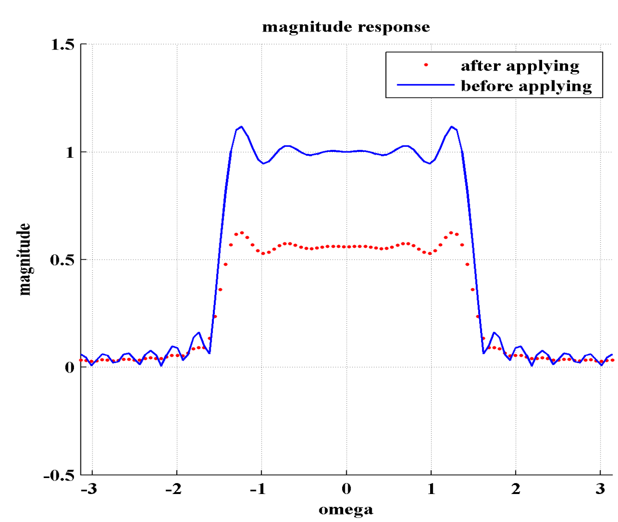

Figure 4: Magnitude response of FIR filter using frequency sampling method through the application of frozen newton method

We then apply the frozen newton method as in (8), considering this as an initial guess and $\Delta\mu$ chosen to minimize the ripples in the stop band. Figure 4 shows the filter response in comparison with the application of the frozen newton method to frequency sampling method of design.

Similar observations as in window method is observed, where application of frozen newton method has reduced the ripples in the stop band. The amplitude variation can be adjusted by using suitable scaling factor.

4.3 Using Wavelet Coefficients

We now consider filter design using the wavelet coefficients, where the cut off is always at $\omega_c = \pi / 2$ and the order of the filter is even. Using the filter coefficients obtained by (6), the response obtained for filter of order N=24 is as shown in figure 5.

Considering this as the initial guess, the result obtained through the application of frozen newton method is as shown in figure 6.

We observed here, that there are no ripples in the response, but there is the shift in the cut off frequency along with the reduction in the magnitude. The shift in the cut off frequency is not possible in the first two methods as they can be designed for any cut off and this shift in cut off frequency can be explored.

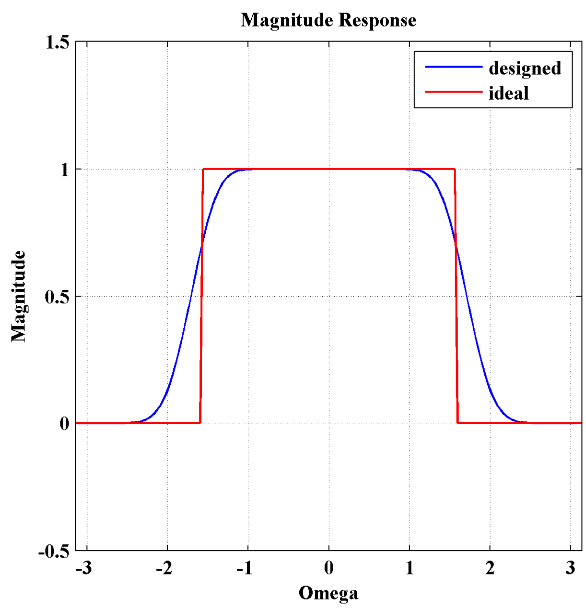

Figure 5: Magnitude response of filter designed using wavelet coefficients

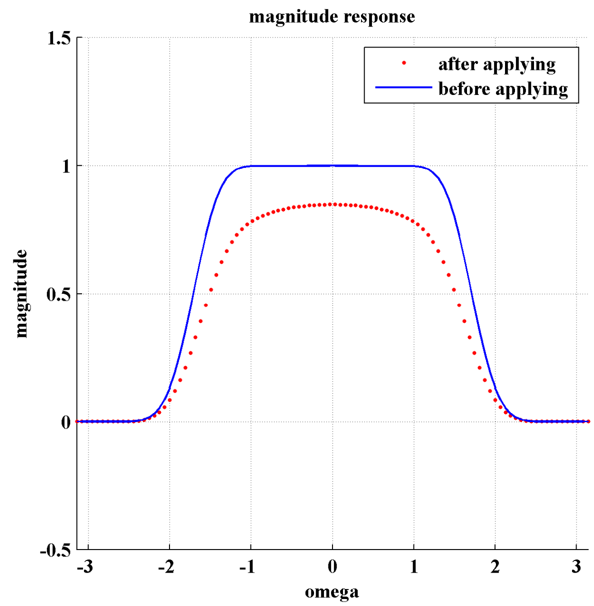

Figure 6: Magnitude response of FIR filter using wavelet coefficients through the application of frozen newton method.

5. CONCLUSION

The work presented here shows the application of Taylor series expansion and the frozen newton method to the design of digital filters. The function of the initial guess and its derivative at the initial guess is identified from the window method, frequency sampling method and using the wavelet coefficients. It is then used for the iterative design of the filter. Applying the frozen newton method up to the first order differentiation, the ripples in stop band are found to be reduced in the window method and the frequency sampling method. Further improvement in the filter response may be expected considering the second order differentiation in the Frozen Newton method. When applied to the design using wavelet coefficients, shift in the cut off frequency is noticed that can be explored. The reduction in the magnitude of the response is observed in all the three cases which can be improved using suitable scaling factor.

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