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Comparison of Cubic Trigonometric Polynomial B-Spline and Extended Uniform Cubic B-Spline in Designing Three-Dimensional Bottle

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ABSTRACT

CAGD has been widely used which brings good impact of computers to industries in manufacture. Cubic trigonometric polynomial B-spline and extended uniform cubic B-spline degree 4 are proposed to generate curve for symmetrical object such as bottle. Then, revolution technique in sweep surface method is used to obtain 3-dimensional bottle's design. Various bottle's design can be formed by adjusting the value of shape parameter and will have different volume. Traditionally, manufacturers produced bottles without knowing the exact volume that the bottles can hold. They resorted to a method known as Water Displacement Method to find the volume they desired. However, this process is time-consuming and complicated as it must undergo trial and error procedure for several times. In this paper, numerical method such as Trapezoidal, Simpson's 1/3 and Simpson's 3/8 are used to calculate the volume of bottle's design constructed by cubic trigonometric polynomial B-spline and extended uniform cubic B-spline degree 4 methods. As the result obtained, the best method use is extended uniform cubic B-spline degree 4 with shape parameter $\lambda = 3$ and the most accurate method to determine the volume is Simpson's 1/3 with 18 sub-intervals. This will be useful to manufacturers as they will have better alternative in making decision to come out with the best design with desired volume.

Key words: Cubic Trigonometric B-spline, Trapezoidal, Simpson's 1/3, Simpson's 3/8.

1. INTRODUCTION

Various surfaces and shapes of bottles have been designed up to this day despite the high competitions among the manufacturers. During the production stages of bottles, the manufacturers and merchants must confront a demanding market and society until consumers are satisfied with the final outcomes. In this research, bottle designs have been chosen as object of symmetrical irregular 3-dimensional. Bottles can be categorized as aesthetic surface as they are mainly generated from 2-dimensional curves which is the most basic element in determining the shapes and silhouettes of other industrial products [1]. Generating curves can be challenging as desired designs cannot be obtained with just a single try. Although bottles are the object of this study, other products involving curves can also be manufactured using the same method. In the recent days, it has been discovered that trigonometric splines and polynomials have been gaining interest in CAGD especially in generating curves [2,3]. The other method that will be ventured upon in this study includes cubic trigonometric polynomial B-spline method and extended cubic uniform B-spline. The cubic trigonometric polynomial B-spline method is known to be a new kind of uniform spline curve which includes both polynomial curves and trigonometric curves [4]. Then, curve is revolving by using revolution technique to form 3-demensional object. The outcome of the revolution technique is that it will have volume. The volume of the irregular symmetrical object is depending on the value of shape parameter use [5]. Several numerical approaches known as Trapezoidal, Simpson's 1/3 and Simpson's 3/8 with a particular subinterval are applied to determine the volume. Hence, the design that best fits the desired volume is chosen as the best method use in designing symmetrical irregular 3-dimensional object.

2. METHODOLOGY

2.1 Cubic Trigonometric B-spline

A cubic trigonometric polynomial B-spline curve with a shape parameter is generated by four consecutive control points and designed using various value of shape parameter. The basis function of cubic trigonometric B-spline with degree 4, with shape parameter $-1 \le \lambda \le 1$, $t \in [0, \frac{\pi}{2}]$ is used as follows [6];

$$basis1 = (1/(4+(2*\lambda^2)+(4*\lambda)))*((1-(\lambda*Sin\,[t]))^2)*\left(\left(1-\left(Sin[t]\right)\right)\right);$$
$$basis2 = (1/(4+(2*\lambda^2)+(4*\lambda)))*((1+(\lambda*Cos\,[t]))^2)*\left(\left(1+\left(Cos[t]\right)\right)\right);$$
$$basis3 = (1/(4+(2*\lambda^2)+(4*\lambda)))*((1+(\lambda*Sin\,[t]))^2)*\left(\left(1+\left(Sin[t]\right)\right)\right);$$
$$basis4 = (1/(4+(2*\lambda^2)+(4*\lambda)))*((1-(\lambda*Cos[t]))^2)*\left(\left(1-\left(Cos[t]\right)\right)\right);$$

The basis function is just a general trigonometric polynomial when $\lambda = 0$ while for $\lambda \neq 0$ the basis function is trigonometric polynomial with shape parameter. The curve can be plotted by using equation bellows;

$$P_i(t) = \sum_{j=0}^{3} B_j(\lambda, t) P_{i+j}\ ,\ t \in [0, \frac{\pi}{2}]$$

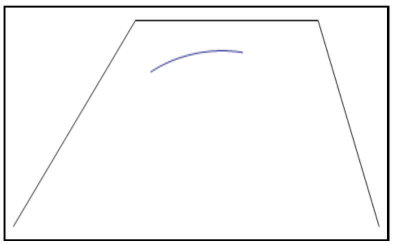


Figure 1: Simple curve of Cubic trigonometric polynomial B-spline.

Figure 1 shows a simple curve of cubic trigonometric polynomial B-spline of with four control points and the curve drawn in a control polygon, respectively.

2.2 Extended Cubic Uniform B-Spline

Xu and Guo-Zhao (2008) stated that extended cubic B-spline is an extension of B-spline with basis function of degree 4. Extension of cubic B-spline was proposed to overcome the problem in cubic B-spline which is once control point are determined, the shape of curve is determined. In extended cubic B-spline method, a free parameter λ is used to alter the shape of generated curve.

For $t \in [0,1]$, the basis function of extended cubic uniform B-spline with degree 4 is as follows:

$$\begin{cases} b_0^4(t) = \dfrac{1}{24}(4-\lambda-3\lambda t)(1-t)^3 \\ b_1^4(t) = \dfrac{1}{24}\left[16+2\lambda-12(2+\lambda)t^2+12(1+\lambda)t^3-3\lambda t^4\right] \\ b_2^4(t) = \dfrac{1}{24}\left[4-\lambda+12t+6(2+\lambda)t^2-12t^3-3\lambda t^4\right] \\ b_3^4(t) = \dfrac{1}{24}\left[4(1-\lambda)+3\lambda t\right]t^3 \end{cases}$$

If the case $\lambda = 0$ occurs, the basis function above will degenerate into cubic uniform B-spline function.

The basis functions $b_i^k(t)$ where $k = 4,5,6$ and $i = 1,2,3,4$ follows the following theorem:

i) $\sum_{i=0}^{3} b_i^k(t) = 1$;

ii) $b_i^k(t) = b_{3-i}^k(1-t)$;

iii) When $-k(k-2) \le \lambda \le 1, b_i^k(t) \ge 0, t \in [0,1]$.

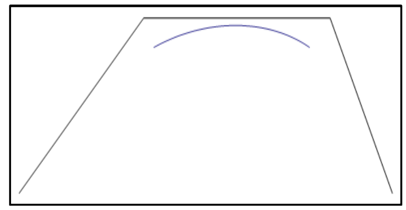


Figure 2: Simple curve of extended uniform cubic B-spline.

Figure 2 shows a simple curve of extended uniform cubic B-spline degree 4 with four control points and the curve drawn in a control polygon.

2.3 Revolution Technique in Sweep Surface Method

2-dimensional bottle's design can be transformed into 3-dimensional object by using revolution technique of sweep surface method. The surface is obtained with the use of the following equation [7],

$$P(t,w) = P(t) \bullet T(w)$$

Where, $P(t,w)$ = Revolution of Surface, $P(t)$ = Cross Section, $T(w)$ = Revolution of Matrix

If the angle of revolution is expressed as w and revolved along axis $r = (r_x, r_y, r_z)$, then it can be computed that the matrix revolution $T(w)$ is as follows in their respective axis, x, y and z. Transformation $T(w)$ is a 3×3 matrix and the expressions below is known as Matrix Revolution $T(w)$ [7].

$$T_x(w) = \begin{bmatrix} \cos w & \sin w & 0 \\ -\sin w & \cos w & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$T_y(w) = \begin{bmatrix} \cos w & 0 & \sin w \\ 0 & 1 & 0 \\ -\sin w & 0 & \cos w \end{bmatrix}$$

$$T_z(w)=\begin{bmatrix}1 & 0 & 0\\ 0 & \cos w & \sin w\\ 0 & -\sin w & \cos w\end{bmatrix}$$

2.4 Volume of Bottle's Design

Numerical method such as Trapezoidal, Simpson's 1/8 and Simpson's 3/8 are used in this paper to calculate the volume of design's bottle. These three respective methods initially are used to compute area, however in this study, some modifications have been applied for the purpose of calculating the volume of each object generated by using the software involved. The formula are as follows [8];

i. Formula to find volume for Trapezoidal:

$$V_T=\frac{D}{2}\left(A_0+2A_1+2A_2+\ldots+2A_{n-1}+2A_n\right)$$

ii. Formula to find volume for Simpson's 1/3:

$$V_{S\frac{1}{3}}=\frac{h}{2}\left(A_0+4A_1+2A_2+4A_3+2A_4+\ldots+A_n\right)$$

iii. Formula to find volume for Simpson's 3/8:

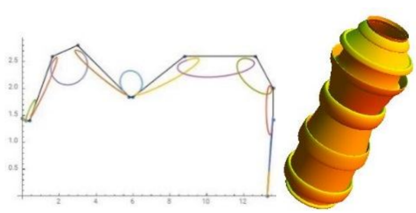
$$V_{S\frac{3}{8}}=\frac{3h}{8}\left(A_0+3A_1+3A_2+2A_3+\ldots+A_n\right)$$

Where D = common distance, A=area , $n=0,1,2,3.....$

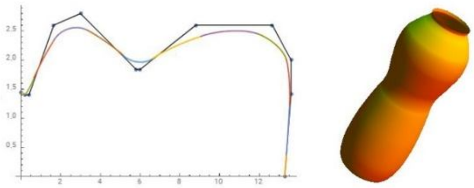
3. RESULTS AND DISCUSSION

3.1 Bottle's design in 2-dimensional and 3-dimensional

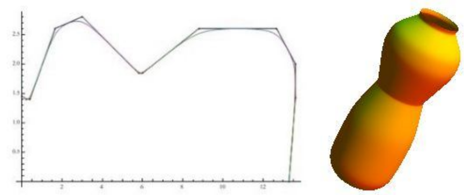
Various Bottle's design can be formed by revolve 2-dimensional cubic trigonometric degree 4 with different shape parameter.



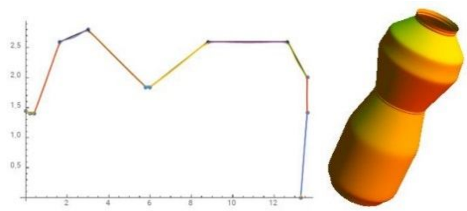
$\lambda=-3$



$\lambda=0$



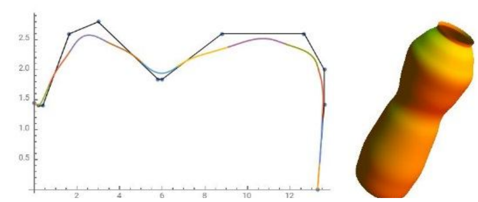
$\lambda=1$



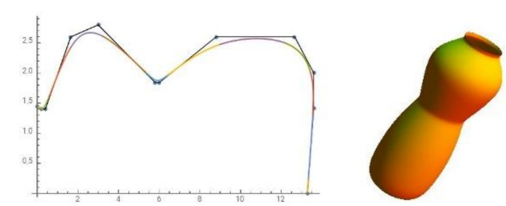
$\lambda=3$

Figure 3: 2-dimensional and 3-dimensional bottle's design by using cubic trigonometric polynomial B-spline with various shape parameters.

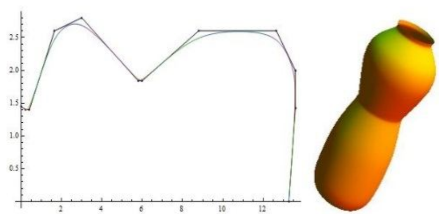
Figure 3 shows that all of the curves generated by implementing cubic trigonometric polynomial B-spline method with shape parameter, $\lambda=-3$, $\lambda=0$, $\lambda=1$ and $\lambda=3$. Then, two-dimensional curves are revolved by using revolution technique to form 3-dimensional design's bottle. The curve and surface formed are smooth by applying $\lambda=0$ and $\lambda=1$. While for $\lambda=3$, the curve and surface are not smooth but it's close to control polygon. Spiral curves are generated by using $\lambda=-3$ and does not satisfy the shape needed.



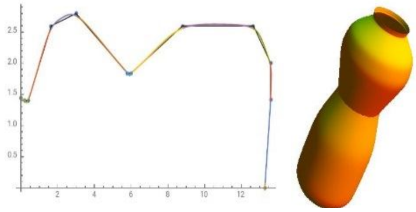
$\lambda=-3$



$\lambda=0$



$\lambda=1$



$\lambda = 3$

Figure 4: 2-dimensional and 3-dimensional bottle's design by using extended uniform cubic B-spline with various shape parameters.

Figure 4 shows all of the curves and bottle's design generated by using extended uniform cubic B-spline method with shape parameter, , $\lambda = -3$, $\lambda = 0$, $\lambda = 1$ and $\lambda = 3$ are put together for the purpose of comparison. From the aspect of smoothness, curve generated by using $\lambda = -3$ is the least smooth curve compared to the other 3 curves. Curve generated by using $\lambda = 0$, $\lambda = 1$ and $\lambda = 3$ are all smooth however, by using $\lambda = 1$ and $\lambda = 3$ the curves are closer to the control polygon. Then, shape of each object formed are similar to the initial object which is a concave-shaped bottle. However, it is shown that object formed when $\lambda = 3$ has a narrowest curve on its body compared to the other objects thus results in more visible intersection line.

3.2. Volume

The table 1 below represents the result of volume obtained for bottle's design of cubic trigonometric polynomial B-spline and extended uniform cubic B-spline with different shape parameter by using Trapezoidal, Simpson's 1/3 and Simpson's 3/8 with different sub-intervals.

Table .1: Approximation of volume for design's bottle of cubic trigonometric polynomial B-spline

Shape parameter, λ	Sub-interval, m	volume (cm^3) Trapezoidal	volume (cm^3) Simpson's 1/3	volume (cm^3) Simpson's 3/8
-3	6	149.7916	164.4712	165.5795
	18	180.0173	181.5223	183.7305
0	6	179.8236	201.2005	199.5031
	18	208.0900	212.8421	211.5206
1	6	196.4994	221.8446	218.6151
	18	225.5596	230.4886	229.2061
3	6	195.8037	219.4001	217.7903
	18	227.2835	231.7308	231.5854

Table .2: Approximation of volume for design's bottle of extended uniform cubic B-spline

Shape parameter, λ	Sub-interval, m	volume (cm^3) Trapezoidal	volume (cm^3) Simpson's 1/3	volume (cm^3) Simpson's 3/8
-3	6	181.6252	204.9317	201.5164
	18	205.6324	209.6419	209.0653
0	6	191.4044	215.1549	212.9098
	18	218.5978	223.5249	221.9862
1	6	193.5349	216.9237	215.1679
	18	222.3809	227.2100	226.2103
3	6	199.5248	224.2354	222.4144
	18	231.4066	236.6818	235.2078

It can clearly be deciphered from observing the table where the volume is decrease as the intervals increase from 6 to 18 which is highly support the theoretical statement where the higher the number of subintervals, the higher the accuracy of the methods [9]. As each of the volume for each shape parameter implemented is analyzed, the best shape parameter use for both method is $\lambda = 3$ and the best method use is extended uniform cubic B-spline. Moreover, the best numerical approach to compute volume is Simpson's 1/3 with 18 sub-intervals which approach to actual volume.

5. CONCLUSION

Value of shape parameter such as $\lambda = -3$, $\lambda = 0$, $\lambda = 1$ and $\lambda = 3$ can be use to form 2-dimensional symmetrical irregular shape such as bottle's design. Then revolution technique is use to transform the curve into 3-dimensional object. Different value of shape parameter will form different design and volume. Trapezoidal, Simpson's 1/3 and Simpson's 3/8 can be used to calculate the volume of bottle's design. However, Simpson's 1/3 with 18 sub-intervals is the best method use because it most approach to actual volume. Then, the best method use to design the irregular shape as well as bottle design is extended uniform cubic B-spline degree 4 with shape parameter $\lambda = 3$.

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