



Numerical Modeling of the Filtration Process During Oil Displacement by Gas

Nozim Kurbonov¹, Komila Ibragimova²

¹ Tashkent University of Information Technologies, Uzbekistan, nozim_kurbonov@mail.ru

² Tashkent University of Information Technologies, Uzbekistan, komila.ibragimova@inbox.ru

ABSTRACT

The paper deals with development of a mathematical model of the process of oil and gas filtering in porous medium with piston extrusion. In order to solve the problem, there was developed a numerical algorithm based on the phase-front straightening and integro-interpolation methods using a conservative finite-difference scheme. The model adequacy was verified by series of computational experiments. The mathematical software allows to analyze parameters of the filtration process in the reservoir system, as well as to forecast and make appropriate decisions in designing and developing of oil and gas fields.

Key words: Computer simulation, numerical method, computational experiment, porous medium, fluid, software, piston extrusion, oil, gas, water.

1. INTRODUCTION

Despite significant progress in the alternative energy sphere, the oil and gas will remain the major energy sources in foreseeable future. Hydrocarbon deposits are gradually decreasing. Moreover, this decreasing is inversely proportional to the increase in the costs of new deposits exploration and development, as well as in extracting hydrocarbons from already explored deposits. Meanwhile, the world output is constantly growing. In order to meet the needs of world economy, there is a requirement of continuous improvement in designing and developing of new oil and gas fields, as well as in the processes of production in existing fields.

Achieving this goal is impossible without developing appropriate mathematical models, effective conservative finite-difference methods and software for a comprehensive study of the processes occurring under a variety of natural and artificial conditions of influence on productive strata.

It should be noted that many researchers are engaged in the problems of modeling in order to study the process of mass transfer in porous media. To date, a number of significant theoretical and applied results have been obtained:

Cueto-Felgueroso L., Fu X. and Juanes R. in their work

[1] considered simulation of flows involving multicomponent mixtures with complex phase behavior. Authors presented a diffuse-interface model of single-component two-phase flow in a porous medium under different wetting conditions. Authors proposed a simplified Darcy-Korteweg model that is appropriate to describe flow in a Hele-Shaw cell or a micromodel, with a gap-averaged velocity. There was studied the ability of the diffuse-interface model to capture capillary pressure and the dynamics of vaporization-condensation fronts and showed that the model reproduces pressure fluctuations that emerge from abrupt interface displacements and from the breakup of wetting films.

Pires A., Bedrikovetsky P. and Shapiro [2] discussed one-dimensional models for two-phase Enhanced Oil Recovery floods. The main result presented by authors is the splitting of the EOR mathematical model into thermodynamical and hydrodynamical parts. The $(n) \times (n)$ conservation law model for two-phase n -component EOR flows in new coordinates is transformed into a reduced $(n - 1) \times (n - 1)$ auxiliary system containing just thermodynamical variables and one lifting equation containing just hydrodynamical parameters. The algorithm to solve analytically the problem includes solution of the reduced auxiliary problem, solution of one lifting hyperbolic equation and inversion of the coordinate transformation. The splitting allows proving the independence of phase transitions occurring during displacement of phase relative permeabilities and viscosities. Reduction of the number of equations allows the generation of new analytical models for EOR. Authors presented the analytical model for displacement of oil by a polymer slug with water drive.

Wenchao L., Jun Y. and Zhangxin C. developed dimensionless mathematical models of one-dimensional flow in the semi-infinite long porous media with threshold pressure gradient which are built for the two cases of constant flow rate and constant production pressure on the inner boundaries [3]. Authors found that the velocity of the moving boundary is proportional to the second derivative of the unknown pressure function with respect to the distance parameter on the moving boundary, which is very different from the classical heat-conduction Stefan problems. The exact analytical solutions of the dimensionless mathematical models were obtained, which can be used for strict validation of approximate analytical solutions, numerical solutions and

pore-scale network modeling for the flow in porous media with threshold pressure gradient.

Yuedong Y., Yu-Shu W. and Ronglei Z. [4] reserved the quadratic term to fully describe the transient fluid flow and developed mathematical models to analyze the transient flow behavior in a double porosity, fractal reservoir with spherical and cylindrical matrix. Authors employed Laplace transformation method to solve these mathematical models and provided the type curves to analyze the pressure transient characteristics. This study indicated that the relative errors in calculated pressure caused by ignoring the quadratic term may amount to 10 % in a fractal reservoir with double porosity, which can't be neglected in general for fractal reservoirs with double porosity at large time scale.

Voskov and Entov proposed a system of equations of one-dimensional flow of a multicomponent mixture with phase transitions through a porous medium [5]. The system describes the processes of enhanced oil recovery by injection of gases. For this system self-similar solutions of the Riemann problem of discontinuity breakdown are constructed by splitting the problem into "physicochemical" and "hydrodynamic" problems. Main elements of the procedure for constructing the solution are illustrated with reference to a four-component system with constant distribution coefficients and the solutions obtained by different methods are compared. It is shown that the approach proposed is also effective for a system with a greater number of components.

In their earlier work [6], they developed a two-step procedure for solving one-dimensional problems of multicomponent two-phase flow generic for EOR processes based on gas injection. First, general mathematical framework underlying this approach is presented briefly. Then its realization is illustrated by examples for 4- and 5-component systems with constant partition coefficients and for oil displacement by CO₂. Directions of future research are also discussed.

Drozdov A.N. et al. carried out researches on physical modelling of processes of water-gas influence on a layer by mixtures for determination of efficiency of high viscosity oils displacement [7]. Authors presented dynamics of an oil displacement by water, gas, water-gas mixture. Observed various viscosity oils final displacement efficiency - displacement water-gas mixtures gas content relationships are offered. It is shown, that use of water-gas mixtures promotes improvement of process of an oil displacement and reduction of a residual oil saturation of rocks.

Korotenko V.A. discussed the process of displacement of oil visco-plastic cold water at reservoir temperature [8]. Authors found that the parameters that influence the advance of the front displacement are liquid water permeability coefficient, the initial pressure gradient and the reduced radius of the well. It was also found that in the displacement of oil by water displacement front hydrodynamic depends not only on the physical properties of the reservoir, but also on the

initial pressure gradient of oil displaced.

In the next paper [9] Korotenko offered the method for determination of the oil displacement coefficient on the basis of studying the structure of the pore. It should be noted that the oil displacement coefficient by water is one of the most important parameters that determine the production potential. Inaccurate assessment of this parameter leads to a distortion of the oil recovery factor and recoverable hydrocarbon reserves, as well as the distortion of technical and economic parameters of the development of both the individual development objects and the whole fields. Satisfactory convergence of results obtained by Korotenko and the results of special stream experiments are shown on the example of West Siberia development objects, that are located in sedimentations of various age. These examples demonstrate the possibility of applying the method to control the results of stream studies in case of sufficient variety of permeability in studied sample collection, if there are absence of direct special laboratory tests on development object.

The literature review shows that most authors are not considering the process of two-way displacement of oil by gas and water from two sides. Thus, this work have been made to fill this gap. As mentioned above, the mathematical model of the considering process must be formulated on the basis of the provisions of the mechanics of multiphase media in the form of a problem like Stefan with unknown phase boundaries.

2. STATEMENT OF THE PROBLEM

In this paper, we study the complex dynamic processes occurring in reservoir conditions when oil is forced out by gas or water in a one-dimensional formulation.

For ease of understanding, let us represent a one-dimensional reservoir in the form shown in figure 1.

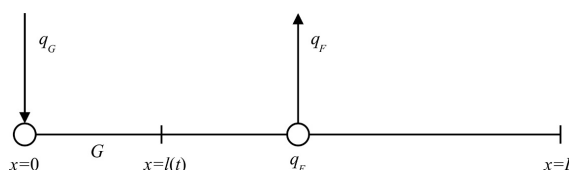


Figure 1: Schematic representation of a one-dimensional reservoir gas-liquid-water

Gas (water) is introduced into the cross section $x = 0$ with intensity q_G . Oil is withdrawn from the cross section $x = \zeta$ with intensity q_F . forming $x = L$ the boundary of the porous reservoir. The boundary between the injected gas (water) and oil is variable $x = l(t)$. In the equations $P_{gas} = P_{gas}(x, t)$, $P_{oil} = P_{oil}(x, t)$, $P = P(x, t)$. Using the laws of gas-hydrodynamics, we can formulate a mathematical model of the process of influencing a reservoir with gas volume and fluid advancement in a reservoir, which can be solved to solve the following system of nonlinear differential equations:

$$\frac{\partial}{\partial x} \left(P_{gas} \frac{K}{\mu_g} \frac{\partial P_{gas}}{\partial x} \right) = m \frac{\partial P_{gas}}{\partial t} \quad \text{with } 0 < x < l(t), \quad (1)$$

$$\frac{\partial}{\partial x} \left(\frac{K}{\mu_o} \frac{\partial P_{oil}}{\partial x} \right) = m \frac{\partial P_{oil}}{\partial t} + F \quad \text{with } l(t) < x < L. \quad (2)$$

Equations (1) and (2) we also write in this form:

$$\frac{\partial}{\partial x} \left(KA \frac{\partial P}{\partial x} \right) = m \frac{\partial P}{\partial t} + F, \quad (3)$$

where

$$K, A, P = \begin{cases} \frac{K}{\mu_g}, P_{gas}, P_{gas}, & 0 \leq x \leq l(t), \\ \frac{K}{\mu_o}, 1, P_{oil}, & l(t) \leq x \leq L, \end{cases} \quad (4)$$

$$F = A_2 q_F \delta(x - \zeta_i). \quad (5)$$

These equations are integrated under the following boundary and internal conditions:

$$\frac{\partial P}{\partial x} \Big|_{x=0} = -A_1 q_g, \quad (6)$$

$$P(x, t) = f(x, t) \quad \text{with } x = L, t > 0. \quad (7)$$

The following conditions are set at the moving interface:

$$S_o \frac{dl}{dt} = -K \frac{\partial P}{\partial x} \Big|_{x=l(t)-0}, \quad (8)$$

$$\frac{K}{\mu_g} \frac{\partial P}{\partial x} \Big|_{x=l(t)-0} = \frac{K}{\mu_o} \frac{\partial P}{\partial x} \Big|_{x=l(t)+0}, \quad (9)$$

$$P_{gas} \Big|_{x=l(t)-0} = P_{oil} \Big|_{x=l(t)+0}. \quad (10)$$

At the beginning of development, the distribution of pressure and phase saturation are known, as well as the position of the interface:

$$P(x, 0) = P_i, \quad l(0) = l^0, \quad 0 < x < L. \quad (11)$$

In formulas (1) - (11), the following notation is used: S_o - saturation of the rock with oil; K - absolute permeability of the rock; μ_g, μ_o - the viscosity of the gas and oil, respectively; $P(x, 0) = P_i$ - initial pressure distribution; ρ_g, ρ_o - the density of gas and oil, respectively; T - absolute temperature; P_{oil}, P_{gas} - the pressure of oil and gas, respectively; ζ_i - internal special point (injection or production well); $l(t)$ - movable interface; L - reservoir length; q_g, q_F - well work intensity; A_1, A_2 - some constant values.

To solve the problem, we first go to dimensionless variables, taking

$$x^* = \frac{x}{L}, \quad l^* = \frac{l(t)}{L}, \quad P^* = \frac{P}{P_o}, \quad t^* = \frac{\rho_o K_o RT}{m \mu_o L^2} t.$$

In the dimensionless form, the boundary-value problem (1) - (11), omitting the asterisks, is rewritten as follows:

$$\frac{\partial}{\partial x} \left(P_{gas} \frac{K}{\mu_g} \frac{\partial P_{gas}}{\partial x} \right) = \frac{\partial P_{gas}}{\partial t}, \quad 0 < x < l(t), \quad (12)$$

$$\frac{\partial}{\partial x} \left(\frac{K}{\mu_o} \frac{\partial P_{oil}}{\partial x} \right) = \frac{\partial P_{oil}}{\partial t} + F \quad \text{with } l(t) < x < L, \quad (13)$$

$$\frac{\partial P}{\partial x} \Big|_{x=0} = -A_1 q_g, \quad (14)$$

$$P(x, t) = f(x, t), \quad x = L, t > 0, \quad (15)$$

$$F = A_2 q_F \delta(x - \zeta_i), \quad (16)$$

$$S_o \frac{dl}{dt} = -\frac{\partial P}{\partial x} \Big|_{x=l(t)+0}, \quad (17)$$

$$P_{gas} \Big|_{x=l(t)-0} = P_{oil} \Big|_{x=l(t)+0}, \quad (18)$$

$$\frac{K}{\mu_g} \frac{\partial P}{\partial x} \Big|_{x=l(t)-0} = \frac{K}{\mu_o} \frac{\partial P}{\partial x} \Big|_{x=l(t)+0}, \quad (19)$$

$$P(x, 0) = P_i, \quad l(0) = l^0, \quad 0 < x < L. \quad (20)$$

Thus, a closed system of nonlinear differential equations was obtained, describing the operation of the "Plast-bore" system. The boundary-value problem describing the filtering process in question relates to tasks like Stefan.

3. SOLUTION METHOD

The numerical solution of problems like (12) - (20) was considered in [10-15], and they can be divided into two essentially different classes.

The first one contains methods with an explicit selection of the boundary and the second one is a joint account.

One of the widely used methods belonging to the first group is the method of straightening phase fronts. Therefore, for the numerical solution of the problem under consideration, we apply the main ideas of the phase-front rectification method.

Let it be required to find functions $P(x, t)$ and $l(t)$, satisfying conditions (12) - (20). To solve this problem, we introduce new independent variables:

$$\xi = \frac{x}{l(t)} \quad \text{and} \quad \zeta = 1 + \frac{x-l(t)}{1-l(t)}, \quad (21)$$

which changes from 0 to 1.

Taking derivatives with respect to variables ξ and ζ equations (12)-(20) can be described as following way :

$$\begin{cases} \frac{\partial^2 P^2}{\partial \xi^2} = \frac{\lambda l(t)}{P} \left[l(t) \frac{\partial P^2}{\partial t} - l'(t) \xi \frac{\partial P^2}{\partial \xi} \right], \\ \frac{\partial}{\partial \zeta} \left(K_o \frac{\partial P}{\partial \zeta} \right) = \\ = B(1-l(t))^2 \left[\frac{\partial(S_o)}{\partial t} - l'(t) \frac{2-\zeta}{1-l(t)} \frac{\partial S_o}{\partial \zeta} \right], \\ S_g + S_o = 1. \end{cases} \quad (22)$$

$$\left. \frac{\partial P^2}{\partial \xi} \right|_{\xi=0} = -A_{1_0} q_G, \quad (23)$$

$$\frac{dl}{dt} = - \left. \frac{\partial P}{l(t) \partial \xi} \right|_{\xi=l-0}, \quad (24)$$

$$D \left. \frac{\partial P}{l(t) \partial \xi} \right|_{\xi=l-0} = \frac{1}{1-l(t)} \left[K \left(\frac{\rho_o K_o}{\mu_o} \right) \right] \left. \frac{\partial P}{\partial \xi} \right|_{\xi=l+0}, \quad (25)$$

where

$$\lambda = \frac{K \rho_o R Z T \mu_G}{K_G P_o \mu_o P}; \quad B = \frac{\rho_o R Z T}{P_o};$$

$$A_{1_0} = A_1 \frac{l(t) L}{P_o}; \quad D = \frac{K_G P_o}{2 \rho_G R Z T};$$

P_o - initial distribution of state function.

The system of equations describing the formulated problem, non-linear with respect to the desired functions. Therefore, to obtain an exact analytical solution of the problem is impossible. To solve it, we use the finite difference method.

The discrete algorithm for solving problem (22) - (25) of analyzing the dynamic states of reservoir filtration systems is based on the use of an integro-interpolation method, which allows us to construct a conservative difference scheme that satisfies the law of conservation of a space-time grid at each node.

Applying the integro-interpolation method for the first equation of the system (22), we get

$$\int_{x_{i-1/2}}^{x_{i+1/2}} \frac{\partial^2 P^2}{\partial \xi^2} d\xi = \lambda l^2 \int_{x_{i-1/2}}^{x_{i+1/2}} \frac{1}{P} \frac{\partial P^2}{\partial t} d\xi - \lambda l l' \int_{x_{i-1/2}}^{x_{i+1/2}} \frac{\xi}{P} \frac{\partial P^2}{\partial \xi} d\xi.$$

As a result, we obtain the following system of finite difference equations:

$$\frac{P_{i-1}^2 - 2P_i^2 + P_{i+1}^2}{h^2} = \lambda \frac{l^2}{P_i^0} \frac{P_i^2 - \bar{P}_i^2}{\Delta t}$$

$$- \lambda \frac{\xi l (l_k - l'_k)}{P_i^0 \Delta t} \frac{P_{i+1}^2 - P_{i-1}^2}{2h}, \quad i = \overline{2, N}.$$

After some transformations, we get

$$\left(1 - \frac{\xi l h \lambda (l_k - \bar{l}_k)}{2 P_i^0 \Delta t} \right) P_{i-1}^2 - \left(2 + \frac{h^2 l^2 \lambda}{\Delta t P_i^0} \right) P_i^2 +$$

$$+ \left(1 - \frac{\xi l h \lambda (l_k - \bar{l}_k)}{2 P_i^0 \Delta t} \right) P_{i+1}^2 = - \frac{h^2 l^2 \lambda}{\Delta t P_i^0} \bar{P}_i^2.$$

As a result, relative to the squares of pressure, we obtain a system of three-point equations

$$a_i P_{i-1}^2 - b_i P_i^2 + c_i P_{i+1}^2 = -d_i, \quad i = \overline{2, N}.$$

Here, the coefficients of three-point equations are determined from the following expressions

$$a_i = 1 - \frac{\xi l h \lambda (l_k - \bar{l}_k)}{2 P_i^0 \Delta t}, \quad b_i = 2 + \frac{h^2 l^2 \lambda}{\Delta t P_i^0},$$

$$c_i = 1 + \frac{\xi l h \lambda (l_k - \bar{l}_k)}{2 P_i^0 \Delta t}, \quad d_i = \frac{h^2 l^2 \lambda}{\Delta t P_i^0} \lambda \bar{P}_i^2.$$

Using the integro-interpolation method, for the second equation of system (22), we get

$$\int_{\zeta_{i-1/2}}^{\zeta_{i+1/2}} \frac{\partial}{\partial \zeta} \left(K_o \frac{\partial P}{\partial \zeta} \right) d\zeta =$$

$$= B(1-l)^2 \int_{\zeta_{i-1/2}}^{\zeta_{i+1/2}} \frac{\partial S_o}{\partial t} d\zeta - \frac{l'}{1-l} \int_{\zeta_{i-1/2}}^{\zeta_{i+1/2}} (2-\zeta) \frac{\partial S_o}{\partial \zeta} d\zeta$$

For the second integral of the right-hand sides, we apply the average theorem and obtain

$$\int_{\zeta_{i-1/2}}^{\zeta_{i+1/2}} (2-\zeta) \frac{\partial S_o}{\partial \zeta} d\zeta =$$

$$= \int_{\zeta_{i-1/2}}^{\zeta_{i-0}} (2-\zeta) \frac{\partial S_o}{\partial \zeta} d\zeta + \int_{\zeta_{i+0}}^{\zeta_{i+1/2}} (2-\zeta) \frac{\partial S_o}{\partial \zeta} d\zeta =$$

$$= (2-\zeta_1) \int_{\zeta_{i-1/2}}^{\zeta_{i-0}} \frac{\partial S_o}{\partial \zeta} d\zeta + (2-\zeta_2) \int_{\zeta_{i+0}}^{\zeta_{i+1/2}} \frac{\partial S_o}{\partial \zeta} d\zeta =$$

$$= (2-\zeta_1) (S_{o_{i-0}} - S_{o_{i-1/2}}) + (2-\zeta_2) (S_{o_{i+1/2}} - S_{o_{i+0}}) =$$

$$= (2-\zeta_1) \left(\frac{S_{o_i} - S_{o_{i-1}}}{2} \right) + (2-\zeta_2) \left(\frac{S_{o_{i+1}} - S_{o_i}}{2} \right) =$$

$$= (2-\zeta_1) \frac{S_{o_i} - S_{o_{i-1}}}{2} + (2-\zeta_2) \frac{S_{o_{i+1}} - S_{o_i}}{2}.$$

Taking into account the computed integral, we obtain the following system of finite difference equations:

$$K_{o_{i+1/2}} \frac{P_{i+1} - P_i}{h} - K_{o_{i-1/2}} \frac{P_i - P_{i-1}}{h} - \delta_j q_o =$$

$$= B h (1-l)^2 \frac{S_{o_i} - \bar{S}_{o_i}}{\Delta t} -$$

$$- B h (1-l) l' (2-\zeta_1) \frac{S_{o_i} - S_{o_{i-1}}}{2} + (2-\zeta_2) \frac{S_{o_{i+1}} - S_{o_i}}{2}.$$

After some transformations, we have

$$\begin{aligned}
 & Bh^2(1-l)l' \frac{2-\zeta_1}{2} S_{o_{i-1}} - \\
 & - \left[Bh^2(1-l)l' \frac{2-\zeta_1}{2} - Bh^2(1-l)l' \frac{2-\zeta_2}{2} - B \frac{h^2(1-l)^2}{\Delta t} \right] S_{o_i} - \\
 & - Bh^2(1-l)l' \frac{2-\zeta_2}{2} S_{o_{i+1}} = \\
 & = K_{o_{i+1/2}} (P_{i+1} - P_i) - K_{o_{i-1/2}} (P_i - P_{i-1}) + B \frac{h^2(1-l)^2}{\Delta t} \bar{S}_{o_i}.
 \end{aligned}$$

As a result, we obtain three-point equations for oil saturation:

$$a_i S_{o_{i-1}} - b_i S_{o_i} + c_i S_{o_{i+1}} = -d_i.$$

Here the coefficients of the resulting equation are determined from the following expressions:

$$a_i = Bh^2(1-l)l' \frac{2-\zeta_1}{2};$$

$$b_i = Bh^2(1-l)l' \frac{2-\zeta_1}{2} - Bh^2(1-l)l' \frac{2-\zeta_2}{2} - B \frac{h^2(1-l)^2}{\Delta t};$$

$$c_i = -Bh^2(1-l)l' \frac{2-\zeta_2}{2};$$

$$d_i = K_{o_{i+1/2}} (P_i - P_{i-1}) - K_{o_{i-1/2}} (P_{i+1} - P_i) - B \frac{h^2(1-l)^2}{\Delta t} \bar{S}_{o_i},$$

where $\zeta_i = [\zeta_{i-1/2}, \zeta_{i-0}]$, $\zeta_{i_2} = [\zeta_{i+0}, \zeta_{i+1/2}]$ and defined

$$\text{as } \zeta_{i_1} = \xi - \frac{h_2}{4}, \zeta_{i_2} = \xi + \frac{h_2}{4}.$$

The size of the error of the scheme approximation is $O(h + \Delta t)$.

Using the conditions under which gas is pumped (23)

$$\frac{-3P_0^2 + 4P_1^2 - P_2^2}{2hl_k} = -A_{1_0} q_g,$$

and at on the border $i = 0$, we find the initial values of the run factors:

$$\alpha_1 = \frac{b_1 - 4c_1}{a_1 - 3c_1},$$

$$\beta_1 = -\frac{c_1}{a_1 - 3c_1} \left(2hl_g A_{1_0} q_g + \frac{d_1}{c_1} \right).$$

The boundary is defined by the following formula:

$$l_k = \sqrt{l_k^2 - \frac{\Delta t}{h} (3P_{\xi_i} - 4P_{\xi_{i-1}} + P_{\xi_{i-2}})}.$$

From condition (25), we deduce

$$\begin{aligned}
 & D \frac{3P_i^2 - 4P_{i-1}^2 + P_{i-2}^2}{2hl_k} = \\
 & = \frac{1}{2h(1-l_k)} \left[\frac{K_g}{\mu_g} + \frac{K_o}{\mu_o} \right] (4P_2 - 3P_1 + P_3).
 \end{aligned}$$

After some transformations, we have

$$\begin{aligned}
 & \frac{(1-l_k)D}{l_k \left(\frac{K_g}{\mu_g} + \frac{K_o}{\mu_o} \right)} [3 - \alpha_l (4 - \alpha_{l-1})] P_l^2 + \\
 & + \frac{(1-l_k)D}{l_k \left(\frac{K_g}{\mu_g} + \frac{K_o}{\mu_o} \right)} [\beta_{l-1} - \beta_l (4 - \alpha_{l-1})] = \\
 & = -[3 - (4 - \alpha_3) \alpha_2] P_1^2 + (4 - \alpha_3) \beta_2 - \beta_3.
 \end{aligned}$$

From condition (23), we get $P_l = P_1 = P$, whence relatively unknown pressures - quadratic equation:

$$D_1 P^2 + D_2 P + D_3 = 0,$$

where

$$D_1 = \frac{(1-l_k)D}{l_k \left(\frac{K_g}{\mu_g} + \frac{K_o}{\mu_o} \right)} [3 - \alpha_l (4 - \alpha_{l-1})],$$

$$D_2 = 3 - (4 - \alpha_3) \alpha_2,$$

$$\begin{aligned}
 D_3 = & \frac{(1-l_k)D}{l_k \left(\frac{K_g}{\mu_g} + \frac{K_o}{\mu_o} \right)} [\beta_{l-1} - \beta_l (4 - \alpha_{l-1})] - \\
 & - (4 - \alpha_3) \beta_2 - \beta_3.
 \end{aligned}$$

Solving the quadratic equation, we find pressure at the points

$$P_{1,2} = \frac{-D_2 \pm \sqrt{D_2^2 - 4D_1 D_3}}{2D_1}.$$

Based on physical considerations, one of them is taken as the solution. The obtained nonlinear system of equations is solved by the sweep method at each time step using the simple iteration method. The convergence conditions for the iterative process are as follows:

$$\max_i \left| P_i^{(s)} - P_i^{(s-1)} \right| \leq \varepsilon_1.$$

where ε_1 - some constant height.

4. RESULTS AND DISCUSSION

In this way, on the basis of the model obtained, a computational algorithm was created for calculating the technological parameters of filtration processes and a software tool was compiled to determine the main parameters and their ranges of changes for the purpose of designing and developing oil and gas fields.

With the help of the program a number of computational experiments were carried out [10,13].

The experiments were carried out with the following input values: $L = 10$ km - the length of the reservoir; $B = 6$ m - power; $H = 700$ m - width; $\mu_o = 4$ cP oil viscosity; the amount of produced fluid - $q_m = 1000$ t / day; $m = 0,2$; $K=0,1$ Darcy, $\mu_G = 0,2$ cP; $P_o = 200$ at;

$$\rho_O = 0,85 \text{ g/sm}^3; \quad R = 8,31 \text{ J/(mol} \cdot \text{K)}; \quad T = 273 \text{ K};$$

$$Z = 40; \quad N = 50; \quad \varepsilon = 0,0001.$$

The results of the performed computational experiments are given in the works and are shown in figure 2-4.

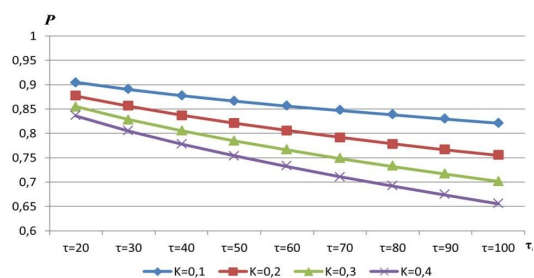


Figure 2: The dynamics of the redistribution of pressure in the reservoir at different values of the filtration coefficient

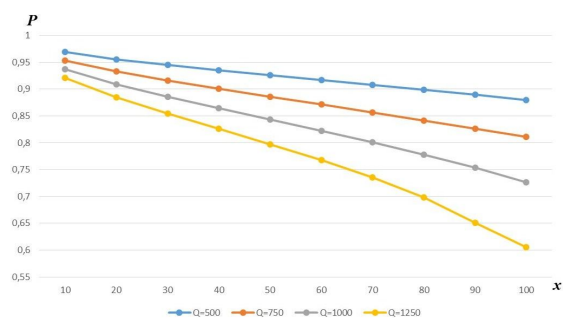


Figure 3: Redistribution of pressure in the reservoir at different well flow rates

The numerical calculations showed that the essential parameters affecting the technology for developing the production of hydrocarbons from reservoir systems are the filtration coefficients (figure 2), viscosities and the structure of porous rocks. Computational experiments were performed at various well flow rates. According to the curves in figure 3, the pressure in the filtration area decreases proportionally with the well flow rate growth.

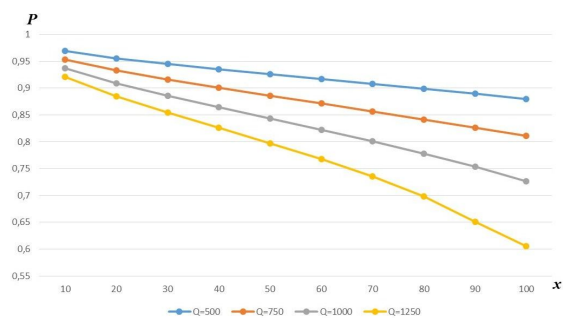


Figure 4: Redistribution of pressure in the reservoir at different formation lengths

In general, it can be noted that the dynamics of the redistribution of the pressure of the reservoir significantly depends on the thickness of the reservoir. With increasing reservoir thickness, the pressure in the well and adjacent points decreases. And the time of exploitation of the productive formation (see figure 4) substantially depends on its length, thickness, number of wells and their flow rates.

5. CONCLUSION

The developed program and the mathematical apparatus implemented in it can be used by specialists of organizations engaged in the extraction of hydrocarbons in order to increase the efficiency of the fields.

REFERENCES

1. L. Cueto-Felgueroso, X. Fu and R. Juanes. **Pore-scale modeling of phase change in porous media**, *Physical review fluids*. – USA, 2018. – vol. 3. – Issue 8. – Pp. 855-863.
2. A. Pires, P. Bedrikovetsky and A. Shapiro. **A splitting technique for analytical modelling of two-phase multicomponent flow in porous media**, *Journal of Petroleum Science and Engineering*. – Elsevier Ltd, 2006. – vol. 51. – Issue 1-2. – Pp. 54-67.
3. L. Wenchao, Y. Jun, C. Zhangxin. **Exact analytical solutions of moving boundary problems of one-dimensional flow in semi-infinite long porous media with threshold pressure gradient** – Elsevier Ltd, 2012. – vol. 55. – Issue 21-22. – Pp. 6017-6022.
4. Y. Yuedong, W. Yu-Shu, Z. Ronglei. **The Transient Flow Analysis of Fluid in a Fractal, Double-Porosity Reservoir**, *Transport in Porous Media*. – Springer, 2012. – vol. 94. – Issue 1. – Pp. 157-187.
5. D.V. Voskov, V.M. Entov. **Problem of Oil Displacement by Gas Mixtures**, *Fluid Dynamics*. – 2011. – № 36. – Pp. 269-248.
6. V.M. Entov, D.V. Voskov. **On oil displacement by gas injection**, *Proc. of 7th European Conference on the Mathematics of Oil Recovery (ECMOR VII)*. – European Association of Geoscientists & Engineers, 2000. – cp-57-00047.
7. A.N. Drozdov et al. **Research of efficiency of high viscosity oil displacement by water-gas mixtures**, *Oil industry*. – 2007. – № 01. – Pp. 58-59.
8. V.A. Korotenko, R.R. Sabitov, N.P. Kushakov, I.V. Sidorov. **Features of water oil replacement viscoplastic** // *Oil and Gas Business*. – 2013. – № 5. – Pp. 190-196.
9. V.A. Korotenko, R.R. Sabitov, N.P. Kushakov, I.V. Sidorov. **Determination Of Oil Displacement Coefficient Using Exploration Of The Pore Space Structure With The Help Of Capillary Studies** // *Oil and gas territory*. – 2017. – № 1. – Pp. 40-50.
10. N. Ravshanov and N.M. Kurbonov. **Computer modeling of processes of fluid filtration in porous media**, *Bulletin of the South Ural State University: Series “Computational mathematics and software engineering”*, Chelyabinsk, 2015. – Vol. 4, № 2. Pp. 89–106. DOI: <http://dx.doi.org/10.14529/cmse150207>.
11. N. Ravshanov, Sh.K. Daliev, and O. Tagaev. **Numerical simulation of two aquarius Horizons**, *International*

Journal of Advanced Trends in Computer Science and Engineering, Vol. 9, pp. 6549-6554, August 2020.

12. A. Ab Aziz, A.R. Zulkefli, and M.A. Khasim. **Finite Different Method and Differential Quadrature Method for Solving Burgers Equation**, *International Journal of Advanced Trends in Computer Science and Engineering*, Vol. 8, pp. 5-8, 2019.
13. N. Ravshanov, N. Kurbonov, A. Mukhamadiev. **An Approximate Analytical Solution of the Problem of Fluid Filtration in the Multilayer Porous Medium**, *International Journal of Computational Methods*. - 2016. - Vol. 13, № 6. — 1650042 [10 pages] DOI: <http://dx.doi.org/10.1142/S0219876216500420>
14. N. Kurbonov, S. Aminov. **Computer modeling of filtration processes with piston extrusion**, *Journal of Physics: Conference Series*. - 2020. - Vol. 1441. - P. 012147. - DOI: 10.1088/1742-6596/1441/1/012147
15. N. Ravshanov, S. Aminov and O. Kravets. **Mathematical model and numerical algorithms to analyze gas filtration process in a porous medium**, *Journal of Physics: Conference Series*. - 2019. - Vol. 1399. - P. 055036. - DOI: 10.1088/1742-6596/1399/5/055036
16. N. Ravshanov, E.Sh. Nazirova and V.M. Pitolin. **Numerical modelling of the liquid filtering process in a porous environment including the mobile boundary of the "oil-water" section**, *Journal of Physics: Conference Series*. - 2019. - Vol. 1399. - P. 022021. - DOI: 10.1088/1742-6596/1399/2/022021