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WOA based Reduced Order Modeling of LTIC Systems

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ABSTRACT

In this paper, the Whale Optimization Algorithm (WOA) has been employed in reduced order modeling of higher order linear time invariant continuous (LTIC) single-input single-output (SISO) systems. The algorithm has considerable features such as; superior performance, fast convergence, trouble-free implementation and gives numerically stable system. The strength of WOA has been tested on three types of systems having complex roots, repeated poles and also on one of the practical IEEE type 1 (DC) excitation model of 4th order. In the present work, WOA minimizes the Integral Square Error (ISE) as an objective function to obtain all the gain parameters of numerator and denominator's polynomials of low order system (LOS). A comparative study has also been included to compare the proposed technique with some of famous/recent techniques existing in the literature for order reduction on the basis of ISE, IAE, IRE and various transient response parameters. The simulation results have been provided in graphical as well as tabular forms.

Key words : Integral Square Error, Integral Absolute Error, Impulse Response Energy, Order Reduction, Stability, Whale Optimization Algorithm.

1. INTRODUCTION

For higher order system (HOS), study and design of practical control systems are very tedious. Therefore, there is a necessity to reduce the order of HOS for which some generalization algorithms are required. These algorithms must be implemented in such away that the reduced equivalent LOS must not lose its original HOS's characteristics. Over many years, various techniques for model order reduction have been invented for simplification such as; system simplification techniques based on balanced truncation [1], whale optimization algorithm [2, 6], combined techniques like: Eigen Spectrum Analysis (ESA) & Pade Approximation [3], ESA & Factor Division [4] and Eigen Permutation & Improved Pade Approximant (IPA) [7], different evolutionary algorithms like: Particle Swarm Optimization (PSO) [5] and Invasive Weed Optimization (IWO) [15].

Further, numerous techniques carried out by the researchers all around the world are available for MOR such as; design of control systems based on LOS [8], comparative analysis of different techniques [9], mixed method [10], Cuckoo Search optimization [11], Big Bang-Big Crunch (BB-BC) algorithm [12, 14], soft computing techniques [16-17], Modified Cuckoo Search (MCS) algorithm [18], Stochastic Fractal Search (SFS) [20], the Routh Approximation [21], Stability Equation [22], differentiation [23], Response Matching Techniques [24], Pole Clustering [25] methods and many more [34-40].

In present work, Whale Optimization Algorithm (WOA) for order reduction of LTIC systems has been explored by minimizing the ISE as an objective function. WOA is based on humpback behavior of whales. The obtained results are also compared with some various recent techniques available in literature.

2. STATEMENT OF PROBLEM

An n^{th} order SISO-LTI-HOS has been defined by the following transfer function (TF):

$$G_n(s) = \frac{a_0 + a_1 s^2 + a_2 s^2 + \dots\dots\dots + a_n s^{n-1}}{b_0 + b_1 s + b_2 s^2 + \dots\dots\dots + b_n s^n} \quad (1)$$

Suppose, the reduced r^{th}order LOS is given as:

$$R_r(s) = \frac{\alpha_0 + \alpha_1 s + \alpha_2 s^2 + \dots\dots\dots + \alpha_r s^{r-1}}{\beta_0 + \beta_1 s + \beta_2 s^2 + \dots\dots\dots + \beta_r s^r} \quad (2)$$

where, r < n……………………………………..
The objective function: ISE is defined as [12, 19, 20, 22]:

$$ISE = \int_0^\infty [y(t) - y_r(t)]^2 \, dt \quad (3)$$

The other performance indices used in present work are given as [19, 22]:

$$IAE = \int_0^\infty |e(t)| \, dt \quad (4)$$

$$IRE = \int_0^{\infty} g^2(t) \; dt \quad (5)$$

where, $g(t)$ is the impulse response of the system.

3. WHALE OPTIMIZATION ALGORITHM (WOA)

A set of random solutions initiate the WOA algorithm in which search agents update their positions with each iteration and best solution obtained so far. In this paper, the application of WOA has been explored in the field of model order reduction and the detailed explanation of WOA along with its steps for implementing the algorithm, pseudo codes, flow chart and mathematical modeling can be found in [2, 6].

4. NUMERICAL EXAMPLES AND SIMULATION RESULTS

Three Simulation Examples have been incorporated for exemplifying the algorithm for the present work.

4.1 Example-1

Let us consider a 9^{th} order system with complex roots be defined as [12, 18]:

$$G_9(s) = \frac{s^4+35s^3+291s^2+1093s+1700}{s^9+9s^8+66s^7+294s^6+1029s^5+2541s^4+4684s^3+5856s^2+4620s+1700} \quad (6)$$

After WOA being run 100 times, the reduced 3^{rd} order denominator's and numerator's polynomials of WOA based LOS are obtained as:

$$D_2(s) = s^3 + 1.6457s^2 + 2.1714s + 1.0602 \quad (7)$$

$$N_2(s) = 0.001s^2 + 0.001s + 1.0602 \quad (8)$$

Therefore, the 3^{rd} order LOS by proposed technique is:

$$R_3(s) = \frac{0.001s^2+0.001s+1.0602}{s^3+1.6457s^2+2.1714s+1.0602} \quad (9)$$

The WOA yields the ISE as 1.632 x 10^{-2} and the reduced model obtained by [18] is given as:

$$R_3(s) = \frac{0.001935s^2+0.005725s+1.073}{s^3+1.681s^2+2.183s+1.073} \quad (10)$$

The transfer function of reduced 3^{rd} order model of the same system by stability equation and Big-Bang Big Crunch method [12] is given by:

$$R_3(s) = \frac{0.0789s^2+0.3142s+0.493}{s^3+1.3s^2+1.34s+0.493} \quad (11)$$

The convergence of objective function; ISE by WOA is shown in Figure 1.

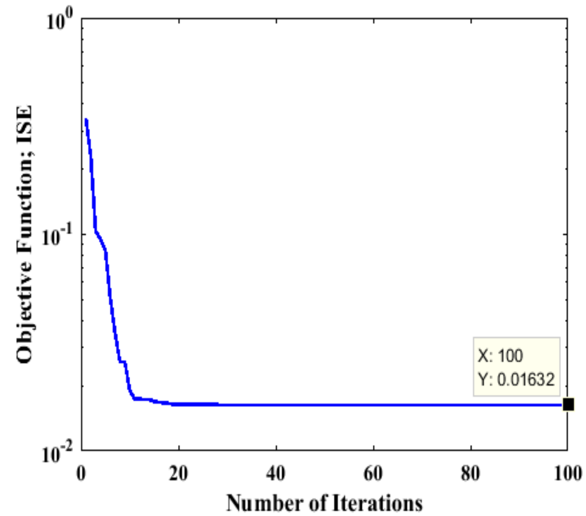

Figure 1: Convergence of objective function for *Example-1*

The comparison of responses, its parameters and performance indices obtained by the proposed WOA and some recent algorithms for 9^{th} order HOS and 3^{rd} order LOS have been shown in Figures2-3 and Tables1(a)-1(b). It is found that (i) A better match of transient response of LOS obtained by WOA with original HOS has been found.(ii)The parameters of 3^{rd} order LOS obtained by WOA and original 9^{th} order HOS are comparable. (iii)The LOS obtained by WOA exhibits the lowest values of performance indices while comparing with some recent algorithms. The same is also shown in Figure 4.

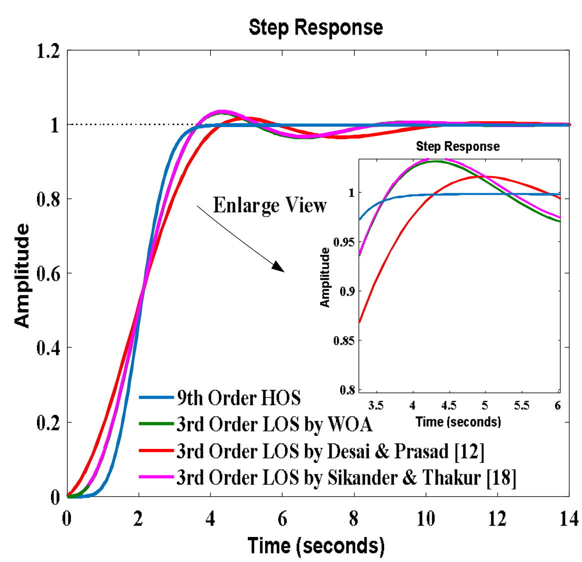

Figure 2: Step response of HOS and LOS for *Example-1*

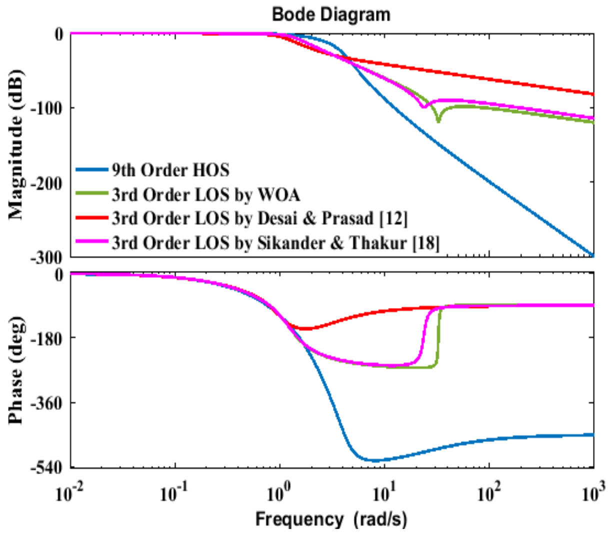

Figure 3: Bode plots of HOS and LOS for *Example-1*

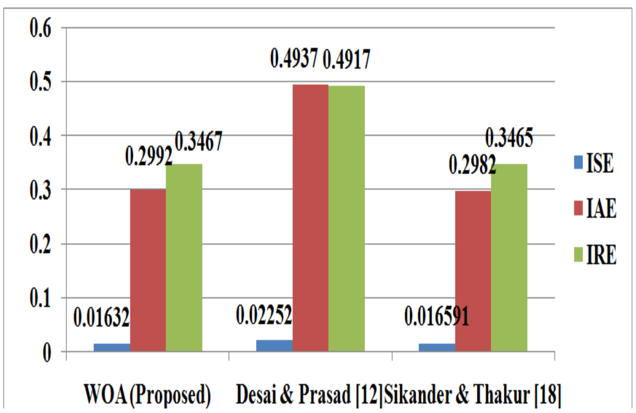

Figure4: Bar chart comparison for *Example-1*

Table 1(a): Comparison for the performance indices obtained by proposed WOA & some recent algorithmsfor *Example-1*

Literature	**ISE**	**IAE**	**IRE**
Original Model	-	-	0.4705
WOA (Proposed)	1.632×10^{-2}	0.2992	0.3467
Desai & Prasad [12]	2.252×10^{-2}	0.4937	0.4917
Sikander& Thakur [18]	1.6591×10^{-2}	0.2982	0.3465
Mukherjee et al. [24]	8.77×10^{-2}	0.9359	0.5085
Vishwakarma& Prasad [25]	5.86×10^{-2}	0.2060	0.6974

Table 1 (b): Comparison for the transient response parameters obtained by proposed WOA & some recent algorithms for *Example-1*

Performance Parameters	Rise Time (sec.)	Settling Time (sec.)	Over Shoot (%)
Original Model	1.52	3.28	0.346
WOA (Proposed)	2.14	7.63	3.16
Desai & Prasad [12]	2.77	9.08	1.60
Sikander& Thakur [18]	2.15	7.62	1.75
Mukherjee et al. [24]	2.92	6.91	0
Vishwakarma& Prasad [25]	2.60	5.15	0

4.2 Example-2

Let a 4^{th} order system with repeated polesbe defined as [18]:

$$G_4(s) = \frac{1}{(s+1)^4} \quad (12)$$

After WOA being applied, the following denominator and numerator's polynomials of LOS are obtained with ISE as 1.495×10^{-2}:

$$D_2(s) = 9.9984s^2 + 6.3484s + 1.6771 \quad (13)$$

$$N_2(s) = 0.001s + 1.6771 \quad (14)$$

Therefore, the 2^{nd} order LOS is:

$$R_2(s) = \frac{0.001s + 1.6771}{9.9984s^2 + 6.3484s + 1.6771} \quad (15)$$

The convergence of ISE by WOA is shown in Figure5

The comparison of responses, its parameters and performance indices obtained by the proposed WOA and some recent algorithms for 4^{th}order HOS and 2^{nd}order LOS have been shown in Figures6-7 and Tables 2(a)-2(b). The same observations (i) through (iii) as in *Example-1* have also been observed for the *Example-2* which shows the superiority of WOA than other recent algorithms. Further, the performance of proposed algorithm is also shown in Figure 8.

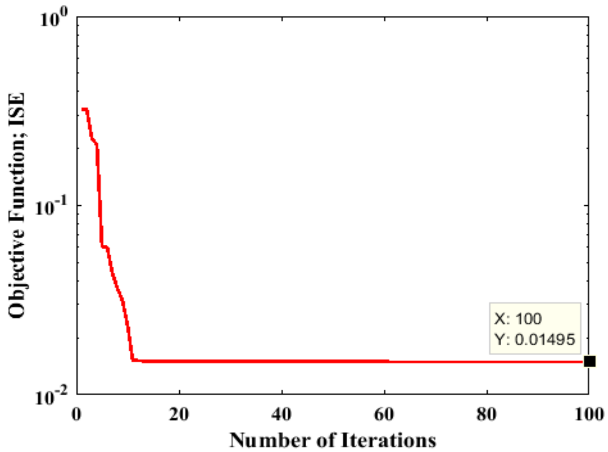

Figure 5: Convergence of objective function for *Example-2*

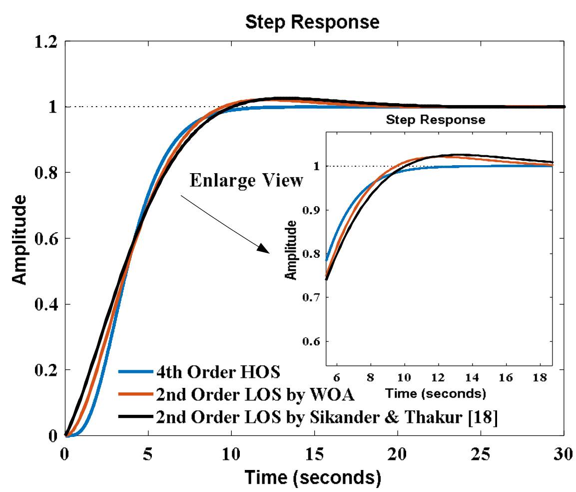

Figure 6: Step response of HOS and LOS for *Example-2*

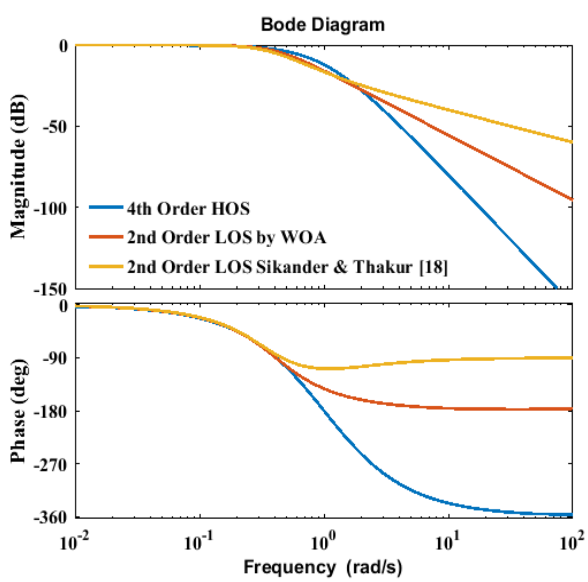

Figure 7: Bode plots of HOS and LOS for *Example-2*

Table 2 (a): Comparison for the performance indices obtained by proposed WOA & some recent algorithms for*Example-2*

Performance Parameters	Reduced Models	IRE	ISE	IAE
Original Model	-	0.1562	-	-
WOA (Proposed)	$\frac{0.001s+1.6771}{9.9984s^2+6.3484s+1.6771}$	0.1319	1.495 x 10^{-2}	0.4133
Sikander& Thakur [18]	$\frac{0.1s+0.1158}{s^2+0.5202s+0.1158}$	0.1219	4.58 x 10^{-2}	0.6764

Table 2 (b): Comparison for the transient response parameters obtained by proposed WOA & some recent algorithms for*Example-2*

Performance Parameters	Rise Time (sec.)	Settling Time (sec.)	Over Shoot (%)
Original Model	4.94	9.08	0
WOA (Proposed)	5.8	13	2.12
Sikander& Thakur [18]	6.56	15.63	2.56

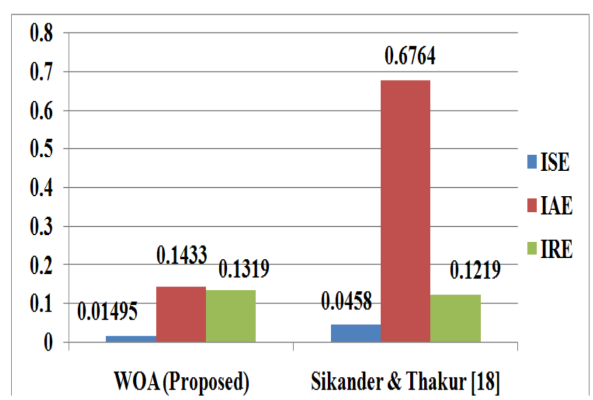

Figure 8: Bar chart comparison for *Example-2*

4.3 Example-3

Let a practical example of IEEE type 1 (DC) excitation model of 4^{th} order be given by [33]:

$$G_4(s) = \frac{400\,s+400}{0.0001s^4+0.03\,s^3+1.36\,s^2+34.32\,s+1.15} \tag{16}$$

After WOA being applied, the following denominator and numerator's polynomials of LOS are obtainedwith ISE as 3.736 x 10^{-1}(Convergence diagram in Figure 9):

$$D_2(s) = s^2 + 21.4829\,s \; + 0.7196 \qquad (17)$$

$$N_2(s) = 252.8464\,s + 250.2952 \qquad (18)$$

Therefore, the 2^{nd} order LOS is:

$$R_2(s) = \frac{252.8464\,s + 250.2952}{s^2 + 21.4829\,s + 0.7196} \qquad (19)$$

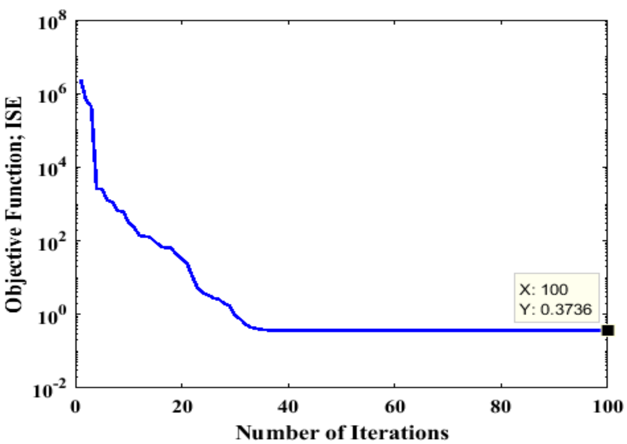

Figure 9: Convergence of ISE for Example-3

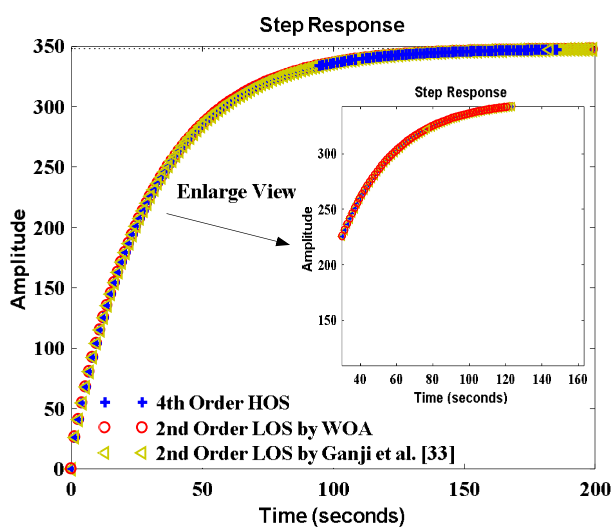

Figure 10: Step response of HOS and LOS for *Example-3*

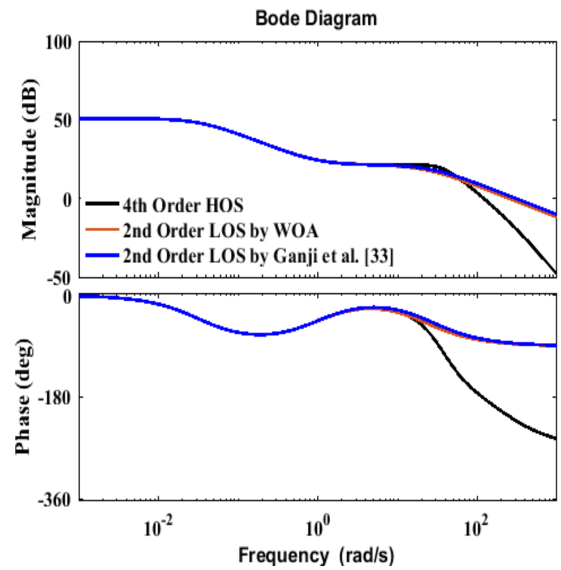

Figure 11: Bode plots of HOS and LOS for *Example-3*

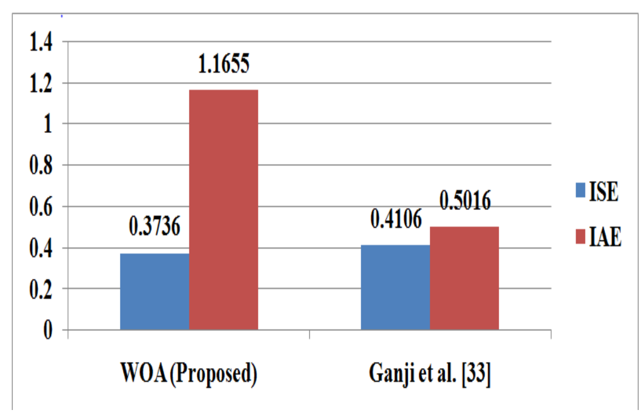

Figure 12: Bar chart comparison for Example-3

Table 3(a) : Comparison for the performance indices obtained by proposed WOA & some recent algorithms for*Example-3*

Literature	**Reduced Models**	IRE	**ISE**	**IAE**
Original Model	-	4028.7	-	-
WOA (Proposed)	$\frac{252.8464s+250.2952}{s^2+21.4829s+0.7196}$	3799.26	3.736 x 10^{-1}	1.1655
Ganji et al. [33]	$\frac{306.565826s+306.681976}{s^2+26.312263s+0.881711}$	4149.217	4.106 x 10^{-1}	0.5016

Table 3 (b): Comparison for the transient response parameters obtained by proposed WOA & some recent algorithms for*Example-3*

Performance Parameters	Rise Time (sec.)	Settling Time (sec.)	Over Shoot (%)
Original Model	65.5	116	2.22×10^{-14}
WOA (Proposed)	65.5	116	0
Ganji et al. [33]	65.5	116	0

The comparison of responses, its parameters and performance indices obtained by the proposed WOA and some recent algorithms for HOS in *Example-3* and 2nd order LOS have been shown in Figures10-11 and Tables 3(a)-3(b). The same observations (i) through (iii) as in *Examples-1*and *2* have also been observed for the Example-3 which shows the superiority of WOA than other recent algorithms. Further, the performance of proposed algorithm is also shown in Figure 12.

5. CONCLUSIONS

In the present work, the stable reduced order models of LTIC systems have been obtained using WOA which evaluates the unknown parameters of numerator and denominator of LOS by minimizing ISE as an OF. Both time and frequency responses have been drawn and it has been found that the reduced order model provides a very close approximation to the original system. It has also been observed that WOA based LOS gives more accurate and stable performance than some recent algorithms available in the literature. Further, the effectiveness and superiority of the proposed algorithm have also been computed by both time response parameters and performance indices. A relative comparison among all parameters obtained by WOA and some recent algorithms have also been carried out which shows the better performance of reduced order model using WOA among aforesaid algorithms without losing all the essential characteristics.

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