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Model Reduction of Linear Time Invariant SISO and MIMO Systems Using Different Optimal Techniques

Parminder Singh[1], Amanpreet Sandhu[2], Parag Nijhawan[3], Simrat Singh Bhutani[4]
[1]Department of Chemical Engineering, Thapar Institute of Engineering and Technology, Patiala 147001, Punjab, India, parmider.singh@thapar.edu
[2]Chitkara University Institute of Engineering and Technology, Chitkara University, 140401, Punjab,India, Amanpreet.sandhu@chitkara.edu.ac.in
[3]Department of Electrical and Instrumentation Engineering, Thapar Institute of Engineering and Technology, Patiala 147001, Punjab, India, parag@thapar.edu
[4]Department of Chemical Engineering, Thapar Institute of Engineering and Technology, Patiala 147001, Punjab, India, simratsingh615@gmail.com

ABSTRACT

In this article, the author analysis various model reduction techniques available in literature to reduce higher order models present in both Single Input and Single Output (SISO) and Multi Input and Multi Output (MIMO) based systems. These reduced models are compared on the basis of integral square error (ISE) between the transient responses of the original system and the reduced model using MATLAB. Frequency response and step response analysis are also carried out so as further compare these techniques on the basis of settling time, rise time, over shoot and peak time.

Keywords: Model Order Reduction, SRAM Method, Balred method, Modified Stability Equation, Continued Fraction Expansion, Modified Cauer Form, MATLAB.

1. INTRODUCTION

The most important subject in engineering is modeling of complex high order systems. A high order model is often very complicated to be used in real practical situations, soan appropriate methodology is adopted based on the physical or mathematical approach to achieve simple models than the original one. So model reduction is very important to engineers and scientists working in different areas of engineering especially for those who are working in the area of process control. In control engineering, model reduction are vital from design of controllers and it also helps to find an approximated models, without incurring too much error. There are many model reduction techniques available in literature [1-8] which reduces the dimensionality of large systems to provide simpler models, which are computationally simpler than the original form. The effort of all these methods aims to obtain a stable reduced order model and also assure that these reduced models represents the same characteristics as that of the original system.

The purpose of this paper is to study such available reduction techniques and compare theiroutcomes based upon response to step input and frequency analysis w.r.t to the actual model. Higher order model studied in the paper, are real linear time invariant systems, taken from both Single Input and Single Output (SISO) and Multi Input and Multi Output (MIMO) based system.

Consider a general higher order model represented by the following transfer function:

$$G(s) = \frac{N(s)}{D(s)} = \frac{\sum_{j=0}^{n-1} b_j s^j}{\sum_{j=0}^{n} a_j s^j} \quad (1)$$

And the reduced order model is represented as follows:

$$G_r(s) = \frac{N_r(s)}{D_r(s)} = \frac{\sum_{j=0}^{m-1} b'_j s^j}{\sum_{j=0}^{m} a'_j s^j} \quad (2)$$

Where G(s) represents a higher order system and $G_r(s)$ represents a reduced order system, and a_j, b_j, a'_j and b'_j are the coefficients of denominator and numerator of G(s) and $G_r(s)$ respectively.

2. MODEL REDUCTION METHODS

2.1 Simplified Routh Approximation Method (SRAM METHOD)

In this method [6] the Routh stability array is prepared using denominator polynomial, whose even and odd parts are written as follows:

$$D(s) = \sum_i b_{1,i+1} s^{n-2i} + \sum_k b_{2,k+1} s^{n-(2k+1)} \quad (3)$$

Where *i= 0,1,2……..n/2 and k = 0,1,2…..(n-2)/2* for n even and *i= 0,1,2……..(n-1)/2 and k = 0,1,2…..(n-1)/2* for n odd.

Routh stability array is prepared as follows:

$$b_{11} b_{12} b_{13} b_{14} \;\ldots\ldots\ldots\ldots\ldots \quad (4)$$
$$b_{21} b_{22} b_{23} b_{24} \;\ldots\ldots\ldots\ldots\ldots$$
$$b_{31} b_{32} b_{33} \;\ldots\ldots\ldots$$
$$b_{41} b_{42} b_{43} \;\ldots\ldots\text{ \& so on….}$$

Where $b_{i,j} = b_{i-2,j+1} - \left[{(b_{i-2} * b_{i-1,j+1})}/{b_{i-1,1}} \right]$

(5)

The parameter h (h_1, h_2, h_3…..) are obtained from the Routh array, where

$h_1 = {b_{11}}/{b_{21}}$, $h_2 = {b_{31}}/{b_{41}}$ and so on…….

Then for reduced model numerator $N_r(s)$ and the denominator $D_r(s)$ is defined as follows:

$N_r(s) = T_1 + T_1 s + T_1 s^2 + \cdots \ldots \ldots \ldots T_t s^{r-1}$, where $T_t = h_1 b_{t-1}$ (6)

And $D_r(s) = s^r + \frac{hr}{b11}\sum_{i=0}^{r-1} b_i s^i$ (7)

2.2 Balanced Truncation Model Reduction (Balred method)

Balred method for higher order models is restricted to finite time intervals, where resulting Lyapunov equations are numerically solved, by using matrix exponential in their inhomogeneities. For small time intervals, a reduced numerical rank of time constraint Gramians are observed, which led to satisfactory results. [9-11]

Consider the higher order system, whose transfer function is given as:

$$G(s) = (sC_2 + C_1)(s^2 M + sD + k)^{-1} B_2 \quad (8)$$

For simplicity, $\mathsf{G} = [\mathsf{M}, \mathsf{D}, \mathsf{K}, \mathsf{B}_2, \mathsf{C}_1, \mathsf{C}_2]$and $\widehat{\mathsf{G}} = [\widehat{\mathsf{M}}, \widehat{\mathsf{D}}, \widehat{\mathsf{K}}, \widehat{\mathsf{B}}_2, \widehat{\mathsf{C}}_1, \widehat{\mathsf{C}}_2]$ are constrained equivalent systems, if there exist nonsingular matrices T_l and T_R, such that $\widehat{\mathsf{M}} = T_l M T_R$,$\widehat{\mathsf{D}} = T_l D T_R$,$\widehat{\mathsf{K}} = T_l K T_R$, $\widehat{\mathsf{B}} = T_l B_2$,$\widehat{\mathsf{C}}_1 = T_l C_1$ and $\widehat{\mathsf{C}}_2 = T_l C_2$, where the pair T_l and T_R are called system equivalence transformation.

2.3 Modified Stability Equation(MSE Method)

In this method the transfer function of reduced orders are obtained directly from the pole zero pattern of the stability equations of the original transfer function [12]. The only advantage of this method is, it yields a stable reduced model, provided the original is also stable. Assume a higher order system as shown below:

$$G(s) = \frac{N(s)}{D(s)} = \frac{a_n s^n + a_{n-1} s^{n-1} + \cdots a_1 s + a_0}{b_m s^m + b_{m-1} s^{m-1} + \cdots b_1 s + b_0} \quad (9)$$

The order of denominator D(s) is m and the order for numerator N(s) is n, such that m>n. the numerator and the denominator are separated in even and odd parts.

$$G(s) = \frac{N(s)}{D(s)} = \frac{N_e(s) + N_o(s)}{D_e(s) + D_o(s)} = \frac{\sum_{i=0,2,4}^{n} a_i s^i + \sum_{i=1,3,5}^{n} a_i s^i}{\sum_{i=0,2,4}^{m} b_i s^i + \sum_{i=1,3,5}^{m} b_i s^i} \quad (10)$$

Further

$$D_e(s) = b_0 + b_2 s^2 + b_4 s^4 + \cdots \quad and\ D_o(s) = b_1 s + 3s^3 + b_5 s^5 + \cdots \quad (11)$$

Or $D_e(s) = b_0 \prod_{i=1}^{k_1}\left(1 + \frac{s^2}{{z_i}^2}\right)$ and $D_o(s) = b_1 s \prod_{i=1}^{k_2}\left(1 + \frac{s^2}{{p_i}^2}\right)$ (12)

Where k_1 and k_2 are the integral parts of n/2 and (n-2)/2 respectively and $z_{12}>p_{12}>z_{22}>p_{22}$…..by discarding the factors with larger magnitude of z_i and p_i.

The reduced denominator of the system is given as:

$$D_r(s) = D_{er}(s) + D_{or}(s) \quad (13)$$

Where $D_{er}(s) = b_0 \prod_{i=1}^{r_1}\left(1 + \frac{s^2}{{z_i}^2}\right)$ and $D_{or}(s) = b_1 \prod_{i=1}^{r_2}\left(1 + \frac{s^2}{{p_i}^2}\right)$ (14)

Where r_1 and r_2 are the integral parts of r/2 and (r-2)/2 respectively. Similarly reduced numerator is represented by equ. (13) as follows:

$$N_r(s) = N_{er}(s) + N_{or}(s) \quad (15)$$

Hence, the complete reduced model of the r^{th} order is represented as follow

$$R(s) = \frac{N_R(s)}{D_R(s)} = \frac{N_{er}(s) + N_{or}(s)}{D_{er}(s) + D_{or}(s)} \quad (16)$$

It may be noted that the zeros and poles with smaller magnitude is more dominate than those zeros and poles of larger magnitude. The reduced model preserve the dominant performance of the original model.

2.4 Continued Fraction Expansion (CFE Method)

Consider the transfer function of the nth order [13] as given below:

$$G(s) = \frac{a_0 + a_1 s + a_2 s^2 + \cdots \ldots a_n s^{n-1}}{a_0 + a_1 s + a_2 s^2 + \cdots \ldots a_n s^n} \quad (17)$$

It’s often convenient to take a_n =1, because a basic control system is a low pass filter in nature and therefore upon simplification, we can handle steady state easily. We can start CFE from the constant term and arrange the polynomial in the ascending order of s and thus the transfer function can be written as:

$$G(s) = \frac{A_{21} + A_{22} s + A_{23} s^2 + \cdots \ldots A_{2,n} s^{n-1}}{A_{11} + A_{12} s + A_{13} s^2 + \cdots \ldots A_{1,(n+1)} s^n} \quad (18)$$

Then an Routh array is prepare using constants of denominator term as the 1st row and the constants of numerator as the entries in the 2nd row, followed by entries in the subsequent rows as

$$A_{j,k} = \frac{A_{j-2,1} A_{j-1,k+1}}{A_{j-1,1}} \quad (19)$$

The simplified model with the denominator of the reduced order r, is derived using the first column values of the Routh array and will be of the general form:

$$R(s) = \frac{A^*_{21} + A^*_{22} s + AA^*_{23} s^2 + \cdots \ldots A^*_{2,n} s^{n-1}}{A^*_{11} + A^*_{12} s + A^*_{13} s^2 + \cdots \ldots A^*_{1,(n+1)} s^n} \quad (20)$$

For the second order model, only 4 quotients of h_i are considered and are given as:

$$R(s) = \frac{h_2 h_3 h_4 + (h_2 + h_4)s}{h_1 h_2 h_3 h_4 + (h_1 h_2 + h_1 h_4 + h_3 h_4)s + s^2} \quad (21)$$

Where $h_1 = \frac{A_{11}}{A_{21}}$; $h_2 = \frac{A_{21}}{A_{31}}$; $h_3 = \frac{A_{31}}{A_{41}}$ $and\ h_j = \frac{A_{j,1}}{A_{j-1,1}}$

2.5 Modified Cauer Form (MCF Method)

Modified Cauer form (MCF) is one of the simplest and most attractive methods for the model reduction of the transfer functions [14]. It has many useful properties such as computational simplicity, the fittings time moments and it also preservation of steady state response for the polynomial inputs of the form $\sum a_i s^i$.

Consider a general higher order model as represented below:

$$G(s) = \frac{b_{11}+b_{12}s+b_{13}s^2+\cdots\ldots b_{1,n}s^{n-1}}{a_{11}+a_{12}s+a_{13}s^2+\cdots\ldots a_{1,(n+1)}s^{n}} \quad (22)$$

Now using the coefficients of both the numerator and denominator of G(s) and Routh Array is prepared. The successive rows are formulated using equ (23)

$$a_{i+1,j} = a_{i,j+1} - h_i b_{i,j+1} \text{ and } b_{i+1,j} = b_{i,j} - H_i a_{i+1,j} \quad (23)$$

Where $h_i = \frac{a_{i,1}}{b_{i,1}}$ and $H_i = \frac{b_{i,n+1-i}}{a_{i+1,n+1-i}}$ and i =1,2,……n.

On complete evaluating of the Routh array the inversion table is constructed to determine the reduced order model.

Elements of the inversion table are evaluated by the following relations for i =1,2,……n.

The values are given by

$$p_{i,j} = p_{i-1,j} + q_{i-,1j}h_i \; for \; j = 1,2,\ldots\ldots\ldots i \quad (24)$$

$$q_{i,j} = q_{i-1,j-1} + p_{i,j}H_i \; for \; j = 1,2,\ldots\ldots\ldots i-1 \quad (25)$$

$$l_{i,j} = l_{i-1,j} + r_{i-1,j}h_i \; for \; j = 1,2,\ldots\ldots\ldots i-1 \quad (26)$$

$$r_{i,j} = r_{i-1,j-1} + l_{i,j}H_i \; for \; j = 1,2,\ldots\ldots\ldots i \quad (27)$$

The coefficients of the reduced order transfer function denominator and numerator are evaluated as follows:

$$a_{1,i} = q_{k,i} \; for \; k = 1,2,3\ldots\ldots\ldots n-1 \; and \; b_{1,i} = r_{k,i} \; for \; i = 1,2,3,\ldots\ldots\ldots k \quad (28)$$

The reduced order model can then be represented as

$$R(s) = \frac{b_{11}+b_{12}s+\cdots\ldots\ldots b_{1,k}s^{k-1}}{a_{11}+a_{12}s+\cdots\ldots\ldots a_{1,k}s^{k-1}+s^k} \quad (29)$$

3. RESULTS AND DISCUSSION

Case 1: Consider a Single input Single output (SISO) system, whose fourth order transfer function is presented in [15] and is shown in equation 30.

$$G(s) = \frac{1.4s^3+24.8s^2+90s+120}{s^4+18s^3+102s^2+180s+120} \quad (30)$$

The proposed reduced models according to the above mentioned reduction techniques are as follows:

SRAM method: $$G(s) = \frac{s+1.3}{s^2+2s+1.3} \quad (31)$$

Balred Method: $$G(s) = \frac{1.3s+1.62}{s^2+2.21s+1.62} \quad (32)$$

MSE Method: $$G(s) = \frac{1.198s+1.253}{s^2+2.138s+1.253} \quad (33)$$

CFE Method: $$G(s) = \frac{1.195s+1.249}{s^2+2.13s+1.249} \quad (34)$$

MCF Method: $$G(s) = \frac{1.2s+1.25}{s^2+2.15s+1.25} \quad (35)$$

Above equations show the comparison of reduced order systems with the original system on the basis of integral square error (ISE) [16]. The accuracy of any reduced model is measured by calculating ISE between the transient responses of the original $(y_o(t))$ and the reduced model $(y_r(t))$ using MATLAB. The ISE should be the minimum for good approximation between the reduced model and the original model, which is given by equation 36.

$$ISE = \int_0^\infty [y_r(t) - y_o(t)]^2 dt \quad (36)$$

Figure 1 represents a block diagram for the comparison of the models, which are simulated in MATLAB/Simulink computer environment.

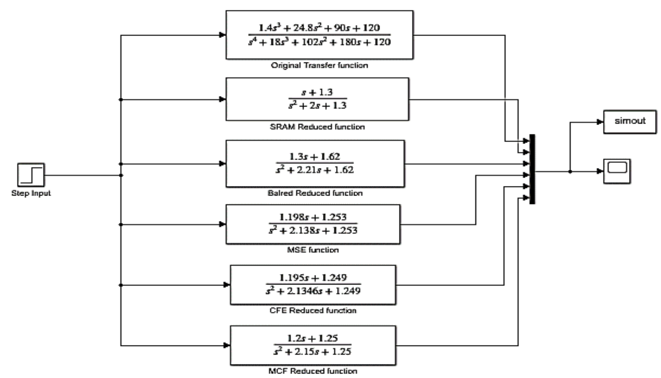

Figure 1: MATLAB/Simulink file for Case 1

Figure 2 shows the comparison of frequency response of the reduced models with original model and the response seems reasonable comparable. And figure 3 shows the step response of the original model and the reduced models. It seems that the responses are matching both in the steady state and in the transient state.

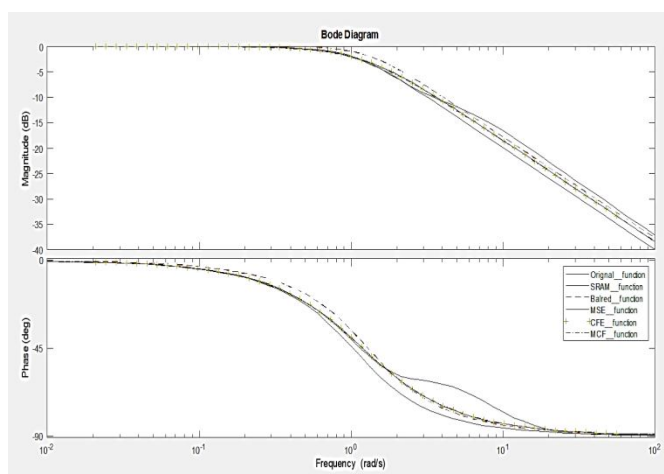

Figure 2: Comparison of frequency response for case 1

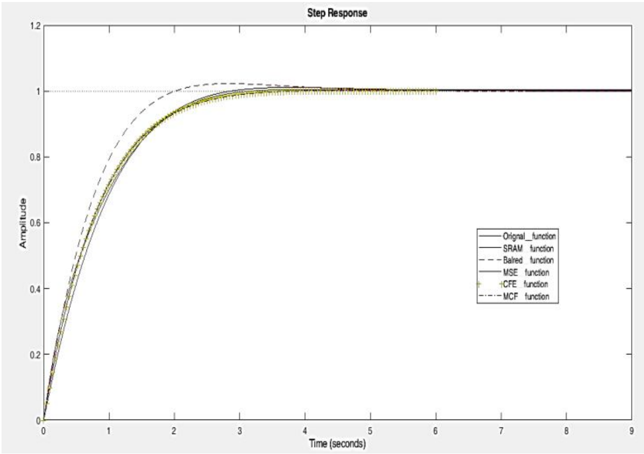

Figure 3: Comparison of step response for case 1

Table 1 shows the comparison of reduced method on the basis of other factors such as rise time, overshoot, peak and peak time, which clearly shows that all these methods give stable response and have minimum steady state error and settling time, but MCF shows much better results in terms of ISE, overshoot and peak attained. Therefore modified Cauer form (MCF) method is very effective in model reduction of LTI – SISO systems.

Table 1: Step response information of original and reduced systems of case 1.

	Rise time	**Settling time**	**Over-shoot**	**Peak**	**Peak time**
Original model	1.692	2.500	0.443	1.004	3.942
SRAM method	1.645	2.444	1.011	1.011	3.776
Balred Method	1.253	3.283	0.655	1.023	2.834
MSE Method	1.635	2.647	0.253	1.003	4.523
CFE Method	1.640	2.655	0.251	1.003	4.531
MCF Method	1.657	2.716	0.178	1.002	3.755

Case 2: Consider the higher order time invariant Multi input Multi output (MIMO) system [17], whose state space model is represented as follows:

$$A = \begin{bmatrix} -4 & -1.5 & -1.5 \\ -5 & -5.5 & -0.5 \\ -1 & 1.5 & -3.5 \end{bmatrix}; B = \begin{bmatrix} 0.5 & 0.55 \\ 1.5 & -1.35 \\ -1.5 & 0.45 \end{bmatrix};$$

$$C = \begin{bmatrix} 3 & 0 & -1 \\ 5.4 & -1.8 & -0.8 \end{bmatrix} \text{ and } D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Therefore the transfer function is given in equation 37:

Table 2 represents the reduced models using the above mentioned methods for all the transfer functions of G(s).

Table 2: Reduced models for all transfer functions

	$G_{11}(s) = \frac{3s^2 + 26}{s^3 + 13s^2 +}$	$G_{12}(s) = G_{21}(s) = \frac{1.2s^2 + 17}{s^3 + 13s^2 +}$	$G_{22}(s) = \frac{5.04s^2 + 56.}{s^3 + 13s^2 +}$
SRAM method	$\frac{2.122s + 3.8}{s^2 + 3.835s + 2}$	$\frac{1.403s + 5.2}{s^2 + 3.835s + 1}$	$\frac{4.589s + 12.01}{s^2 + 3.835s + 2.8}$
Balred method	$\frac{2.993s + 38.0}{s^2 + 6.961s + 6}$	$\frac{1.196s + 8.3}{s^2 + 5.582s + 4}$	$\frac{5.038s + 21.32}{s^2 + 6.048s + 5.0}$
MSE method	$\frac{2.962s + 7.7}{s^2 + 6.739s + 5}$	$\frac{1.186s + 8.6}{s^2 + 5.743s + 4}$	$\frac{5.038s + 21.20}{s^2 + 6.048s + 5.0}$
CFE method	$\frac{15.268s - 1}{9.144s^2 + 10.5}$	$\frac{0.1331s + 1.8}{1.002s^2 + 1.7s}$	$\frac{39.91s + 4.0}{15.242s^2 + 11.1}$
MCF method	$\frac{3s + 11.75}{s^2 + 7.485s + 8}$	$\frac{3s + 12.66}{s^2 + 4.848s + 6}$	$\frac{3.04s + 2.126}{s^2 + 0.415s + 0.5}$

$$G(s) = \begin{bmatrix} \frac{3s^2+26s+47}{s^3+13s^2+47s+35} & \frac{1.2s^2+17.2s+64}{s^3+13s^2+47s+35} \\ \frac{1.2s^2+17.2s+64}{s^3+13s^2+47s+35} & \frac{5.04s^2+56.24s+147.2}{s^3+13s^2+47s+35} \end{bmatrix} \quad (37)$$

Where the following transfer functions represents 3rd order systems.

$$G_{11}(s) = \frac{3s^2+26s+47}{s^3+13s^2+47s+35} \quad (38)$$

$$G_{12}(s) = G_{21}(s) = \frac{1.2s^2+17.2s+64}{s^3+13s^2+47s+35} \quad (39)$$

$$G_{21}(s) = \frac{5.04s^2+56.24s+147.2}{s^3+13s^2+47s+35} \quad (40)$$

Table 3(a),(b) & (c) shows the comparison of all reduced models with their respective original model on the basis of Integral square error (ISE).

Table 3(a): ISE comparison between various reduced order systems for $G_{11}(s)$

Method of order reduction	**Reduced system of** $G_{11}(s) = \frac{3s^2+26s+47}{s^3+13s^2+47s+35}$	**ISE**
SRAM method	$\frac{2.122s + 3.835}{s^2 + 3.835s + 2.856}$	0.3113
Balred Method	$\frac{2.993s + 38.073}{s^2 + 6.961s + 6.017}$	0.0077
MSE Method	$\frac{2.962s + 7.708}{s^2 + 6.739s + 5.741}$	0.0034
CFE Method	$\frac{15.268s - 1.343}{9.144s^2 + 10.578s - 1}$	0.9502
MCF Method	$\frac{3s + 11.75}{s^2 + 7.485s + 8.753}$	0.0950

Table 3(b): ISE comparison between various reduced order systems for $G_{12}(s) = G_{21}(s)$

Method of order reduction	Reduced system of $G_{12}(s) = G_{21}(s) = \frac{1.2s^2+17.2s+64}{s^3+13s^2+47s+35}$	ISE
SRAM method	$\frac{1.403s + 5.222}{s^2 + 3.835s + 2.856}$	0.0807
Balred Method	$\frac{1.196s + 8.396}{s^2 + 5.582s + 4.593}$	0.7001
MSE Method	$\frac{1.186s + 8.658}{s^2 + 5.743s + 4.734}$	0.0014
CFE Method	$\frac{-0.1331s + 1.828}{1.002s^2 + 0.0017s + 1}$	0.563
MCF Method	$\frac{3s + 12.662}{s^2 + 4.848s + 6.922}$	1.111

Table 3(c): ISE comparison between various reduced order systems for $G_{22}(s)$

Method of order reduction	Reduced system of $G_{22}(s) = \frac{5.04s^2+56.24s+147.2}{s^3+13s^2+47s+35}$	ISE
SRAM method	$\frac{4.589s + 12.01}{s^2 + 3.835s + 2.856}$	0.0521
Balred Method	$\frac{5.038s + 21.32}{s^2 + 6.048s + 5.069}$	0.0203
MSE Method	$\frac{5.038s + 21.20}{s^2 + 6.048s + 5.047}$	0.0016
CFE Method	$\frac{39.91s + 4.042}{15.242s^2 + 11.1303s + 1}$	0.216
MCF Method	$\frac{3.04s + 2.126}{s^2 + 0.415s + 0.504}$	0.363

Figure 4(a), (b) & (c) shows the comparison of frequency response of the reduced models with original model and the response seems reasonable comparable.

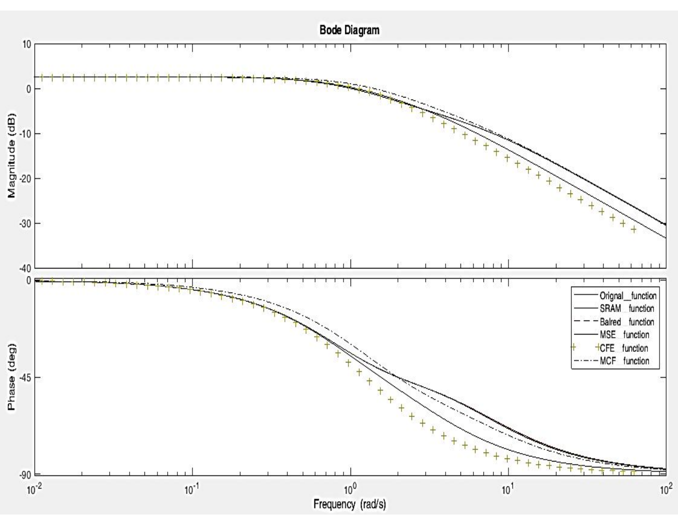

Figure 4(a): Comparison of frequency response of various reduced order systems for $G_{11}(s)$

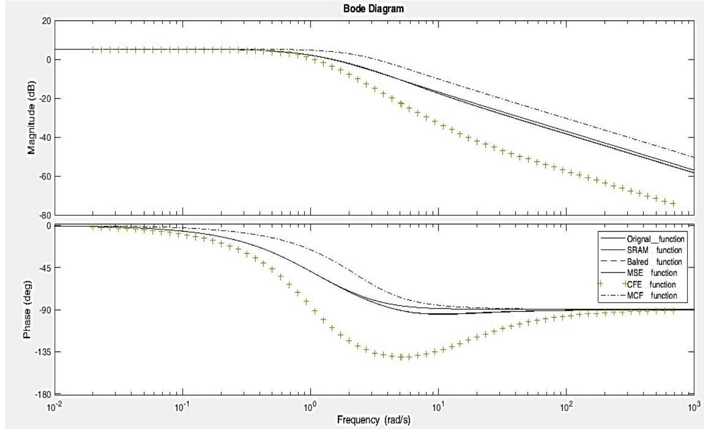

Figure 4(b): Comparison of frequency response of various reduced order systems for $G_{12}(s)$

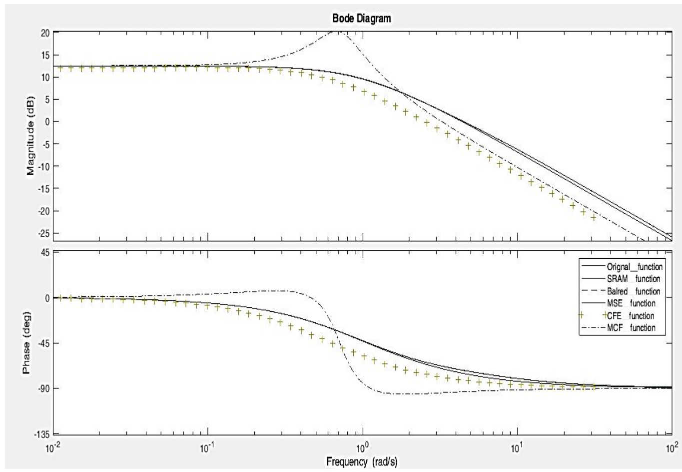

Figure 4(c): Comparison of frequency response of various reduced order systems for $G_{22}(s)$

Figure 5(a), (b) & (c) shows the step response of the original model and the reduced models. It seems that the responses are matching both in the steady state and in the transient state.

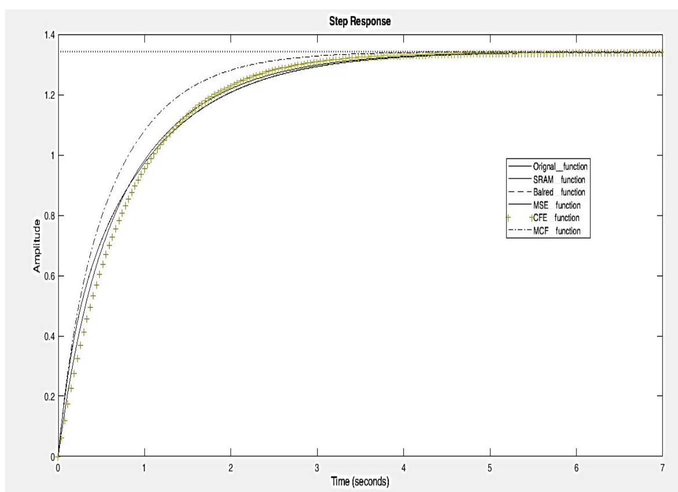

Figure 5(a): Comparison of step response for $G_{11}(s)$

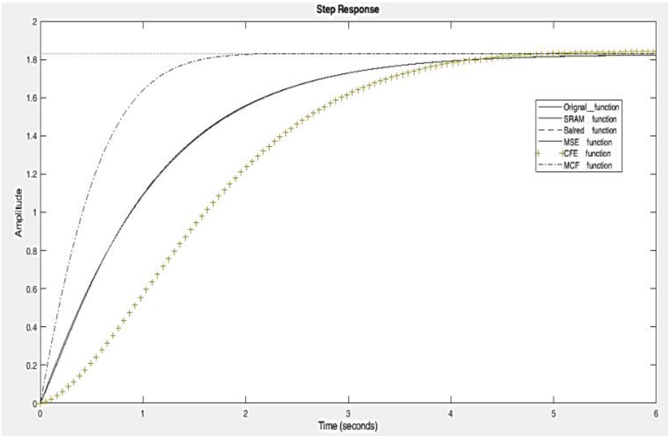

Figure 5(b): Comparison of step response for $G_{12}(s)$

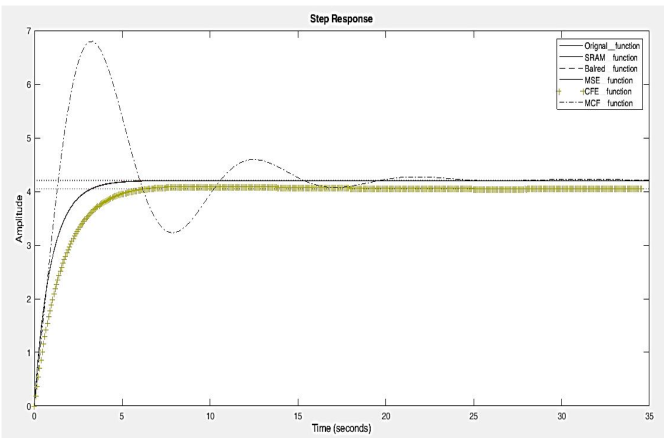

Figure 5(c): Comparison of step response for $G_{22}(s)$

Table 4(a), (b) & (c)again shows the comparison of reduced method on the basis of rise time, overshoot, peak and peak time, which clearly shows that all these methods give stable response and have minimum steady state error and settling time, but MSE shows much better results in terms of ISE, overshoot and peak attained. Therefore modified stability equation (MSE) method is very effective in model reduction of LTI – MIMO systems.

Table 4(a): Step response information of original and reduced systems of $G_{11}(s)$

	Rise time	**Settling time**	**Over-shoot**	**Peak**	**Peak time**
Original model	1.958	3.617	0.000	1.341	6.303
SRAM method	1.85	3.496	0.000	1.342	7.47
Balred Method	1.947	3.588	0.000	1.339	5.867
MSE Method	1.41	2.57	0.000	1.34	4.26
CFE Method	1.77	3.2	0.000	1.34	7.14
MCF Method	1.958	3.62	0.000	1.343	6.083

Table 4(b): Step response information of original and reduced systems of $G_{12}(s)$

	Rise time	**Settling time**	**Over-shoot**	**Peak**	**Peak time**
Original model	2.248	4.000	0.000	1.827	7.320
SRAM method	2.268	3.990	0.000	1.870	7.140
Balred Method	2.245	3.900	0.000	1.830	7.299
MSE Method	0.950	1.510	0.097	1.810	2.560
CFE Method	2.660	4.110	0.000	1.840	5.860
MCF Method	2.254	4.010	0.000	1.830	7.340

Table 4(c): Step response information of original and reduced systems of $G_{21}(s)$

	Rise time	**Settling time**	**Over-shoot**	**Peak**	**Peak time**
Original model	2.162	3.860	0.000	4.202	6.862
SRAM method	2.130	3.820	0.000	4.200	15.43
Balred Method	2.152	3.844	0.000	4.202	6.830
MSE Method	1.070	3.820	0.000	4.190	3.330
CFE Method	3.162	5.007	1.320	4.090	9.570
MCF Method	2.160	18.600	61.110	6.790	6.860

4. CONCLUSION

The higher order system in linear time variant in SISO and in MIMO systems are analyzed for its reduced order model by using various reduction techniques. The reduced model given by various methods approximately matches the time moments of the original system. The step response information with reduced systems is comparable to original higher order systems in terms of rise time, settling time, overshoot and peak time. Furthermore these reduction techniques preserve the stability in a lower order system. It's worth noticing that although all the methods are stable but it may turn out to be non-minimum phase in both SISO and MIMO systems.

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