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Study of Schrodinger Equation with Quantum Deformation for Three-Dimensional Harmonic Oscillator plus Inverse Quadratic Potential by Hypergeometric Method

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ABSTRACT

The Schrodinger equation with quantum deformation for three-dimensional Harmonic Oscillator plus Inverse Quadratic potential was obtained the solution by Hypergeometric method with substituting variable and parameter to reduce Schrodinger equation into second order Hypergeometric differential equation. The non-relativistic energy was obtained from condition a=-n and wave function was expressed by Hypergeometric function. The results were calculated and visualized by Matlab R2013a software. The energy spectrum and wave function depend on the quantum deformation and quantum number parameters.

Key words : Hypergeometric Method, Oscillator plus Inverse Quadratic potential, Quantum deformation, Schrodinger equation.

1. INTRODUCTION

The behavior of particle could be described by Schrodinger equation [1]. In Schrodinger equation, a particle was considered as electromagnetic wave where the total of energy was quantized as energy of one photon [2]. The Schrodinger equation was influenced by potential, shape invariance potential, to obtain the solution exactly. The potential was position function that was used to reduce the Schrodinger equation into differential equation of Hypergeometric function. Some potentials were used to solve the Schrodinger equation such as Kratzer, Hulthen, Coulomb, Harmonic Oscillator, Pöschl-Teller, Rosen Morse, Manning Rosen, Eckart and Woods-Saxon potentials [3-7].

Some methods were often used to solve the Schrodinger equation with shape invariance potential such as Nikiforov-Uvarov (NU), Asymptotic Iteration Method (AIM), Supersymmetry Quantum Mechanics and Hypergeometric methods [3,4,8-12].

In quantum mechanics, the anharmonic vibrational motion of molecule was the motion of harmonic mathematically. The anharmonic oscillator in quantum system consist of harmonic oscillator and inverse quadratic [14]. Several researchers had investigated the q-deformed quantum Harmonic Oscillator that describe about the vibrational and rotational motions of particles [15]. The deformation was applied in mathematical, physics and chemistry, such as nuclear and high energy physics, quantum hall effect, black holes, cosmic string, conductance of metals and semiconductors, and analyzed of phonon spectrum [16-18].

The paper is organized as follows. In section 2, the basic theory of Schrodinger equation, q-deformed quantum, and Harmonic Oscillator plus Inverse Quadratic potential. In section 3, study about Hypergeometric method, results and discussion in section 4, and finally conclusion in section 5.

2. BASIC THEORY

2.1 Schrodinger Equation

The general of time-independent Schrodinger equation with mas m and vector potential $V(r)$ was written by [10,19-20]

$$\left(-\frac{\hbar^2}{2m}\nabla^2+V(r)\right)\phi(r,\theta,\varphi)=E\phi(r,\theta,\varphi) \tag{1}$$

with the Laplacian operator was given as [13]

$$\frac{1}{r^2}\left(\frac{\partial}{\partial r}\left(r^2\frac{\partial\phi}{\partial r}\right)+\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\phi}{\partial\theta}\right)+\frac{1}{\sin^2\theta}\frac{\partial^2\phi}{\partial\varphi^2}\right) \tag{2}$$

and wave function [10,19]

$$\phi(r,\theta,\varphi)=\frac{G(r)}{r}Y(\theta,\varphi) \tag{3}$$

By inserting equations (2) and (3) into (1), the radial part of Schrodinger equation was written as

$$\left(\frac{\partial^2}{\partial r^2}-\frac{l(l+1)}{r^2}+\frac{2m}{\hbar^2}(E+V)\right)G=0 \tag{4}$$

2.2 Quantum Deformation

The form of Schrodinger equation with q-deformed quantum mechanics in equation (4) was written as

$$\left(\hat{p}^2-\frac{l(l+1)}{r^2}+\frac{2m}{\hbar^2}(E+V)\right)G=0 \tag{5}$$

where the operator of momentum and position as [17,18,21]

$$\hat{p}=-i\hbar D_r=-i\hbar\left(1+qr^2\right)\frac{d}{dr},\hat{x}=r \tag{6}$$

By using equation (6), the form of Schrodinger equation in equation (5) becomes

$$\left(D_r^2-\frac{l(l+1)}{r^2}+\frac{2m}{\hbar^2}(E+V)\right)G=0 \tag{7}$$

with

$$D_r^{q^2}=\left(1+qr^2\right)\frac{\partial^2}{\partial r^2}+2qr\left(1+qr^2\right)\frac{\partial}{\partial r} \tag{8}$$

Set the variable as [18]

$$x=\sqrt{q}r\rightarrow dr=\frac{dx}{\sqrt{q}} \tag{9}$$

By inserting equation (9) into equation (8), we have

$$D_r^{q^2}=q\left(1+x^2\right)^2\frac{\partial^2}{\partial x^2}+2qx\left(1+x^2\right)\frac{\partial}{\partial x}=\frac{\partial^2}{\partial r^2} \tag{10}$$

Equation (10) was the general equation of q-deformed quantum deformation in radial part.

2.3 Three-Dimensional Harmonic Oscillator plus Inverse Quadratic Potential

The Three-dimensional Harmonic Oscillator plus Inverse Quadratic potential was defined as [14,19,22-24]

$$V(r)=ar^2+\frac{b}{r^2} \tag{11}$$

with $a=\frac{1}{2}m\omega^2$ and b was force of external field.

By Changing the r variable in equation (11) into equation (9), the three-dimensional Harmonic Oscillator plus Inverse Quadratic potential becomes

$$V(x)=\frac{ax^2}{q}+\frac{b}{x^2}q \tag{12}$$

By inserting equations (10) and (12) into (4), we have

$$(1+x^2)^2\frac{d^2G}{dx^2}+2x(1+x^2)\frac{dG}{dx}-\frac{l(l+1)}{x^2}G$$
$$+\left(\frac{2mE}{q\hbar^2}+\frac{2m}{\hbar^2}\left(\frac{ax^2}{q^2}+\frac{b}{x^2}\right)\right)G=0 \tag{13}$$

Equation (13) would be reduced into the Schrodinger equation with variable substitution by using Hypergeometric method.

3. HYPERGEOMETRIC METHOD

The general form of Hypergeometric function was described by [4,25]

$$s(1-s)g''(s)+\left(c'-(a'+b'+1)s\right)g'(s)-a'b'g(s)=0 \tag{14}$$

The solution of wave function of Hypergeometric function was given as [1,8-9]

$$g(s)={}_2F_1(a',b'c';s)=\sum_{s=0}^{\infty}\frac{(a')_n(\mathrm{b}')_n}{(\mathrm{c}')_n}\frac{s^n}{n!} \tag{15}$$

The solution of equation (15) has the form of polynomial rank n with $a'=-n$ or $b'=-n$.

4. RESULT AND DISCUSSION

The equation (13) would be obtained the solution by changing the variable as

$$x=\tan y\rightarrow dx=\sec^2 ydy \tag{16}$$

Equation (13) becomes

$$\frac{d^2G}{dy^2}+\left(\frac{\left(l(l+1)+\frac{2mb}{\hbar^2}\right)}{\tan^2 y}+\frac{2ma}{q\hbar^2}\tan^2 y+\frac{2mE}{q\hbar^2}\right)G=0 \tag{17}$$

or

$$\frac{d^2G}{dy^2}-\left(\frac{\left(l(l+1)-\frac{2mb}{\hbar^2}\right)}{\sin^2 y}-\frac{\frac{2ma}{q\hbar^2}}{\cos^2 y}\right)G$$
$$-\left(l(l+1)-\frac{2mb}{\hbar^2}+\frac{2ma}{q\hbar^2}+\frac{2mE}{q\hbar^2}\right)G=0 \tag{18}$$

By setting

$$\eta(\eta-1)=\left(l(l+1)-\frac{2mb}{\hbar^2}\right)\rightarrow\eta=\frac{1\pm\sqrt{1+4\left(l(l+1)-\frac{2mb}{\hbar^2}\right)}}{2} \tag{19}$$

$$\upsilon(\upsilon-1)=\frac{2ma}{q\hbar^2}\rightarrow\upsilon=\frac{1\pm\sqrt{1+4\left(\frac{2ma}{\hbar^2}\right)}}{2} \tag{20}$$

$$E'=\left(l(l+1)-\frac{2mb}{\hbar^2}+\frac{2ma}{q\hbar^2}+\frac{2mE}{q\hbar^2}\right) \tag{21}$$

Equation (18) becomes

$$\frac{d^2G}{dy^2}-\left(\frac{\eta(\eta-1)}{\sin^2 y}-\frac{\upsilon(\upsilon-1)}{\cos^2 y}\right)G-E'G=0 \tag{22}$$

The form of equation (22) like the general form of second order differential Schrodinger equation with Pöschl-Teller potential [1,3] that could be solved by Hypergeometric method.

By using variable approximation as

$$\cos^2 y=s\rightarrow\frac{ds}{dy}=-2\sin y\cos y=-2\sqrt{s(1-s)} \tag{23}$$

By inserting equations (23) into (22), we have

$$s(1-s)\frac{d^2G}{ds^2}+(1-2s)\frac{dG}{ds}+\left(\frac{E'}{4}-\frac{\eta(\eta-1)}{4(s-1)}-\frac{\upsilon(\upsilon-1)}{s}\right)G=0 \tag{24}$$

The wave function of Hypergeometric was given as

$$G=s^{\alpha}(1-s)^{\beta}g(s) \tag{25}$$

By substituting equations (25) into (24), we have

$$s(1-s)g''+\left(\left(2\alpha+\frac{1}{2}\right)-(2\alpha+2\beta+1)s\right)g'+\left(\frac{E'}{4}-(\alpha+\beta)^2\right)g=0 \tag{26}$$

By comparing equations (14) and (26), we have the parameters

$$a'+b'=2\alpha+2\beta \tag{27}$$

$$a'=(\alpha+\beta)-\frac{\sqrt{E}}{2} \tag{28}$$

$$b'=(\alpha+\beta)+\frac{\sqrt{E}}{2} \tag{29}$$

$$c=2\alpha+\frac{1}{2} \tag{30}$$

$$\alpha=\frac{\eta}{2}=\frac{1\pm\sqrt{1+4\left(l(l+1)-\frac{2mb}{\hbar^2}\right)}}{4} \tag{31}$$

$$\beta=\frac{\upsilon}{2}=\frac{1\pm\sqrt{1+4\left(\frac{2ma}{\hbar^2}\right)}}{4} \tag{32}$$

The energy equation was obtained as

$$E_n=(\eta+\upsilon+2n)^2 \tag{33}$$

with $E'=E_n$ in equation (21), equation (33) becomes

$$E=\frac{q\hbar^2}{2m}\left(\begin{array}{l}\left(\left(\frac{1\pm\sqrt{1+4\left(l(l+1)-\frac{2mb}{\hbar^2}\right)}}{2}\right)+\left(\frac{1\pm\sqrt{1+4\left(\frac{2ma}{\hbar^2}\right)}}{2}\right)+2n\right)^2\\ -l(l+1)+\frac{2mb}{\hbar^2}-\frac{2ma}{q\hbar^2}\end{array}\right) \tag{34}$$

Equation (34) was analytic energy equation for solution of Schrodinger equation with quantum deformation on Three-dimensional Harmonic Oscillator plus Inverse Quadratic potential. The numerical result of energy equation in equation (34) was calculated by Matlab R2013a software that were showed in Table 1.

Table 1: The Energy for Schrodinger Equation with Quantum Deformation on Three-Dimensional Harmonic Oscillator plus Inverse Quadratic Potential with $a=b=0.05$

q	n	E	n	E	n	E
0.01	1	0.23448	2	0.40784	3	0.62119
0.04		0.66221		1.23218		1.96215
0.07		1.04547		1.99214		3.21880
0.1		1.41423		2.73109		4.44796

0.4		4.92280		9.86152		16.40023
0.7		8.36378		16.89408		28.22438
1		11.79115		23.90695		40.02274

In Table 1 is shown that the energy of the system that is affected by q-deformed quantum are the positive values. In this case, the increase of radial quantum number causes the increase of energy spectrum. The increase of quantum deformation causes the increase of energy spectrum.

Furthermore, the wave function was obtained by inserting equations (23), (38-32) into (25) that was given as

$$G=\left(\cos^2 y\right)^{\left(\frac{1+\sqrt{1+4\left(l(l+1)-\frac{2mb}{\hbar^2}\right)}}{4}\right)}\times \left(\sin^2 y\right)^{\left(\frac{1+\sqrt{1+4\left(\frac{2ma}{\hbar^2}\right)}}{4}\right)} {}_2F_1\left(-\mathrm{n},2\alpha+2\beta+1,2\alpha+\frac{1}{2};\cos^2 y\right) \tag{35}$$

with a and b were parameters of potential.

The wave function equation was expressed as a function of quantum number that is shown in Table 2.

Table 1: The Wave Function for Schrodinger Equation with Quantum Deformation on Three-Dimensional Harmonic Oscillator plus Inverse Quadratic Potential with Quantum Number Variations

n	G_n
0	$G_0=\left(\cos^2 y\right)^{\left(\frac{1+\sqrt{1+4\left(l(l+1)-\frac{2mb}{\hbar^2}\right)}}{4}\right)}\times \left(\sin^2 y\right)^{\left(\frac{1+\sqrt{1+4\left(\frac{2ma}{\hbar^2}\right)}}{4}\right)} C'$
1	$G_1=-\left(\cos^2 y\right)^{\left(\frac{1+\sqrt{1+4\left(l(l+1)-\frac{2mb}{\hbar^2}\right)}}{4}\right)}\times \left(\sin^2 y\right)^{\left(\frac{1+\sqrt{1+4\left(\frac{2ma}{\hbar^2}\right)}}{4}\right)} C'\left(2\alpha+\frac{1}{2}\right)\left(1-\frac{(2\alpha+2\beta+2)}{\left(2\alpha+\frac{1}{2}\right)}\left(\cos^2 y\right)\right)$
2	$G_2=\left(\cos^2 y\right)^{\left(\frac{1+\sqrt{1+4\left(l(l+1)-\frac{2mb}{\hbar^2}\right)}}{4}\right)}\times \left(\sin^2 y\right)^{\left(\frac{1+\sqrt{1+4\left(\frac{2ma}{\hbar^2}\right)}}{4}\right)} C'\left(2\alpha+\frac{1}{2}\right)\left(2\alpha+\frac{3}{2}\right)\times \left(1-\frac{2(2\alpha+2\beta+3)}{\left(2\alpha+\frac{1}{2}\right)}\left(\cos^2 y\right)+\frac{(2\alpha+2\beta+3)(2\alpha+2\beta+4)}{\left(2\alpha+\frac{1}{2}\right)\left(2\alpha+\frac{3}{2}\right)}\left(\cos^2 y\right)^2\right)$

and the wave function are visualized by Matlab R2013a software that shown in Figure 1., Figure 2., and Figure 3.

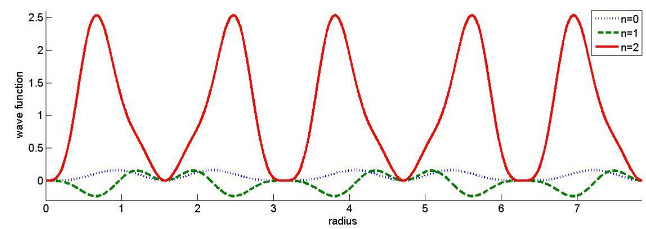

Figure 1: Wave Function of Schrodinger Equation for Three-Dimensional Harmonic Oscillator plus Inverse Quadratic Potential with $q = 0.01$

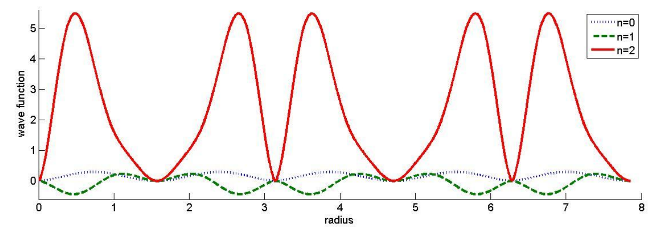

Figure 2: Wave Function of Schrodinger Equation for Three-Dimensional Harmonic Oscillator plus Inverse Quadratic Potential with $q = 0.1$

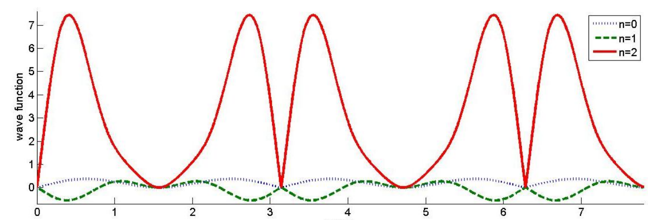

Figure 3: Wave Function of Schrodinger Equation for Three-Dimensional Harmonic Oscillator plus Inverse Quadratic Potential with $q = 1$

Figure 1, Figure 2 and Figure 3 show about the visualization of wave function for Schrodinger equation with radial quantum number and quantum deformation variations. The wave functions have uniform period.

The form of wave function is influenced by potential. The effect of Inverse Quadratic potential is higher than the Harmonic Oscillator potential. There is a splitting the wave function in the upper and lower parts. The wide of wave function for upper and lower parts were different. The wide of half wavelength of the upper part is small but the lower part is large. Both the increase of radial quantum number and quantum deformation cause the increase of amplitude. The amplitude of wave function in the lower part is higher than the upper part.

5. CONCLUSION

The Schrodinger equation for Three-Dimensional Harmonic Oscillator plus Inverse Quadratic potential with quantum deformation was solved by using Hypergeometric method. The energy spectrum of system and wave function was obtained from the nonrelativistic energy equation and hypergeometric function equation, respectively, which were calculated numerically and visualized by using Matlab R2013a software. The increase of radial quantum number and quantum deformation cause the increase of energy spectrum, while the wave function was influenced by potential, quantum deformation and radial quantum number parameters.

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