ISSN 2278-3091
Volume 9 No.2, March - April 2020
International Journal of Advanced Trends in Computer Science and Engineering
Available Online at http://www.warse.org/IJATCSE/static/pdf/file/ijatcse150922020.pdf
https://doi.org/10.30534/ijatcse/2020/150922020

A High-Performance Computing of Internal Rate of Return using a Centroid-based Newton-Raphson Iterative Algorithm

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ABSTRACT

A popular financial metric in estimating the profitability of a project or investment is the internal rate of return. However, the IRR variable cannot be easily isolated from the equation. This is effectively solved by using iterative root-finding algorithms, some of the most frequently used of which are secant, bisection, false position, and Newton-Raphson algorithm. Although the Newton-Raphson method is considered to be the fastest to converge and the most popular method, it still requires an initial guess value from the user, which could result in the algorithm to not converge to the root if the user input is far from the actual root. This issue is addressed by a midpoint-based Newton-Raphson technique, which sets the midpoint of cash flows as the initial guess input. However, the midpoint technique is static as it does not adjust with unequal cash flows. This study presents a centroid-based Newton-Raphson algorithm in estimating IRR, which dynamically takes into consideration the values of cash flows. The experimental results show that the proposed algorithm ensures convergence by producing an initial IRR with an accuracy of 91.41%. This indicates that it is 26.75% more accurate in approximating the initial IRR than the midpoint-based Newton-Raphson algorithm. It also reduced the required iterations of convergence by 35.33% over the midpoint-based Newton-Raphson algorithm. These findings show that the employment of the centroid-based Newton-Raphson algorithm in approximating IRR provides a significantly better approach in evaluating investments than the current method.

Key words: Convergence, initialization, IRR, Newton-Raphson algorithm, Root-finding algorithm

1. INTRODUCTION

One of the most notable benefactors of fast-paced innovation in technology is the finance sector [20]. With so many options of projects and ventures for investment, it is necessary to have an informed decision to achieve the maximum profit possible [6], [8]. This is why financial metrics are used to estimate the viability of an investment, and one of the most popular investment productivity measuring tools is the internal rate of return (IRR) [15]. IRR estimates the attractiveness of a potential investment venture by calculating the minimum rate where the net present value of the said investment is zero [22]. However, there is no analytical method that could find the IRR value. Researchers have employed different techniques in finding solutions for nonlinear functions such as the IRR function. A simplified closed-form approach was recommended in solving IRR [17], however, it returned a low accuracy rate.

One study [23] devised a free n-point iterative technique of optimal order of convergence using rational interpolate. However, this technique requires a primary value from the user which could lead to non-convergence when the value is far from the true root. Another study [21] used the Halley technique, the Newton approach, and the combination of the Newton technique, the Newton inverse scheme, and the Halley technique in dealing with nonlinear functions. Although the latter method is composed of three techniques, the likelihood of a divergence still exists because it still requires a primary value from the user. A modified regula-falsi method (MRFM) was proposed [2] which was applied to photovoltaic applications and showed it is has a fast convergence. However, MRFM can still cause non-convergence due to the method's requirement of two initial guess inputs.

In the paper [10], an exponential interpolation and corrected secant formulas were introduced which achieved accurate results and can estimate the unknown. However, the formulas are complex for solving IRR, by hand calculation or computer program. Another mathematical formula [3] was devised which claimed to precisely calculate IRR. Nevertheless, such a method assumes just a single amount of the initial project return and a single amount of cost, which is not always true in real-world scenarios where almost always there is more than one value for each year in the project's life.

Given the complexity of solving function IRR, a fuzzy approach was proposed [5]. However, the result of the study could not be scrutinized as the authors did not present the speed and accuracy of the said method. Another approach used Particle Swarm Optimization (PSO) and Ant Colony Optimization (ACO) algorithms to approximate IRR[16], but the resulting speeds and accuracy levels were not good enough. Another technique in determining IRR is by applying bisection and secant methods [18], however, both these algorithms are slow in converging, if they converge at all.

The study of [11] developed a solution in finding IRR and called it a simplified IRR approach. It presupposes cash inflows to be identical and positive, which is practically not the case in real scenarios. Moreover, the scheme is essentially complicated and would still require a root-finding method.
Prominent root-finding algorithms are bisection, false position, and secant, and Newton-Raphson method[1], [9], [18]. Among these methods, Newton-Raphson is the most preferred technique because of its quick convergence and level of accuracy rate [7], [19]. However, this technique requires an initial value from the user which could lead to division by zero and uncertain convergence when the value is far from the true root [4], [12], [15]. The study of [14] tried to address this matter by generating the midpoint of cash flows and set it as the primary guess IRR. This approach proved to be effective in improving the speed and accuracy of the newton-raphson algorithm. However, the midpoint approach is static and the accuracy, as well as the number of iterations, can still be further improved using centroid.

Thus, in the next section, we present an alternative way of setting the initial IRR ina newton-raphson algorithm that further enhances its computation speed and accuracy.

2. METHODOLOGY

This study shows the performance of the centroid-based Newton-Raphson algorithm (CNRA), an enhancement of the midpoint-based Newton-Raphson algorithm (MNRA), in real scenarios.

2.1 Setting up the Centroid-Based Newton-Raphson Algorithm

Figure 1: Generic Newton-Raphson algorithm for calculating IRR

A generic process of a newton-raphson algorithm in estimating IRR, as shown in Figure 1, would require an initial guessed IRR from the user and then employ the equation [7],

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)} \tag{1}$$

MNRA simplified the process of calculating for the IRR, presented in Figure 2, by setting the initial guess IRR with the midpoint of cash flows instead of requiring input from the user [14].

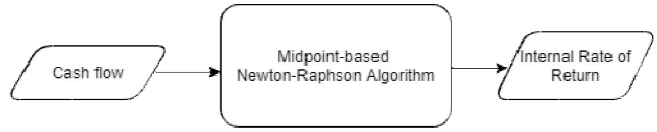

Figure 1: MNRA process for calculating IRR

To calculate for the initial IRR, MNRA employs $(((n - 1)) \ / \ 2) + 1$ of the midpoint of periods of cash flows which is then supplied to the equation,

$$IRR \leftarrow \frac{\sum_{i=1}^{n} C_i}{|C_0|}^{\frac{1}{((n-1)/2)+1}} - 1 \tag{2}$$

where:
n= number of cash flows;
C_i= subsequent cash flows;
C_0= initial investment.

This method optimized the original newton-raphson by giving results that are close to the true solution with fast convergence and high accuracy. Despite this, the midpoint becomes static when there are more varying values of cash flows. This is easily addressed by using the centroid of cash flows instead of its midpoint. The MNRA served as the reference point of CNRA.

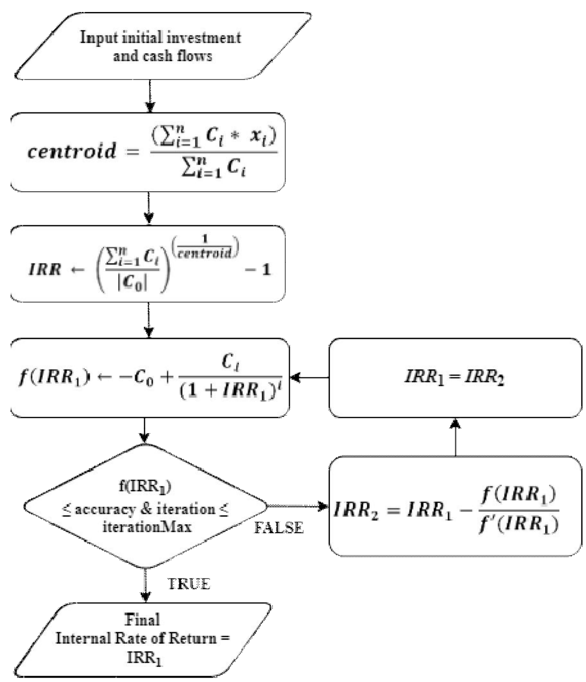

Figure2. Process of the Newton-Raphson algorithm with a centroid-based approach in calculating IRR [13]

Based on the process of the centroid approach presented in Figure 3, the centroid of time periods of cash flows is calculated with the equation,

$$centroid \leftarrow \frac{(\sum_{i=1}^{n} C_i * x_i)}{\sum_{i=1}^{n} C_i} \tag{3}$$

where:
C_i= subsequent cash flows
x_i = distance of C_i from the time of first investment.

The centroid value is then supplied to the equation to calculate the initial IRR as displayed in the equation below,

$$IRR_1 \leftarrow \left(\frac{\sum_{i=1}^{n} C_i}{|C_0|}\right)^{\left(\frac{1}{centroid}\right)} - 1 \quad (4)$$

where:
C_0 = initial investment.

2.2 Test Case Analysis

The simulation for this study uses two test cases. The first test case is a residential farm lot amortization, a part of the dataset of which is presented in Table 1. It is a 6-year installment plan, where payments are made at the end of each month for 72 months, of land priced at 820,000.00 in Philippine peso. This amount is the value for variable $C_0$and will be set to negative. The remainder of the data in the said table consists of the values for variables C_1 to C_{41}, respectively.

Table 1: Part of Residential Farm Lot Amortization Dataset (Test case A)

Month	Payment (in PHP)		Month	Payment (in PHP)
0	-820,000.00		21	12,000.00
1	20,000.00		22	12,000.00
2	20,000.00		23	12,000.00
3	20,000.00		24	12,000.00
4	20,000.00		25	15,000.00
5	20,000.00		26	15,000.00
6	20,000.00		27	15,000.00
7	12,000.00		28	15,000.00
8	12,000.00		29	15,000.00
9	12,000.00		30	15,000.00
10	12,000.00		31	15,000.00
11	12,000.00		32	15,000.00
12	12,000.00		33	15,000.00
13	12,000.00		34	15,000.00
14	12,000.00		35	15,000.00
15	12,000.00		36	15,000.00
16	12,000.00		37	12,000.00
17	12,000.00		38	12,000.00
18	12,000.00		39	12,000.00
19	12,000.00		40	12,000.00
20	12,000.00		41	12,000.00

The rest of the data, which are not included in the table, are as follows; a payment of 12,000.00 for each month until the 44th month, which are the values for variables C_{42} to C_{44}; C_{45} to C_{56} have the value 10,000.00 each; the amount 9,000.00 is the value for variables C_{57} to C_{70}, which are the payments for the 57th to 70th months, respectively; lastly, the amount 5,000.00 is for the 71st and 72nd months, which are the values of variables C_{71} to C_{72}.

The second test case, shown in Table 2, is from the cash flows of a real company named herein, for confidentiality, ABC Company for 40 years. The cash flow is represented by the variable C_i where *i* is the year of each cash flow.

Table 2: ABC Company Cash Flow Dataset (Test Case B)

Year	Cash Flow (in PHP)		Year	Cash Flow (in PHP)
0	-2,400,000.00		21	7,440,000.00
1	-1,500,000.00		22	7,440,000.00
2	-1,500,000.00		23	7,440,000.00
3	-1,200,000.00		24	7,440,000.00
4	-1,200,000.00		25	7,440,000.00
5	2,000,000.00		26	11,940,000.00
6	1,800,000.00		27	11,940,000.00
7	1,800,000.00		28	11,940,000.00
8	1,800,000.00		29	11,940,000.00
9	1,800,000.00		30	11,940,000.00
10	2,640,000.00		31	33,600,000.00
11	2,640,000.00		32	33,600,000.00
12	2,640,000.00		33	33,600,000.00
13	2,640,000.00		34	33,600,000.00
14	4,200,000.00		35	40,320,000.00
15	4,200,000.00		36	48,384,000.00
16	4,200,000.00		37	58,060,800.00
17	4,200,000.00		38	69,672,960.00
18	4,200,000.00		39	83,607,552.00
19	4,200,000.00		40	90,000,000.00
20	4,200,000.00			

Unlike the dataset in Table 1, which is of amortization, the dataset as shown in Table 2 is of investment and income. The negative values represent investments or losses, while the positive values are those of income and are gradually increasing.

3. RESULTS AND DISCUSSION

This section presents the simulation results of the CNRAand its comparison toMNRA, in terms of accuracy and number of iterations.The percentage of error in calculating the initial IRR is presented by computing the difference between the initial IRR and the final IRR over said final IRR.

$$Initial\ IRR\ Error = \frac{|IRR_1 - IRR_f|}{IRR_f} \quad (5)$$

3.1 Results on Accuracy

Based on Figures 4(a) and 4(b), the calculated centroid for test case A is 32.16820276497696 while the midpoint formula generated a value of 37, and the computed centroid for test case B is 33.64441721373592 while the midpoint is 21.0, respectively. This indicates that, in the centroid approach, the bigger the amount of cash flow, the more the center goes toward such cash flow. This centroid method is dependent on the amounts of cash flows and not simply on the midpoint of the periods, regardless of the sizes of cash flows, as in the case of the midpoint approach.

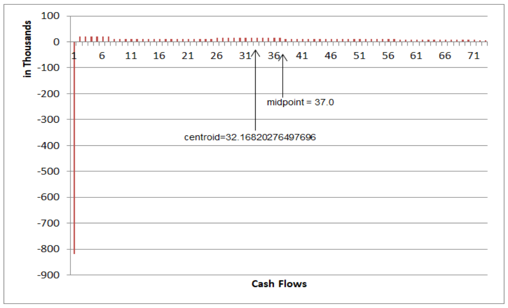

Figure 4(a): Centroid vs. Midpoint of Test Case A

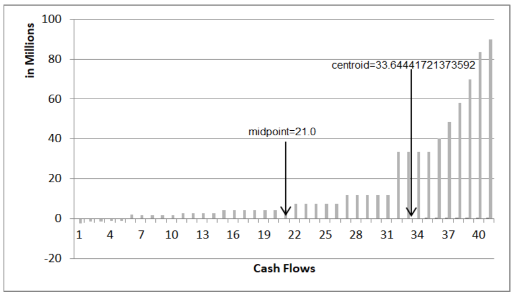

Figure 4(b): Centroid vs. Midpoint of Test Case B

As shown in Table 3, CNRA generated an initial IRR of 0.00176999956426726 which is much closer to the final IRR than the initial IRR of MNRA which is 0.00153867915934657. With its low error of initial IRR, CNRA garnered a higher accuracy rate than MNRA. It is also improved the accuracy rate by 15%.

Table 3: Accuracy of the Computation of Initial IRR of Test Case A

Method	IRR	Initial IRR	Error of Initial *IRR*	Accuracy
MNRA	0.00179047641749015	0.00153867915934657	0.14063143	85.94%
CNRA	0.00179047641748805	0.00176999956426726	0.01143654	98.86%
Initial IRR Accuracy Improvement Percentage				15%

In the case of test case B, presented in Table 4, MNRA produced an error of 0.4170646079 in calculating the initial IRR while CNRA caused a much lower error value of 0.1604498968. CNRA also delivered a much higher accuracy of 83.96%, in producing the initial IRR, than MNRA which only garnered 58.29%.

Table 4: Accuracy of the Computation of Initial IRR of Test Case B

Method	IRR	Initial IRR	Error of Initial *IRR*	Accuracy
MNRA	0.216735223	0.30712781386997	0.4170646079	58.29%
CNRA	0.21673522304876416	0.18196007887465182	0.1604498968	83.96%
Initial IRR Accuracy Improvement Percentage				44%

In either case, the CNRA has better accuracy than the MNRA. At an average, the centroid approach calculates the initial IRR closer to the true IRR with an accuracy of 91.41% compared to the MNRA's accuracy rate of 72.12%. This indicates that CNRA is 26.75% better than the midpoint approach in estimating the initial IRR. This study, however, does not make a general claim of convergence for all test problems.

3.2 Results on Iteration

Comparing the resulting iterations needed for both methods in generating the initial IRR, as shown in Table 5, the centroid approach delivers a 33% improvement in test case A as well as a 37.5% improvement in test case 2. This indicates that at an average, CNRA reduced the iteration by 35.25% over MNRA.

Table 5: Iteration of the Computation of Initial IRR of Midpoint Approach and Centroid Approach

Test Case	Method	Iterations
Farm Lot Amortization (Test case A)	MNRA	3
	CNRA	2
ABC Company (Test case B)	MNRA	8
	CNRA	5
Average iteration improvement of CNRA		35.25%

4. CONCLUSION AND FUTURE WORK

The experimental results showed that the centroid-based Newton-Rapshon algorithm outperformed the midpoint-based Newton-Raphson regarding the accuracy in generating the initial IRR in both test cases. CNRA further ensures convergence as the generated initial IRR is closer to the true root, at an average, by26.75% than the MNRA. Furthermore, the iterations required for convergence was reduced at an average rate of 35.35%. The results also showed that CNRA performed even better when there are more varying values in the dataset because of its ability to adjust to the changes in cash flows.

This study recommends further experimentation of the CNRA algorithm in other test scenarios, for instance, in investment decision making and in determining realistic interest rates in compliance with the Truth in Lending Act. The algorithm could also be incorporated in a decision-support system for investment evaluation. Further research may also be made where a variable step length is considered.

ACKNOWLEDGEMENT

We would like to thank the Commission of Higher Education (CHED) of the Philippines for their support for this scholarly study.

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