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Mathematical Investigation and Computers Simulation of Plant Protection Process Taking into Account the Temporary Age Structure and Arbitrary Tropic Functions

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ABSTRACT

The problem of describing the process of protecting agricultural crops in mathematical modeling is relevant. Today, the problem of plant protection attracts the attention of a large circle of scientists in connection with its promising use in priority areas of development of science, technology and agriculture. Mathematical modeling of the process of protecting the planned crop is one of the main tools in predicting the state of natural systems and managing them. One of the most important national economic, social and environmental problems at present is the improvement of systems for protecting agricultural crops from pests. Mathematical modeling of the process of protecting the planned harvest obliges to improve regional systems for protecting cotton on the basis of a targeted ecological and biological study, revealing the specifics of the formation and development of agro ecosystems in intensive crop production. The main task of the integrated method of combating agricultural pests of the plant protection process is the management of agrocenoses of “harmful insects” and “beneficial insects” of species based on the use of biological, chemical methods as a means or a management tool.The use of mathematical methods and computer software products to solve the problem of protecting the planned crop significantly increases the efficiency of planned and economic work and ensure optimal results. In this paper, we consider a mathematical model of the plant protection process, taking into account the temporal age structure and arbitrary trophic functions, formulate the task of plant protection. Necessary and sufficient conditions for the solvability of the plant protection problem with arbitrary trophic functions are found.

Key words: arbitrary tropic function, bio-system, mathematical and computer modeling, plant protection

1. INTRODUCTION

Protecting the planned agricultural crop around the world is one of the most important government tasks. The development of methods for protecting crops from agricultural pests naturally requires forecasting the dynamics of biological populations, communities and ecosystems under certain anthropogenic influences [2]-[3]. At the same time, experiments on real systems are very expensive, lengthy and often unacceptable, so there is a need to develop various kinds of mathematical models. With the help of mathematical models, it became possible to experimentally study the consequences of certain planned measures affecting the functioning of natural systems, direct experiments with which are unacceptable [4]-[5].

It is known that one of the main problems of agriculture is the effective control of agricultural pests, which includes two tasks [16]. The first - based on the available information about agrocenosis, the thresholds of harmfulness of pests and the efficiency levels of entomophages are determined [8]-[9]. Moreover, these parameters are determined locally by entomologists according to the accounting of a certain field (usually on 100 plants). The results obtained then apply to the remaining areas. The second task is the use of pesticides to suppress pest numbers. It is clear that such a method for determining the parameters of the integrated control method (a combination of agrotechnical, biological and chemical methods of control) due to lack of information is not accurate and will not reflect the real picture in the studied agrocenosis [12]. Therefore, the task arises of formalizing the process of determining the thresholds of harmfulness of pests and the efficiency levels of entomophages (in practice, this task is usually called a preparatory task in relation to other control methods). Mathematical modeling of the dynamics of the number of biological populations has a rather long history [1]-[6].

The construction and study of mathematical models is one of the most common methods of scientific knowledge. Today mathematical modeling is becoming an effective research tool in almost every field of science. Using this method, interesting and important results were obtained in biophysics, biochemistry, microbiology, population genetics, and ecology, which are reflected in numerous articles and monographs [13].

Statement of problems related to the construction of mathematical models of the dynamics of the number of biological populations and the stability of biological communities a number of works by domestic and foreign scientists are devoted to [14]-[15]. The remarkable work of a number of other scientists should also be noted here Vito Volterra, A.D. Lotki, R. May, Y. Svirezheva, D.O. Logofeta, M.K. Yunusi [7]-[10] and a number of other scientists.

2. MATHEMATICAL MODEL OF PLANT PROTECTION, TAKING INTO ACCOUNTTHE TIME-AGE STRUCTURE AND WITH ARBITRARY TROPHIC FUNCTIONS

Consider a model biosystem with three trophic levels of the plant type - “harmful insects” - “beneficial insects”, into which an external resource N_0 (fertilizer or water used for irrigation, or solar energy) enters at a speed of Q. The biomass (or abundance) of the corresponding levels will be denoted by $N_i,\ N_i = N_i(t),\ i=\overline{0,3},$ where $N_0 = N_0(t) -$ is the mass of the external resource at the moment of time t, $N_1 = N_1(t) -$ the biomass of the crop plants at the moment of time, $N_i = N_i(a,t) -$ the number $(i=2)$ of harmful and $(i=3)$ beneficial insects of age a at the moment of time.

Suppose that the state of the model agrocenosis is described using the following relationships [11]-[17]:

$$\begin{cases} \dfrac{dN_0}{dt} = Q - \alpha_0 N_0 N_1, \\ \dfrac{dN_1}{dt} = k_0\alpha_0 N_0 N_1 - V_1(N_1)\tilde{N}_2 - m_1 N_1, \\ \dfrac{\partial N_2}{\partial t} + \dfrac{\partial N_2}{\partial a} = k_1 V_1(N_1) N_2 - V_2(N_2)\tilde{N}_3 - m_2 N_2, \\ \dfrac{\partial N_3}{\partial t} + \dfrac{\partial N_3}{\partial a} = k_2 V_2(N_2) N_3 - \varepsilon N_3^2 - m_3 N_3, \\ N_i\big|_{t=0} = N_i^0(a), \qquad N_i(0,t) = \displaystyle\int_{\alpha_i}^{\beta_i} B_i(\xi) N_i(\xi,t)\, d\xi, \quad i=2,3, \\ \tilde{N}_i = \displaystyle\int_{\bar{\alpha}_i}^{\bar{\bar{\alpha}}_i} N_i(a,t)\, da, \quad i=2,3. \end{cases} \tag{1}$$

Where Q, α_0, m_i , k_i, ε are the biological parameters of the populations included in the agrocenosis (Q = Q (t) is the rate of external resource intake; m_i – are the averaged coefficients of natural mortality, , $i = 1,2,3$; k_i -are the fractions of consumed biomass for reproductive metabolism and growth; α_0 – is the coefficient of trophic functions; ε- is the coefficient of self-limitation of the population of beneficial insects), $V_i(.)$ -is a trophic function with the following properties: $\frac{dv_i(N)}{dN} > 0$, $\frac{d^2v_i(N)}{dN^2} \le 0$, $i = 1,2$. . $B_i(.)$ Are the birth rates of harmful and beneficial insects, i =2,3, $\tilde{N}_i = \tilde{N}_i(t)$ are the total numbers of harmful and beneficial insects, respectively as in (1), for those ages that damage crops and destroy pests (for point models $\tilde{N}_i = N_i(t)$) i =2,3, t is the time $t \in [0, t_k], t_k = const < \infty, a$ a is the age, $a \in [0, \infty)$.

Suppose that B_i (.)≥ 0, $i = 2,3$.

$\tilde{N}_i = \tilde{N}_i(t) = \int_{\alpha_i}^{\beta_i} N_i(a,t)da$, $\alpha_i, \beta_i = const > 0$, $i = 2,3$.

Definition. The average biomass of the plant (or the average yield) at time τ is the value $N_1^\tau = \frac{1}{\tau}\int_0^\tau N_1(t)dt$, $i = 2,3$, $\tau > 0$.

Following the works [6]-[10], we formulate the problem of plant protection in terms of model agrocenosis (1).

We introduce the notation $\tilde{N}_i(t) = \dfrac{1}{\tau}\int_0^\tau \tilde{N}_i(t)dt$, $i = 2,3$, $\tau > 0$.

It is required to find the number N_2^p, N_3^p, such that $\frac{1}{\tau}\int_0^\tau \tilde{N}_2(t)dt \le N_2^P$, $\frac{1}{\tau}\int_0^\tau \tilde{N}_3(t)dt \ge N_3^P$ and for which the condition $\frac{1}{\tau}\int_0^\tau N_1(t)dt \ge N_1^P$, $N_1^P \in [N_1^{min}, N_1^{max}]$, where

N_1^p –the set value of the planned crop plant biomass, N_2^p, N_3^p – the corresponding thresholds of harmfulness of pests and efficiency levels of beneficial insects (entomophage).

The following theorem holds.

Theorem. To have a condition

$$\frac{1}{\tau}\int_0^\tau N_1(t)dt \ge N_1^p, \quad N_1^p \in \left[N_1^{\min},\ N_1^{\max}\right]$$

at $V_i(\cdot) \ge 0, \quad \dfrac{dV_i}{dN} > 0, \quad \dfrac{d^2V_i}{dN^2} \le 0$ and

$$0<\min_{\substack{0\le a<\infty \\ 0\le t\le\tau}}\frac{V_1(N_1(t))}{N_1(t)}=\bar{\alpha}_1<\infty,\quad 0<\max_{\substack{0\le a<\infty \\ 0\le t\le\tau}}\frac{V_2(N_2(a,t))}{N_2(a,t)}=\bar{\alpha}_2<\infty,$$

$\bar{\alpha}_1\cdot\bar{\alpha}_2=const,\ \ i=1,2$

the fulfillment of inequality is necessary and sufficient

$$\begin{cases} N_0(t)\le\dfrac{Q}{\alpha_0 N_1^p},\quad 0\le t\le\tau, \\ \dfrac{1}{\tau}\displaystyle\int_0^\tau \tilde{N}_2(t)dt\le N_2^p, \\ \dfrac{1}{\tau}\displaystyle\int_0^\tau \tilde{N}_3(t)dt\ge N_3^p. \end{cases}\qquad (2)$$

where, $N_2^p=\dfrac{k_0Q}{\bar{\alpha}_1N_1^p}-\dfrac{m_1}{\bar{\alpha}_1}-\dfrac{1}{\bar{\alpha}_1\tau}\ln\dfrac{N_1(\tau)}{N_1(0)}$

$$N_3^p=\frac{k_1\bar{\alpha}_1}{\bar{\alpha}_2}N_1^p-\frac{m_2}{\bar{\alpha}_2}-\frac{1}{\bar{\alpha}_2\tau}\max_a\ ln\frac{N_2(a,\tau)}{N_2(a,0)}.$$

Proof. Let the condition be satisfied $\dfrac{1}{\tau}\displaystyle\int_0^\tau N_1(t)dt\ge N_1^p,\ N_1^p\in\left[N_1^{\min},N_1^{\max}\right].$

Let us prove the validity of (2).

By the first equation of (1), we obtain

$$N_0(t)=N_0(0)exp(-\alpha_0)\int_0^\tau N_1(\zeta)d\zeta+Q\int_0^\tau exp(-\alpha_0\int_\tau^\zeta N_1(\zeta)d\zeta)d\zeta\le$$

$$\le\left[N_0(0)-\frac{Q}{\alpha_0N_1^p}\right]exp(-\alpha_0N_1^pt)+\frac{Q}{\alpha_0N_1^p}$$

from here

$$N_0(t)\le\frac{Q}{\alpha_0N_1^p},\quad 0\le t\le\tau,\qquad (N_0(0)=\frac{Q}{\alpha_0N_1^p})$$

By virtue of the 2nd equation (1), we have

$$\frac{dN_1}{dt}=k_0\alpha_0N_0N_1-V_1(N_1)\tilde{N}_2-m_1N_1,$$

$$\frac{d(ln N_1)}{dt}=k_0\alpha_0N_0-\frac{V_1(N_1)}{N_1}\tilde{N}_2-m_1$$

$$\frac{V_1(N_1)}{N_1}\tilde{N}_2(t)=k_0\alpha_0N_0-m_1-\frac{d(ln\ N_1)}{dt}$$

$$\frac{V_1(N_1)}{N_1}\tilde{N}_2(t)=\frac{k_0Q}{N_1^p}-m_1-\frac{d(ln N_1)}{dt}$$

Integrating the last equality over t from 0 to τ, we obtain

$$\frac{1}{\tau}\int_0^\tau\frac{V_1(N_1(t))}{N_1(t)}\tilde{N}_2(t)dt=\frac{k_0Q}{N_1^p}-m_1-\frac{1}{\tau}\ln\frac{N_1(\tau)}{N_1(0)}$$

$$0<min_{0\le t\le\tau}\frac{V_1(N_1(t))}{N_1(t)}\tilde{N}_2(t)\le\frac{k_0Q}{N_1^p}-m_1-\frac{1}{\tau}ln\frac{N_1(\tau)}{N_1(0)},$$

being that $0<min_{0\le t\le\tau}\dfrac{V_1(N_1(t))}{N_1(t)}=\bar{\alpha}_1<\infty,$

$$\tilde{N}_2(t)\le\frac{K_0Q}{\bar{\alpha}_1N_1^p}-\frac{m_1}{\bar{\alpha}_1}-\frac{1}{\bar{\alpha}_1\tau}ln\frac{N_1(\tau)}{N_1(0)}=N_2^p$$

$$\tilde{N}_2(t)\le N_2^p$$

On the basis of the 3rd equation (1) we get

$$\frac{\partial N_2}{\partial t}+\frac{\partial N_2}{\partial a}=k_1V_1(N_1)N_2-V_2(N_2)\tilde{N}_3-m_2N_2,$$

We make the notation $a=t+\xi,\ \ \varphi(t,\xi)=N_2(a,t)$

Then $\dfrac{\partial\varphi}{\partial t}=\dfrac{\partial N_2}{\partial t}+\dfrac{\partial N_2}{\partial a}$

and get $\dfrac{\partial}{\partial t}\ln\varphi=k_1V_1(N_1)-m_2-\dfrac{V_2(N_2)\tilde{N}_3}{N_2},$

$$\frac{V_2(N_2)}{N_2}\tilde{N}_3(\mathrm{t})=k_1V_1(N_1)-m_2-\frac{\partial}{\partial t}\ln\varphi.$$

Integrating the last equation in t from 0 to τ, we obtain

$$\frac{1}{\tau}\int_0^\tau\frac{V_2(N_2)}{N_2}\tilde{N}_3(t)dt=k\bar{\alpha}_1N_1^p-m_2-\frac{1}{\tau}ln\frac{N_2(a,\tau)}{N_2(a,0)}$$

,

$$0<\max_{0\le t\le\tau}\frac{V_2(N_2)}{N_2}\tilde{N}_3(\mathrm{t})\ge k_1\bar{\alpha}_1N_1^p-m_2-\frac{1}{\tau}\ln\frac{N_2(a,\tau)}{N_2(a,0)}$$

$$,0<\max_{0\le t\le\tau}\frac{V_2(N_2(a,t))}{N_2(a,t)}=\bar{\alpha}_2<\infty,$$

$$\tilde{N}_3(\mathrm{t})\ \overline{\alpha}_2 \geq k_1\overline{\alpha}_1 N_1^p - m_2 - \frac{1}{\tau}\ln\frac{N_2(a,\tau)}{N_2(a,0)}$$, and

therefore

$$\tilde{N}_3(\mathrm{t}) \geq \frac{k_1\overline{\alpha}_1}{\overline{\alpha}_2}N_1^p - \frac{m_2}{\overline{\alpha}_2} - \frac{1}{\tau\overline{\alpha}_2}\max_a \ln\frac{N_2(a,\tau)}{N_2(a,0)} = N_3^p$$

, $\tilde{N}_3(t) \geq N_3^p$.

Adequacy. Let inequalities (2) hold.

Let us prove that the condition $\frac{1}{\tau}\int_0^\tau N_1(t)dt \geq N_1^p$.

From the first equation (1) we find

$$N_0(t) = N_0(0) + Qt - \alpha_0\int_0^\tau N_0 N_1(t)dt$$

from here $N_0(t) \geq N_0(0) + Qt - \frac{Q}{N_1^p}\int_0^\tau N_1(t)dt$

and therefore

$$\frac{1}{\tau}\int_0^\tau N_1(t)dt - N_1^p \geq \frac{(N_0(0) - \dfrac{Q}{\alpha_0 N_1^p})N_1^p}{Qt}$$ from

here

$$\frac{1}{\tau}\int_0^\tau N_1(t)dt \geq N_1^p.$$

As N_2^p, N_3^p non-negative then $N_1^{\min} \leq N_1^p \leq N_1^{\max}$.

Conclusion. The results of the study can be used in the design of measures to protect plants from agricultural pests.

3. COMPUTER SIMULATION RESULTS

This section shows the results of computer programs. Computer programs are developed in the C++ programming language, which is widely used in scientific, engineering, mathematical and computer fields. The program is designed to calculate the dynamics of the number of harmful and beneficial insects and makes it possible to predict the dynamics of insect development for the considered time interval. The results of the program can be obtained in the form of a table and in the form of a graph. The calculations were carried out in several versions based on preliminary data. Each option is performed at different initial values of the number of insects and different time intervals. The lifespan of one generation is 13 days, and the number of generations per season is 3.

When calculating each option, the Figure 1 and 2 shows the results that were get:

time, days	harmful insects	beneficial insects
13	4419	764
26	6527	854
39	10805	4983

time, days	harmful insects	beneficial insects
13	4703	3983
26	6753	5737
39	10862	9350

time, days	harmful insects	beneficial insects
13	4869	2446
26	7082	3683
39	11529	6270

Figure 1: Results Obtained by the Program "Dynamics of the Number of Harmful and Beneficial Insects"

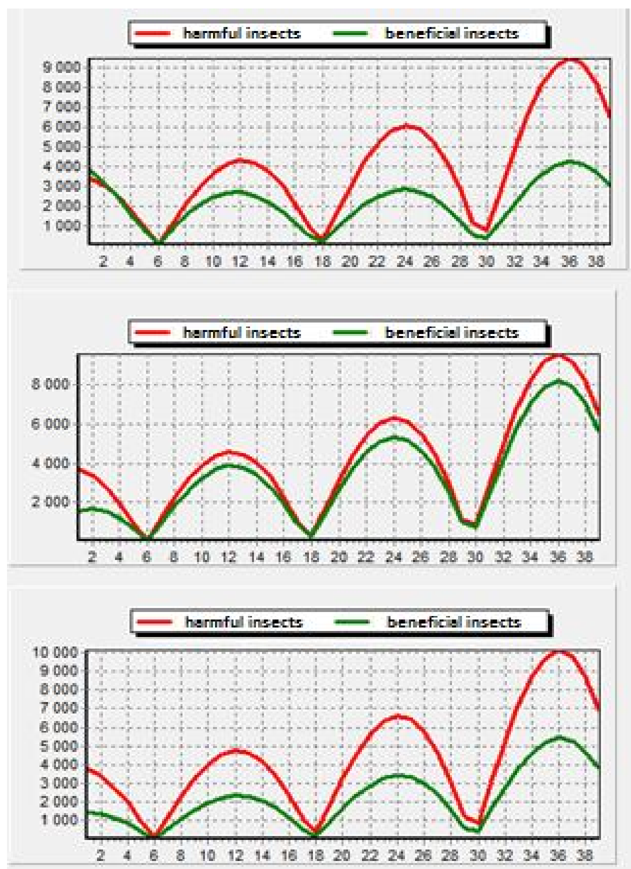

Figure 2: Screenshots of Results Obtained by the Program "Dynamics of the Number of Harmful and Beneficial Insects"

4. CONCLUSION

The research results can be used to solve the problem of forecasting and planning, conducting field experiments for specific populations, biological communities and ecological systems.

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