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# The Image Models of Combined Correlation-Extreme Navigation System of Flying Robots 

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#### Abstract

The results of the development of a model for the functioning of combined correlation-extremal navigation systems (CENS) of flying robots (FR) with radiometric and optical-electronic (OE) sensors are presented. In general, an expression is obtained for the probability of correct localization of an object binding (OB), and an expression is given for the probability of correct localization of an OB for a single binding session. The main factors that lead to distortions of the decisive function (DF) formed by CENS are identified. The results of determining the influence of the spatial position of the FR on the current image (CI) generated by CENS are given.


Key words: flying robot, correlation-extremal navigation system, binding object, current image, signal model.

## 1. INTRODUCTION

At present, combined CENS with matrix radiometric and optical-electronic sensors are widely used for navigation FR. A feature of such navigation systems is their autonomy, passivity due to the absence of radiation, and, accordingly, high noise immunity. At the same time, these navigation systems must ensure high accuracy of FR positioning with allowance for possible CI distortions.

### 1.1 Problem analysis

When solving problems of navigation, the efficiency of such systems may be insufficient, due to the impossibility of
forming a unimodal DF due to the influence of a number of factors leading to CI distortions and, accordingly, to CI mismatch to a pre-formed reference image (RI) [1,2]. The need to ensure high accuracy of CENS positioning in areas with various objects and in conditions of changing the geometry of sight necessitates further studies of the DF formation process and analysis of the factors that have the greatest influence on the formed CENS CI, and accounting of these influencing factors to the output signals $[3,4]$.

## The aim of research

The aim of the article is to develop CI models formed by the combined CENS sensors taking into account the shape of the real antenna directivity diagram (ADD), receiver inertia and image blur due to the FR movement during the frame exposure time.

## 2. Main Material

The basic model of the combined CENSes functioning process, we will understood as some idealized model of the system, which describes the conversion signal patterns arising from the solvable tasks, but does not take into account the interference characteristic specifics of radiometric reception or reception by optical-electronic sensors. It is assumed that the only interference to receiving signals is the internal noise of the receiver, and various destabilizing factors distorting the CI structure (changing weather conditions, underlying surface conditions, etc.) are absent.

### 2.1. The main objectives and model of signal processing in the RM channel CENS

In accordance with the theory of thermal radiation in the RM channel, the selection of information about the OB is based on the difference in emissivity of materials and underlying surfaces. Therefore, the recorded (informative) signal parameter used for detection, identification, recognition, mapping, etc., is the intensity of the received signal.

The main problem solved by the RM channel at each session of the binding trajectory is to estimate the coordinates of the OB characteristic point $\left(\mathrm{X}_{\mathrm{OB}}, \mathrm{Y}_{\mathrm{OB}}\right)$ based on the processing of the heat thermal relief $T(x, y)$ in the view field $S \subset \mathbf{R}^{2}$ and using a priory information in the form of RI about one of the view field parts, i.e. display formation $\mathrm{S} \rightarrow\left(\mathrm{X}_{\mathrm{OB}}, \mathrm{Y}_{\mathrm{OB}}\right)$. This task is divided into a number of sub tasks:

1) the review of the underlying surface in the $O B$ area;
2) the formation in a certain subject plane associated with the FR coordinate system , a radio-thermal relief of the area within the view field, i.e. radiometric imaging (RMI);
3) RMI analysis in order to identify anomalies due to the presence of the OB, based on its comparison with the RI, determining the coordinates of the selected anomaly characteristic point, and issuing target indications of the CS.

On the basis of these data and information about the height at the time of the frame pickup received from the altimeter, the coordinates for the FR mass center and the mismatch angles of the FR velocity vector with the OB direction are calculated. This information, in the form of target indications, enters the control system and is used to correct the FR trajectory.

Let in the coordinate system ( $x, y$ ) on the earth surface be given the field of the heat (radio) thermal relief $T(x, y)$. A multi path antenna converts radio-thermal radiation with an intensity $T(x, y)$ into a combination of processes $\left\{\mathrm{u}_{\mathrm{S}_{\mathrm{ij}}}(\mathrm{t})\right\}_{\mathrm{i}=1, \mathrm{j}=1}^{\mathrm{N}_{1}, \mathrm{~N}_{2}}$ with two-sided spectral power densities:

$$
\begin{equation*}
\mathrm{S}_{\mathrm{s}_{\mathrm{ij}}}(\mathrm{f})=\mathrm{kT} \mathrm{~s}_{\mathrm{ij}} / 2, \tag{1}
\end{equation*}
$$

and the antenna temperature $\mathrm{T}_{\mathrm{s}_{\mathrm{ij}}}$ for each channel is determined by the expression:

$$
\begin{gather*}
\mathrm{T}_{\mathrm{sij}}=\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \mathrm{T}(\mathrm{x}, \mathrm{y}) \mathrm{G}\left(\mathrm{x}_{\mathrm{ij}}-\mathrm{x}, \mathrm{y}_{\mathrm{ij}}-\mathrm{y}\right) \mathrm{dxdy}  \tag{2}\\
\mathrm{i} \in \overline{1, \mathrm{~N}_{1}}, \mathrm{j} \in \overline{1, \mathrm{~N}_{2}}
\end{gather*}
$$

where $G\left(x_{i j}-x, y_{i j}-y\right)-i t$ is the function describing the $i j-$ th partial ADD, converted to a coordinate system on the surface of the earth, which axis intersects the earth surface at a point $\left(\mathrm{x}_{\mathrm{ij}}, \mathrm{y}_{\mathrm{ij}}\right)$.

Thus, the heat radio thermal relief distribution undergoes the information loss due to image discretization following the finiteness of the processing channels number and the image "blurring" of the final the width of each partial ADD, which leads to a decrease in the accuracy and reliability of the positioning.

Let further signals be processed by a multi-channel matrix radiometer, which uses independent radiometric channels. In each channel, the signal is amplified in the amplification path to the level necessary for normal operation of the quadratic detector. Before amplification, the signal is summed with interference due to the antenna noise, the input circuits and the intrinsic noise of the amplification path. Assuming that the frequency path response is rectangular with a bandwidth $\Delta f$ and center frequency $f_{0}$, the signal at its output can be represented as:

$$
\begin{equation*}
\mathrm{u}_{\mathrm{ij}}(\mathrm{t})=\mathrm{u}_{\mathrm{s}_{\mathrm{ij}}}(\mathrm{t})+\mathrm{u}_{\mathrm{n}_{\mathrm{ij}}}(\mathrm{t}), \tag{3}
\end{equation*}
$$

where $\mathrm{u}_{\mathrm{s}_{\mathrm{ij}}}(\mathrm{t}), \mathrm{u}_{\mathrm{n}_{\mathrm{ij}}}(\mathrm{t})$ - they are the band Gaussian random processes with spectral power densities

$$
\begin{align*}
& \mathrm{S}_{\mathrm{sij}}(\mathrm{f})=\mathrm{kT}_{\mathrm{S}_{\mathrm{ij}}}\left[\operatorname{rect}\left(\mathrm{f}+\mathrm{f}_{0}, \Delta \mathrm{f}\right)+\operatorname{rect}\left(\mathrm{f}-\mathrm{f}_{0}, \Delta \mathrm{f}\right)\right] / 2 \\
& \mathrm{~S}_{\mathrm{n}_{\mathrm{ij}}}(\mathrm{f})=\mathrm{kT}_{\mathrm{n}_{\mathrm{ij}}}\left[\operatorname{rect}\left(\mathrm{f}+\mathrm{f}_{0}, \Delta \mathrm{f}\right)+\operatorname{rect}\left(\mathrm{f}-\mathrm{f}_{0}, \Delta \mathrm{f}\right)\right] / 2, \tag{4}
\end{align*}
$$

where $\mathrm{k}=1,38 \cdot 10^{-23} \mathrm{~J} /$ hail is the Boltzmann constant;

$$
\operatorname{rect}(\mathrm{f}, \Delta \mathrm{f})=\left\{\begin{array}{l}
1,|\mathrm{f}| \leq \Delta \mathrm{f} / 2  \tag{5}\\
0,|\mathrm{f}|>\Delta \mathrm{f} / 2
\end{array}\right.
$$

$\mathrm{T}_{\mathrm{n}_{\mathrm{ij}}}$ - equivalent temperature of internal noise.

The signal processing algorithm, for example, in an ideal compensation matrix radiometer is as follows:

$$
\begin{gather*}
\left\{\mathrm{u}_{\mathrm{ij}}(\mathrm{t}) \mapsto \hat{\mathrm{T}}_{\mathrm{s}_{\mathrm{ij}}}\right\} \hat{\mathrm{T}}_{\mathrm{s}_{\mathrm{ij}}}=\frac{1}{\mathrm{k} \Delta \mathrm{f} \tau} \int_{0}^{\tau} \mathrm{u}_{\mathrm{ij}}^{2}(\mathrm{t}) \mathrm{dt}-\mathrm{T}_{\mathrm{n}_{\mathrm{ij}}}  \tag{6}\\
i \\
\mathrm{i} \in \overline{1, \mathrm{~N}_{1}}, \mathrm{j} \in \overline{1, \mathrm{~N}_{2}}
\end{gather*}
$$

where $\hat{\mathrm{T}}_{\mathrm{s}_{\mathrm{ij}}}$ - is the antenna temperature estimate for the ij -th channel.

In the signals optimal reception theory with fluctuation noise, it is customary to use the square root of the mean square deviation of the measured value (estimate) of the antenna temperature $\hat{\mathrm{T}}_{\mathrm{s}}$ from its true value as an indicator of noise immunity $\mathrm{T}_{\mathrm{s}_{\mathrm{ij}}}$, i.e.

$$
\begin{equation*}
\delta \mathrm{T}_{\mathrm{ij}}=\left[\mathbf{M}\left(\hat{\mathrm{T}}_{\mathrm{s}_{\mathrm{ij}}}-\mathrm{T}_{\mathrm{s}_{\mathrm{ij}}}\right)^{2}\right]^{1 / 2} \tag{7}
\end{equation*}
$$

By a direct averaging of (6), we can verify that the estimate $\hat{\mathrm{T}}_{\mathrm{s}_{\mathrm{ij}}}$ is unbiased, i.e. $\mathbf{M} \hat{\mathrm{T}}_{\mathrm{s}_{\mathrm{ij}}}=\mathrm{T}_{\mathrm{s}_{\mathrm{ij}}}$. Then the noise immunity index (7) coincides with the standard deviation $\hat{\mathrm{T}}_{\mathrm{s}_{\mathrm{ij}}}$

$$
\begin{equation*}
\delta \mathrm{T}_{\mathrm{ij}}=\left(\mathbf{D} \hat{\mathrm{T}}_{\mathrm{s}_{\mathrm{ij}}}\right)^{1 / 2} \tag{8}
\end{equation*}
$$

Using (6), direct calculations can show that

$$
\begin{equation*}
\delta \mathrm{T}_{\mathrm{ij}}=\frac{\mathrm{T}_{\mathrm{s}_{\mathrm{ij}}}+\mathrm{T}_{\mathrm{n}_{\mathrm{ij}}}}{\sqrt{\Delta \mathrm{f} \tau}} \tag{9}
\end{equation*}
$$

Thus, the signal at the output of the radiometric channel can be represented as

$$
\begin{equation*}
\hat{\mathrm{T}}_{\mathrm{s}_{\mathrm{ij}}}=\mathrm{T}_{\mathrm{s}_{\mathrm{ij}}}+\mathrm{n}_{\mathrm{ij}} \tag{10}
\end{equation*}
$$

where $\mathrm{n}_{\mathrm{ij}}$ - where it is the fluctuation component of the output signal. Since the integrator in (6) is a low-pass filter, by virtue of the well-known theorem on the random process normalization at the output of a narrow-band filter, $\mathrm{n}_{\mathrm{ij}}$ it is a Gaussian random variable with zero mean value and the standard deviation (9).

The set of output signals of radiometric channels forms $\mathrm{N}_{1} \times \mathrm{N}_{2}$ - CI matrix $\left\{\hat{\mathrm{T}}_{\mathrm{s}_{\mathrm{ij}}}\right\}$, on the basis of comparing fragments with the RI matrix $\mathbf{E}=\left[\mathrm{e}_{\mathrm{ij}}\right], \mathrm{i} \in \overline{1, \mathrm{M}_{1}}, \mathrm{j} \in \overline{1, \mathrm{M}_{2}}, \mathrm{M}_{1}<\mathrm{N}_{1}, \mathrm{M}_{1}<\mathrm{N}_{1}, \quad$ in the secondary processing device using an appropriate algorithm, it is decided to localize the OB by searching for the CI fragment most relevant to RI.

Fragment by the author called the fragment CI, which actually corresponds to RI. For this fragment, the relation (10) takes the form:

$$
\begin{equation*}
\hat{\mathrm{T}}_{\mathrm{s}_{\mathrm{ij}}}^{\mathrm{kl}}=\mathrm{e}_{\mathrm{ij}}+\mathrm{n}_{\mathrm{ij}}^{\mathrm{kl}} \tag{11}
\end{equation*}
$$

where $(k, l)$ - are the coordinates of the author fragment.
Thus, the secondary processing system of the RM channel forms the mapping $\left\{\hat{\mathrm{T}}_{\mathrm{ij}}\right\} \mapsto(\hat{\mathrm{k}}, \hat{\mathrm{l}})$ of the CI matrix into the coordinate estimate of the author fragment.

As the indicator of a secondary processing device noise immunity, we will use the probability of the OB correct localization $\mathrm{P}_{\mathrm{CL}}$. Let us reveal the meaning of this concept.

Let it on the i-th session of the trajectory binding ( $\mathrm{i} \in \overline{1, \mathrm{~N}_{\mathrm{t}}}, \mathrm{N}_{\mathrm{t}}-$ the total number of binding sessions (frames)) the RI estimated position $\hat{\mathrm{k}}_{\mathrm{i}}, \hat{l}_{\mathrm{i}}$ is obtained relative to $C I$, and the true values are equal $\mathrm{k}_{\mathrm{i}}, \mathrm{l}_{\mathrm{i}},\left(\mathrm{k}_{\mathrm{i}} \in \overline{1, \mathrm{~N}_{1}-\mathrm{M}_{1}+1}, \mathrm{l}_{\mathrm{i}} \in \overline{1, \mathrm{~N}_{2}-\mathrm{M}_{2}+1}\right)$. Let us denote by the $\mathrm{A}_{\mathrm{i}}$ event that the i -th binding session for the absolute $\operatorname{error}\left(\Delta \mathrm{k}_{\mathrm{i}}=\hat{\mathrm{k}}_{\mathrm{i}}-\mathrm{k}_{\mathrm{i}}, \Delta \mathrm{l}_{\mathrm{i}}=\hat{\mathrm{l}}_{\mathrm{i}}-\mathrm{l}_{\mathrm{i}}\right)$ satisfies the condition:

$$
\begin{equation*}
\left|\Delta \mathrm{k}_{\mathrm{i}}\right|<1,\left|\Delta \mathrm{l}_{\mathrm{i}}\right|<1, \tag{12}
\end{equation*}
$$

Then this event will be called the correct localization of the OB at the i-th session of the trajectory binding. When this event is performed, the accuracy of the trajectory reference will be no worse than the resolution element on the ground, determined by the width of the beam, the height of the FR at the time of the shot and the angle of OB sight.

For the first binding session, we have $\mathrm{P}_{\mathrm{CL}_{1}}=\mathrm{P}\left(\mathrm{A}_{1}\right)$. In the second session, the correct localization probability of the correct OB is the probability of combining events $\mathrm{A}_{1}$ and $\mathrm{A}_{2}$, which is determined by the multiplication rule of probabilities $\mathrm{P}_{\mathrm{CL}_{2}}=\mathrm{P}\left(\mathrm{A}_{1} \bigcap \mathrm{~A}_{2}\right)=\mathrm{P}\left(\mathrm{A}_{1}\right) \mathrm{P}\left(\mathrm{A}_{2} \mid \mathrm{A}_{1}\right)$. Similarly, at the i-th $\operatorname{session} P_{C L i}=P\left(\bigcap_{j=1}^{i} A_{j}\right)=P\left(A_{1}\right) P\left(A_{2} \mid A_{1}\right) \cdots P\left(A_{i} \mid \bigcap_{j=1}^{i-1} A_{j}\right)$.

Then the final probability of the correct OB localization will be determined by the expression:

$$
\begin{equation*}
P_{C L}=P_{C L N_{t}}=P\left(\bigcap_{j=1}^{N_{t}} A_{j}\right)=P\left(A_{1}\right) P\left(A_{2} \mid A_{1}\right) \cdots P\left(A_{N_{t}} \mid \bigcap_{j=1}^{N_{t}-1} A_{j}\right) \tag{13}
\end{equation*}
$$

Let us estimate the signal influence and interference components in the image (10) on the probability of the correct OB localization. For simplicity, we will consider events $A_{i}$ mutually independent. Then (13) takes the form

$$
\begin{equation*}
\mathrm{P}_{\mathrm{CL}}=\prod_{\mathrm{i}=1}^{\mathrm{N}_{\mathrm{t}}} \mathrm{P}\left(\mathrm{~A}_{\mathrm{i}}\right), \tag{14}
\end{equation*}
$$

It can be shown that the probabilities of the correct $O B$ localization on one binding session under the following assumptions and assumptions [5,6]:

1) RI is a sample of a one-dimensional Gaussian ergodic process with zero mean and variance $\sigma_{\mathrm{e}}^{2}$;
2) an additive model of interaction of CI with the form noise (11) is used $\mathrm{z}_{\mathrm{i}}=\mathrm{e}_{\mathrm{i}}+\mathrm{n}_{\mathrm{i}}$;
3) noise $n_{i}$ has the same characteristics as RI, except for dispersion, which is equal $\sigma_{\mathrm{n}}^{2}$ and the same for all channels;
4) the processes z and n are independent;
5) to compare the fragments of RI with CI, the mean absolute difference algorithm is used, whose criteria function is

$$
\begin{equation*}
\mathrm{D}_{\mathrm{j}}=\frac{1}{\mathrm{~N}} \sum_{\mathrm{i}=1}^{\mathrm{N}}\left|\mathrm{e}_{\mathrm{i}+\mathrm{j}}-\mathrm{z}_{\mathrm{i}}\right| \tag{15}
\end{equation*}
$$

where N - is the number of elements CI ; expressions will be determined

$$
\mathrm{P}_{\mathrm{CL}}=\frac{1}{2^{\mathrm{M}} \sqrt{\pi}} \int_{-\infty}^{\infty} \mathrm{e}^{-\mathrm{x}^{2}}\left[1+\operatorname{erf}\left(\frac{\mathrm{x}+\sqrt{\frac{\mathrm{N}}{\pi-2}}(\sqrt{2 \mathrm{q}+1}-1)}{\sqrt{2 \mathrm{q}+1}}\right)\right]^{\mathrm{M}} \mathrm{dx},(16)
$$

where M - is the number of independent samples of the criterion function;

N - is the number of independent images samples
$\mathrm{q}=\sigma_{\mathrm{e}}^{2} / \sigma_{\mathrm{n}}^{2}-$ signal-to-noise ratio;
$\operatorname{erf}(\mathrm{x})=\frac{2}{\sqrt{\pi}} \int_{0}^{\mathrm{x}} \mathrm{e}^{-\mathrm{t}^{2}} \mathrm{dt}$.
From the expression (16) it follows that:

1) the probability of the correct localization of the OB increases with increasing signal-to-noise ratio;
2) the probability of correct localization of OB increases with increasing N and decreases with increasing M .

However, the expression (16) does not take into account a number of factors that will lead to a decrease in the probability of correct OB localization.

### 2.2 Analysis of factors that lead to distortions in a decisive function formed by a correlation-extreme navigation system

The task solution of high-precision FR autonomous positioning that use combined CENS should be carried out primarily in the informative parameters bundle of the system "three-dimensional shape of objects on CI - sensors of different physical nature (SDPN) - geometric position of FR, taking into account its random change" under the influence:

- various types of CI interference and distortion, which may be natural or artificial;
propagation medium (PM) of radio waves and the interference effect on the state of the navigation system (NS).

The CENS FR efficiency functioning is determined by the decisive function (the team for the correction of the spatial FR position) and is estimated by the accuracy parameters and positioning probability [7].

The FR locating probability using the CENS in the k -th session of the binding is determined by the expression:

$$
\begin{equation*}
\mathrm{P}_{\mathrm{Mi}_{\mathrm{i}}}=\mathrm{P}_{\mathrm{CL}_{\mathrm{i}}} \cdot \mathrm{P}_{\mathrm{C}_{\mathrm{i}}} \tag{17}
\end{equation*}
$$

where $\mathrm{P}_{\mathrm{CL}_{\mathrm{i}}}$ - the probability of the correct OB localization on the CI ;
$\mathrm{P}_{\mathrm{C}_{\mathrm{i}}}=\mathrm{P}_{\mathrm{W}} \cdot \mathrm{P}_{\mathrm{IP}} \cdot \mathrm{P}_{\mathrm{CS}}$ - the performing probability of the FR flight path correction at the k -th session of the binding, which is determined by the following parameters:
$\mathrm{P}_{\mathrm{W}}$ - the influence probability of weather conditions on the control system (CS) LR;
$\mathrm{P}_{\mathrm{IP}}$ - the interference possibility on the FR (CS) functioning;
$\mathrm{P}_{\mathrm{CS}}-$ the failure-free operation probability of the FR CS.

The accuracy index (positioning error) of the CENS is characterized by the standard deviation (SD) of the real FR coordinates after performing the k - th correction with respect to the given one [5, 6].

Accuracy index CENS FR can be represented by the expression:

$$
\begin{equation*}
\sigma_{\mathrm{k}}=\sqrt{\sigma_{\mathrm{CL}_{\mathrm{i}}}^{2}+\sigma_{\mathrm{C}_{\mathrm{k}}}^{2}+\sigma_{\mathrm{CS}_{\mathrm{k}}}^{2}} \tag{18}
\end{equation*}
$$

where $\sigma_{\mathrm{CL}_{\mathrm{i}}}=\mathrm{f}\left(\sigma_{\mathrm{CI}}, \sigma_{\mathrm{x}_{\mathrm{i}} \mathrm{y}_{\mathrm{i}} \mathrm{z}_{\mathrm{i}}}\right)$ - is the SD of the OB localization on the CI at the k -th binding session, which depends on the RI manufacturing accuracy $\sigma_{\text {RI }}$ and the errors in determining the spatial position of the FR, arising under the influence of random factors $\sigma_{\mathrm{x}_{\mathrm{i}} \mathrm{y}_{\mathrm{i}} \mathrm{z}_{\mathrm{i}}}$;
$\sigma_{\mathrm{C}_{\mathrm{k}}}$ - the coordinate deviation of the coordinates after performing the correction in the k session of the reference CENS;
$\sigma_{\mathrm{CS}_{\mathrm{k}}}$ - the SD of testing control signals after the correction FR flight path.

The random factors' influence that lead to errors in determining the FR spatial position requires the appropriate methods development on the formation of the CENS DF and the FR positioning.

Probability $\sigma_{\mathrm{CL}_{\mathrm{i}}}$ is determined by many factors. These include the three-dimensional shape of the sight surface (SS) objects, as well as the influence of the FR spatial position instability, which determine the quality of CI, is formed by the CENS sensors [5].

The need to take into account the three-dimensional shape of the SS objects, especially in the conditions of changing the sighting geometry, is due to the appearance on the object saturated they contours. These factors, as well as the rapid change of the FR spatial position, caused by gusts of wind, air holes and air currents, can also contribute to the distortion of the DF, which in turn leads to a decrease in the CENS functioning efficiency.

### 2.3 The changes impact analysis in the FR spatial position on the CI formation

Preserving the reasoning integrity intact, we will refine the CI model, which is formed by the channels of the combined CENS using the example of a RM channel with an informative parameter - a radio-brightness temperature, followed by the generalization to an optical-electronic channel [8].

The CI model building will be carried out taking into account the following conditions:

1) The FR flies evenly and straightforwardly at the $V$ speed at an angle $\varphi$ to the vertical;
2) The ADD is approximated by a Gaussian surface;
3) The CI frame formation is carried out by a multi path (matrix) system;
4) The CI is formed in accordance with (11) in each channel under the influence of additive noise;
5) The effect of changing the FR spatial position on the CI is carried out by the yaw angle $\psi^{\prime}=\psi \pm \Delta \psi$;
6) the angles of pitch and roll do not change.

Suppose that the FR moves in the $x z$ plane of the $z, y, z$ coordinate system associated with the SS (Fig. 1.1-a)). The position of each partial ADD is characterized by angles $\beta$ and $\alpha$. The aperture angle of the ADD at the half power level is equal $\theta_{\mathrm{x}}$ in the plane along the elevation angle and $\theta_{\mathrm{y}}$ - in the azimuth plane. For the ADD Gaussian approximation, its section by the xy plane is an ellipse.

The CI is formed as a matrix of M rows and N columns. The inclination of the plane in which the ADD are located along the axes is given by the angle $\beta_{\mathrm{i}, \mathrm{j}} \in \overline{1, \mathrm{M}}$ relative to the velocity vector V , and the position of the axis of each ADD in the row is characterized by the angle $\alpha_{\mathrm{ij}}$.


Figure 1: Geometric conditions for the formation of CENS CI

The distortions of the CI frame can be found from the motion equation of the centers and sizes of the principal semi-axes of half-power ellipses for each beam of the ADD.

In accordance with the motion direction (Fig. 1), the FR spatial position at the moment of time $t$ is represented as:

$$
\left\{\begin{array}{l}
x(t)=x_{0}+V \cdot\left(t-t_{0}\right) \sin \varphi  \tag{19}\\
y(t)=0 \\
z(t)=z_{0}-V \cdot\left(t-t_{0}\right) \sin \varphi
\end{array}\right.
$$

In view of (19), the motion equation of the ellipse center for the ij -th partial ADD is written as:

$$
\left\{\begin{array}{l}
\mathrm{x}_{\mathrm{ij}}(\mathrm{t})=\mathrm{k}_{\mathrm{ij}} \mathrm{x}_{\mathrm{ij}}^{\prime}(\mathrm{t}) ;  \tag{20}\\
\mathrm{y}_{\mathrm{ij}}(\mathrm{t})=\mathrm{k}_{\mathrm{ij}} \mathrm{y}_{\mathrm{ij}}^{\prime}(\mathrm{t}),
\end{array}\right.
$$

where:

$$
\begin{gathered}
x_{i j}^{\prime}(t)=x_{0}+z_{0} \operatorname{tg}\left(\varphi-\beta_{i}\right)+V\left(t-t_{0}\right) \sin \beta_{i} \sec \left(\varphi-\beta_{i}\right) ; \\
y_{i j}^{\prime}(t)=z(t) \operatorname{tg}\left(\alpha_{i j}\right) \sec \left(\varphi-\beta_{i}\right) ;
\end{gathered}
$$

$$
\mathrm{k}_{\mathrm{ij}}=\left\{1-\left[\frac{\sin \left(\theta_{\mathrm{x}_{\mathrm{ij}}} / 2\right)}{\cos \alpha_{\mathrm{ij}} \cos \left(\varphi-\beta_{\mathrm{i}}\right)}\right]^{2}\right\}^{-1} .
$$

The dimensions of the main ellipse semi axes are determined by the relations [9]:

1) in a plane that passes through points $\left(\mathrm{x}_{0}, 0, \mathrm{z}_{0}\right),\left(\mathrm{x}_{0}, 0,0\right),\left(\mathrm{x}_{\mathrm{ij}}, \mathrm{y}_{\mathrm{ij}}, 0\right)$ :

$$
\begin{equation*}
\Delta \mathrm{x}_{\mathrm{ij}}(\mathrm{t})=\mathrm{z}(\mathrm{t})\left[\mathrm{k}_{\mathrm{ij}}(\mathrm{t})-1\right] \operatorname{ctg}\left(\theta_{\mathrm{x}_{\mathrm{ij}}} / 2\right) \tag{21}
\end{equation*}
$$

2) in the orthogonal plane:

$$
\begin{equation*}
\Delta y_{\mathrm{ij}}(\mathrm{t})=\mathrm{z}(\mathrm{t}) \operatorname{tg} \frac{\theta_{\mathrm{y}_{\mathrm{ij}}}}{2} \sqrt{\left(\mathrm{k}_{\mathrm{ij}}-1\right)\left(\mathrm{k}_{\mathrm{ij}} \operatorname{ctg}^{2} \frac{\theta_{\mathrm{x}_{\mathrm{ij}}}}{2}-1\right)} \tag{22}
\end{equation*}
$$

Assume that the principal axes of the half-power ellipses are parallel to the x , y axes. For these conditions, the normalized ADD at a point $\left(\mathrm{x}_{0}, 0, \mathrm{z}_{0}\right)$, which axis intersects the xz plane at a point $\left(\mathrm{x}_{\mathrm{ij}}^{0}, \mathrm{y}_{\mathrm{ij}}^{0}\right)$, can be represented as follows [4]:

$$
\begin{equation*}
\mathrm{G}\left(\mathrm{x}, \mathrm{y}, \mathrm{x}_{\mathrm{ij}}^{0}, \mathrm{y}_{\mathrm{ij}}^{0}\right)=\frac{1}{2 \pi \delta_{\mathrm{x}_{\mathrm{ij}}} \delta_{\mathrm{y}_{\mathrm{ij}}}} \exp \left\{-\left[\frac{\left(\mathrm{x}-\mathrm{x}_{\mathrm{ij}}^{0}\right)^{2}}{2 \delta_{\mathrm{x}_{\mathrm{ij}}}^{2}}+\frac{\left(\mathrm{y}-\mathrm{y}_{\mathrm{ij}}^{0}\right)^{2}}{2 \delta_{\mathrm{y}_{\mathrm{ij}}}^{2}}\right]\right\} \tag{23}
\end{equation*}
$$

We select the parameters $\delta_{\mathrm{x}_{\mathrm{ij}}}$, $\delta_{\mathrm{y}_{\mathrm{ij}}}$ so that the main ellipse semi-axes dimensions of the ADD half power coincide with the dimensions, which are determined by the (21), (22):

$$
\begin{equation*}
\frac{\left(x-x_{i j}^{0}\right)^{2}}{2 \ln 2 \delta_{x_{i j}}^{2}}+\frac{\left(y-y_{i j}^{0}\right)^{2}}{2 \ln 2 \delta_{y_{i j}}^{2}}=1 \tag{24}
\end{equation*}
$$

As a result, we get

$$
\begin{equation*}
\delta_{\mathrm{x}_{\mathrm{ij}}}=\frac{\Delta \mathrm{x}_{\mathrm{ij}}}{\sqrt{2 \ln 2}} ; \delta_{\mathrm{y}_{\mathrm{ij}}}=\frac{\Delta \mathrm{y}_{\mathrm{ij}}}{\sqrt{2 \ln 2}} \tag{25}
\end{equation*}
$$

Let's find the dependence of the brightness temperature in time at the output of a separate RM channel. To do this, suppose there are K zones with brightness temperatures $\mathrm{T}_{\mathrm{m}}$, on the CI that are distributed on a uniform background with the temperature $\mathrm{T}_{\mathrm{B}}$. Then, in the xy plane, the following brightness distribution temperatures will occur:

$$
T_{B r}(x, y)=\left\{\begin{array}{ll}
T_{m}, & x, y \in S_{m}, m \in \overline{1, K}  \tag{26}\\
T_{B}, & x, y \notin S_{m}=\bigcup_{m=1}^{K} S_{m}
\end{array},\right.
$$

where $\mathrm{S}_{\mathrm{m}} \bigcap \mathrm{S}_{\mathrm{n}}=\varnothing, \mathrm{m} \neq \mathrm{n}$.

Let's take into account that the radiometer low-pass filter (LPF) has an impulse response $\mathrm{h}_{\mathrm{ij}}(\mathrm{t})=\frac{1}{\tau_{\mathrm{ij}}} \exp \left(-\mathrm{t} / \tau_{\mathrm{ij}}\right)$. Then the signal at the output of the RM channel is represented in the form:

$$
\begin{equation*}
\mathrm{T}_{\mathrm{S}_{\mathrm{ij}}}^{\mathrm{r}}=\mathrm{e}^{-\left(\mathrm{t}-\mathrm{t}_{0}\right) / \tau_{\mathrm{ij}}} \mathrm{~T}_{\mathrm{S}_{\mathrm{eff}}}\left(\mathrm{t}_{0}\right)+\frac{1}{\tau_{\mathrm{ij}}} \mathrm{~T}_{0}^{\mathrm{t}} \mathrm{e}^{\eta / \tau_{\mathrm{ij}}} \mathrm{~T}_{\mathrm{S}_{\mathrm{eff}} \mathrm{ij}}\left(\eta-\mathrm{t}_{0}\right) \mathrm{d} \eta \tag{27}
\end{equation*}
$$

where $\mathrm{T}_{\mathrm{S}_{\text {eff }} \mathrm{ij}}-\mathrm{it}$ is the antenna temperature at the ij -th input; $\tau_{\mathrm{ij}}$ - time constant of the ij -th channel.

Taking into account that the antenna system parameters $\mathrm{T}_{\mathrm{S}_{\mathrm{ij}}}$ can be represented as follows [6]:

$$
\begin{equation*}
\mathrm{T}_{\mathrm{S}_{\mathrm{ij}}}(\mathrm{t})=\underset{\mathrm{R}^{2}}{\mathrm{~T}} \mathrm{~T}_{\mathrm{S}_{\mathrm{eff}}}(\mathrm{x}, \mathrm{y}) \mathrm{G}\left(\mathrm{x}, \mathrm{y}, \mathrm{x}_{\mathrm{ij}}(\mathrm{t}), \mathrm{y}_{\mathrm{ij}}(\mathrm{t})\right) \mathrm{dxdy} . \tag{28}
\end{equation*}
$$

Taking into account (26), we write (28) as follows:

$$
\begin{equation*}
\mathrm{T}_{\mathrm{S}_{\mathrm{ij}}}(\mathrm{t})=\mathrm{T}_{\mathrm{b}}+\underset{\mathrm{m}-1}{\mathrm{e}} \underset{\mathrm{~m}}{\mathrm{e}}\left(\mathrm{~T}_{\mathrm{m}}-\mathrm{T}_{\mathrm{b}}\right) \underset{\mathrm{S}_{\mathrm{m}}}{\mathrm{~T}} \mathrm{G}\left(\mathrm{x}, \mathrm{y}, \mathrm{x}_{\mathrm{ij}}(\mathrm{t}), \mathrm{y}_{\mathrm{ij}}(\mathrm{t})\right) \mathrm{dxdy} \tag{29}
\end{equation*}
$$

Taking into account (23), (27), (28), after integration in the range from $t_{0}=t-3 \tau$ to $t$ we obtain:

$$
\begin{align*}
\mathrm{T}_{\mathrm{S}_{\mathrm{ij}}}^{\mathrm{r}}(\mathrm{t}) & =\mathrm{T}_{\mathrm{b}}+\left.\underset{\mathrm{m}-1}{\mathrm{e}}\left(\mathrm{~T}_{\mathrm{m}}-\mathrm{T}_{\mathrm{b}}\right) \Phi \frac{\xi-\mathrm{y}_{\mathrm{ij}}\left(\mathrm{t}_{0}\right)}{\delta \mathrm{y}}\right|_{\xi=\mathrm{c}_{\mathrm{m}}} ^{\mathrm{d}_{\mathrm{m}}} \times \\
& \times \Phi \frac{\xi-\mathrm{x}_{\mathrm{ij}}\left(\mathrm{t}_{0}\right)}{\delta \mathrm{x}}-\left.\mathrm{B}_{\mathrm{ij}}(\mathrm{t} \xi)\right|_{\xi=\mathrm{a}_{\mathrm{m}}} ^{\mathrm{b}_{\mathrm{m}}}, \tag{30}
\end{align*}
$$

where

$$
\mathrm{B}_{\mathrm{ij}}(\mathrm{t}, \xi)=\exp \left(\frac{\mathrm{r}_{\mathrm{ij}}^{2}}{2}+\frac{\xi-\mathrm{x}_{\mathrm{ij}}\left(\mathrm{t}_{0}\right)}{\delta \mathrm{x}} \mathrm{r}_{\mathrm{ij}}-\frac{\mathrm{t}-\mathrm{t}_{0}}{\tau_{\mathrm{ij}}}\right) \times
$$

$$
\times\left[\Phi\left(\frac{\xi-\mathrm{x}_{\mathrm{ij}}(\eta)}{\delta \mathrm{x}}+\mathrm{r}_{\mathrm{ij}}\right)\right]_{\eta=\mathrm{t}_{0}}^{\mathrm{t}}
$$

$\mathrm{r}_{\mathrm{ij}}=\frac{\delta_{\mathrm{x}_{\mathrm{ij}}}}{\mathrm{V}_{\mathrm{x}_{\mathrm{ij}}} \tau_{\mathrm{ij}}} ; \Phi(\xi)=\frac{1}{\sqrt{2 \pi}} \int_{-\infty}^{\xi} \mathrm{e}^{-\theta^{2} / 2} \mathrm{~d} \theta ; \mathrm{i} \in \overline{1, \mathrm{M}} ; \mathrm{j} \in \overline{1, \mathrm{~N}}$.

Expression (30) is a signal model that is formed by a separate RM channel depending on the spatial position of the FR and its orientation, without taking into account the channel noise. The expression that describes the CI model taking into account the noise in the radiometric channel can be represented as [9]:

$$
\begin{gather*}
S_{R M}=\left\|S_{R_{i}, j}\right\|_{\substack{i=\overline{1 \ldots . . . . N}}},  \tag{31}\\
S_{R M}(i, j)=T_{b}+\left.\sum_{m=1}^{K}\left(T_{m}-T_{b}\right) \Phi\left(\frac{\xi-y_{i j}\left(t_{0}\right)}{\delta y}\right)\right|_{\xi=c_{m}} ^{d_{m}} \times
\end{gather*}
$$

where

$$
\left.\times\left[\Phi\left(\frac{\xi-\mathrm{x}_{\mathrm{ij}}(\mathrm{t})}{\delta \mathrm{x}}\right)-\mathrm{B}_{\mathrm{ij}}(\mathrm{t} \xi)\right] \right\rvert\,{ }_{\xi=\mathrm{a}_{\mathrm{m}}}+\mathrm{n}(\mathrm{t}) .
$$

The CI model in the optic electronic channel in accordance with (31) can be represented as follows:

$$
\begin{align*}
& \mathbf{S}_{\mathrm{OE}}=\left\|\mathrm{S}_{\mathrm{OE}_{\mathrm{i}, \mathrm{j}}}\right\|_{\substack{\mathrm{i}=\overline{\mathrm{j}=\ldots \mathrm{M}},=1 \ldots \mathrm{~N}}},  \tag{32}\\
& \mathrm{~S}_{\mathrm{OE}}(\mathrm{i}, \mathrm{j})=\mathrm{B}_{\mathrm{b}}+\sum_{\mathrm{m}=1}^{\mathrm{K}}\left(\mathrm{~B}_{\mathrm{Br}_{\mathrm{m}}}-\mathrm{B}_{\mathrm{Br}_{\mathrm{b}}}\right) \Phi\left(\frac{\xi-\mathrm{y}_{\mathrm{i}, \mathrm{j}}\left(\mathrm{t}_{0}\right)}{\delta \mathrm{y}}\right) \mathrm{d}_{\mathrm{m}} \mathrm{~d}_{\xi=\mathrm{c}_{\mathrm{m}}} \times
\end{align*}
$$

where

$$
\times\left[\Phi\left(\frac{\xi-\mathrm{x}_{\mathrm{i}, \mathrm{j}}\left(\mathrm{t}_{0}\right)}{\delta \mathrm{x}}\right)-\mathrm{B}_{\mathrm{i}, \mathrm{j}}(\mathrm{t} \xi)\right] \prod_{\mathrm{b}_{\mathrm{m}}=\mathrm{a}_{\mathrm{m}}}+\mathrm{n}(\mathrm{t})
$$

The signal brightness $\mathrm{B}_{\mathrm{Br}}(\mathrm{i}, \mathrm{j}, \mathrm{t}, \varepsilon, \mu, \varpi)$, which is received by the CENS OE sensor from the SS element at a point ( $\mathrm{i}, \mathrm{j}$ ) in time $t$ described by the expression [9]:

$$
\mathrm{B}_{\mathrm{Br}}(\mathrm{i}, \mathrm{j}, \mathrm{t}, \varepsilon, \mu, \varpi)=\mathrm{E}(\mathrm{i}, \mathrm{j}, \mathrm{t}, \varepsilon, \mu, \varpi) \mathrm{r}_{\mathrm{Br}}(\mathrm{i}, \mathrm{j}, \mathrm{t}, \varepsilon, \mu, \varpi),(33)
$$

where $E(i, j, t, \varepsilon, \mu, \varpi)$ - is the spectral illumination field, which is created by the image element ( $\mathrm{i}, \mathrm{j}$ ) ;
$\mathrm{r}_{\mathrm{Br}}(\mathrm{i}, \mathrm{j}, \mathrm{t}, \varepsilon, \mu, \varpi)-$ the spectral brightness coefficient;
$\varpi$ - the observation and illumination conditions vector:

$$
\begin{equation*}
\varpi=\left\|\varphi \phi \omega \psi \mathrm{E}_{\mathrm{dir}} / \mathrm{E}_{\mathrm{dif}}\right\| ; \tag{34}
\end{equation*}
$$

$\phi$ and $\varphi$ - the angles of the SS element observation;
$\omega$ and $\psi-$ the angles of the SS element illumination;
$\mathrm{E}_{\text {dir }}$ and $\mathrm{E}_{\text {dif }}$ - - random illumination fields, which are created by direct and diffuse radiation.

Taking into account (33), the model of the image of the SS formed by the OE sensor can be represented as:

$$
\begin{equation*}
\mathrm{S}_{\mathrm{SS}_{\mathrm{OE}}}(\mathrm{i}, \mathrm{j})=\mathrm{B}_{\mathrm{Br}}\left(\mathrm{i}, \mathrm{j}, \mathrm{t}, \varepsilon, \mu, \varphi, \phi, \omega, \psi, \mathrm{E}_{\mathrm{dir}}, \mathrm{E}_{\mathrm{dif}}\right), \tag{35}
\end{equation*}
$$

where $\varepsilon, \mu-$ it is the dielectric and magnetic permeability of covers and SS materials.

## 3. CONCLUSION

A formalized basic model has been developed for describing the operation of combined CENS FR, which describes the scheme of signal conversion without taking into account the specific features of interference typical of radiometric reception or reception by optical-electronic sensors. It is proposed to use the probability of correct localization of the OB as an indicator of the noise immunity of the secondary processing system. Analytical expressions for the probability of correct OB localization are obtained. Developed CI models, formed by sensors of combined CENS. The peculiarity of the models is taking into account the shape of the real ADD, the inertia of the sensors and the blurring of the image due to the FR movement during the frame exposure time. The models allow to calculate the output signals of the sensors of the combined CENS without the intrinsic noise of the channels.

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